Analysis of Influence of Non-Uniform Grids Method on the Self – Teleportation of Fields Method

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Abstract - **This work is based on the Self-Teleportation of Fields Method. This method consists on the cancellation of an incident wave by a negative copy of itself. However, this cancellation is not perfect when one applies the spatial and temporal discretization of the FDTD Method [1]. This error comes from phase difference between the incident and the copied waves. This work describes how to minimize this error by the use of the Non-Uniform Grids Method. From the achieved results, it is observed that the more refined the grid is, the more attenuated the incident wave will be, in despite of the increasing.**

Keywords – **FDTD method, absorbing boundaries, equivalent source, subgridding.**

I. Introduction

In the simulation of electromagnetic devices using the FDTD Method it is necessary to avoid reflected waves by the domain's boundaries. This is obtained by absorbing boundary condition (ABC) methods. The current ABC methods are: the Mur's method and the Perfectly Matched Layer (PML) method.

The ABC method developed by Mur [2] is based on radiation boundary conditions. This method provides a maximal attenuation of 40 dB and is easy to implement.

The ABC method developed by Berenger [3] and improved by Gedney [4] is based on anechoic chamber's materials. It is called Perfected Matched Layer (PML) and provides much more attenuation than Mur's method. However, it is hard to implement and demands a higher computational cost.

Over the years, improvements had been made in these methods without significantly changing their basis. However, more recently a new ABC method was developed by Diaz and Scherbatko [5, 6] and was called "Self-Teleportation of Fields". This method is a new version of radiation boundary condition like Mur's method and provides attenuation comparable with the same.

However in this method, many boundaries can be stacked, witch increases its attenuation capacity to that of PML method but with less complexity and computational cost.

This method consists in making a negative copy of the electric and magnetic fields at a position of FDTD grid at another position of same FDTD grid aiming at canceling the total field.

If these positions are coincident, it is possible to make a perfect cancellation by addition of the original fields to the copied fields with inverted signal. However, to avoid feedback instabilities, these positions cannot be coincident. For this position difference, the wave cancellation is not perfect, because there are amplitude and phase errors between the original and the copied fields.

The aim of this work is at applying the Non-Uniform Grids Method [7 - 9] to divide the FDTD domain in such a way to bring these positions as close as possible to minimize the errors. This paper also analyzes the influence of this subdivision on the wave cancellation.

II. Used methods

In the following there is a brief description of the two methods used in this work: the Self-Teleportation of Fields Method and the Non-Uniform Grid Method.

II.a. Self-Teleportation of Fields Method [4, 5]

This method is a discrete version of Schelkunoff's equivalence theorem. According to this theorem, the field outside a source region limited by an imaginary closed surface S can be obtained by introduction, over this surface, of electric current densities K_e and magnetic current densities K_m, with the elimination of the original source.

These current densities are chosen in such way that the fields inside S are zero and outside S are equal to the original ones. These currents are obtained by expressions (1) e (2).

$$\overrightarrow{K}_e = \hat{n} \times \overrightarrow{H}_{tot} \quad (1)$$

$$\overrightarrow{K}_m = -\hat{n} \times \overrightarrow{E}_{tot} \quad (2)$$

where $\hat{n}$ is the outward normal to S.

The implementation of this method using FDTD consists in creating an imaginary plane S that acts as a buffer for the current densities K_e and K_m and the creation of an imaginary plane S' that will dispose these mentioned current densities, as illustrated in Fig.1.

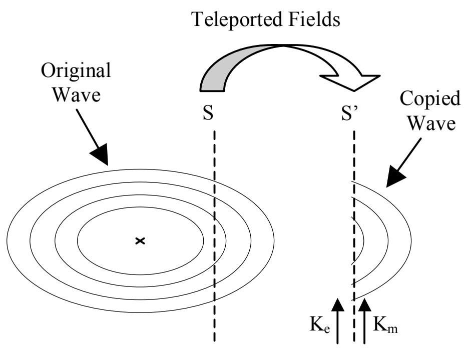

Fig. 1. Illustration of equivalence theorem in FDTD Method.

If the surfaces S and S' are spatial and temporal coincident, and if we subtracted the original field from the copied one, we will have a perfect cancellation of the incident wave on this surface. However, due to the feedback of the fields in S to generate the current densities K_e and K_m in S', the surfaces S and S' cannot be coincident in FDTD Method. Usually, S' is located one cell apart of S and its fields are delayed one time step in the simulations.

These currents densities can be introduced into FDTD Method by equations (3) and (4).

$$\mu \frac{\partial \overrightarrow{H}}{\partial t} = -\vec{\nabla} \times \vec{E} - \frac{\overrightarrow{K}_m}{ds} \quad (3)$$

$$\varepsilon \frac{\partial \overrightarrow{E}}{\partial t} = \vec{\nabla} \times \vec{H} - \frac{\overrightarrow{K}_e}{ds} \quad (4)$$

where ds is the area element.

II.b. Non-Uniform Grids Method [7 - 9]

For an accurate numerical computation, the grid discretization should have a maximum value. This value depends on the device's geometry to be analyzed. Therefore, the most refined the grid is, the most accurate the result will be. However, this refinement increases the necessary memory and increases the processing time.

To avoid this problem, and still obtain a good result, is to make this refinement only in some essential areas, such as discontinuities, edges and others. This technique reported in [10, 11], is called "Subgridding" Method.

However, if we want to increase the resolution only one direction, axis y for example, we need to apply the "Non-Uniform Grids" Method.

In FDTD, the Non-Uniform Grids equations are obtained by the Maxwell's equations in the integral form, as shown in (5) and (6).

$$\oint_{\Gamma} \overrightarrow{H} \cdot \overrightarrow{dl} = \frac{\partial}{\partial t} \left(\iint_{\Sigma} \varepsilon \overrightarrow{E} \cdot \overrightarrow{ds} \right) \quad (5)$$

$$\oint_{\Gamma} \overrightarrow{E} \cdot \overrightarrow{dl} = -\frac{\partial}{\partial t} \left(\iint_{\Sigma} \mu \overrightarrow{H} \cdot \overrightarrow{ds} \right) \quad (6)$$

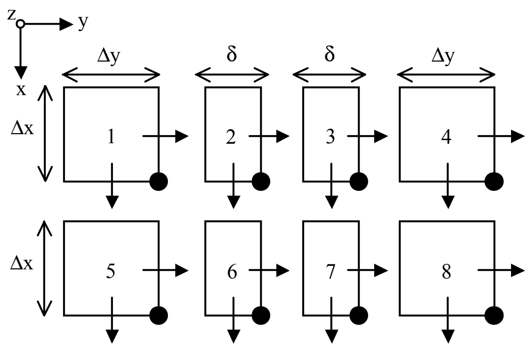

Fig. 2. Illustration of FDTD grid for TE mode.

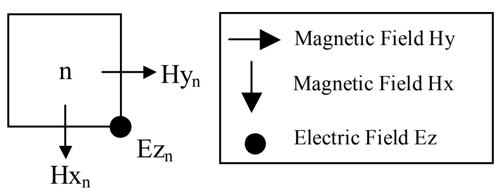

Fig. 3. Illustration of Yee's cell for TE mode.

The electric as well the magnetic fields can be obtained, in each case, by Faraday's Law or by Ampere's Law. Fig. 4 exemplifies how to calculate the E_z field by Ampere's Law.

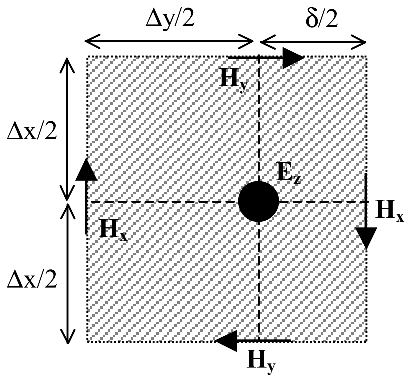

Fig. 4. Illustration of Ampere's law.

Using equation (5) in the cells of Fig. 2, we can calculate the electric fields in (7), (8) and (9) from cells 1, 2 and 3 respectively.

$$\varepsilon \cdot \frac{E_{Z1}^{n+1} - E_{Z1}^{n-1}}{\Delta t} = \frac{\left(H_{Y5}^{n+1/2} - H_{Y1}^{n+1/2}\right)}{\Delta x} + \frac{\left(H_{X1}^{n+1/2} - H_{X2}^{n+1/2}\right)}{\left(\frac{\Delta y + \delta}{2}\right)} \tag{7}$$

$$\varepsilon \cdot \frac{E_{Z2}^{n+1} - E_{Z1}^{n-1}}{\Delta t} = \frac{\left(H_{Y6}^{n+1/2} - H_{Y2}^{n+1/2}\right)}{\Delta x} + \frac{\left(H_{X2}^{n+1/2} - H_{X3}^{n+1/2}\right)}{\left(\frac{\delta + \delta}{2}\right)} \tag{8}$$

$$\varepsilon \cdot \frac{E_{Z3}^{n+1} - E_{Z3}^{n-1}}{\Delta t} = \frac{\left(H_{Y7}^{n+1/2} - H_{Y3}^{n+1/2}\right)}{\Delta x} + \frac{\left(H_{X3}^{n+1/2} - H_{X4}^{n+1/2}\right)}{\left(\frac{\Delta y + \delta}{2}\right)} \tag{9}$$

Now, using (6) in the cells of Fig.2, we can calculated the magnetic fields in (10), (11) and (12) from cells 2, 3 and 4 respectively.

$$-\mu \cdot \frac{H_{X2}^{n+1/2} - H_{X2}^{n-1/2}}{\Delta t} = \frac{\left(E_{Z2}^{n} - E_{Z1}^{n}\right)}{\delta} \tag{10}$$

$$-\mu \cdot \frac{H_{X3}^{n+1/2} - H_{X3}^{n-1/2}}{\Delta t} = \frac{\left(E_{Z3}^{n} - E_{Z2}^{n}\right)}{\delta} \tag{11}$$

$$-\mu \cdot \frac{H_{X4}^{n+1/2} - H_{X4}^{n-1/2}}{\Delta t} = \frac{\left(E_{Z4}^{n} - E_{Z3}^{n}\right)}{\Delta y} \tag{12}$$

To avoid divergence in FDTD Method, by the Courant condition, we have to chose the minimal size of the cells in the axis x and the minimal size in the axis y. With this, the Courant condition becomes (13).

$$c \cdot \Delta t \leq \left[\left(\frac{1}{\Delta x_{\min}}\right)^2 + \left(\frac{1}{\Delta y_{\min}}\right)^2\right]^{-1/2} \tag{13}$$

For $\Delta x = \Delta$ and $\Delta y = \Delta / m$, the increase in the processing will be

$$N'_{\max} = N_{\max} \cdot \sqrt{\frac{1 + m^2}{2}} \tag{14}$$

where $N_{\max}$ is the maximum computational time for $\Delta x = \Delta$ and $\Delta y = \Delta$.

III. Numerical Results

This section presents simulations' results showing the cancellation of an incident wave after crossing a teleportation boundary for four kinds of Non-Uniform Grids.

Fig.5 shows how a gaussian pulse snapshot located at $x = 100\Delta x$ and $y = 75\Delta y$ propagates to beyond the teleportation boundary. Located at $y = 85\Delta y$.

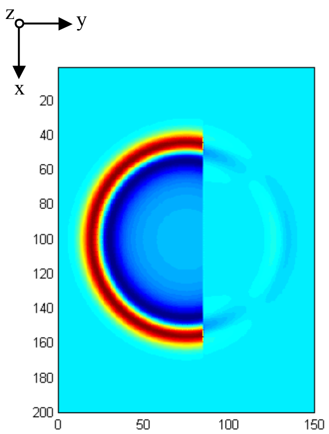

Fig. 5. Cylindrical wave snapshot.

The results were obtained using a domain of $\{200 \times 150\}$ Yee cells with $\Delta x = \Delta y = \Delta = 1mm$ of discretization.

Fig. 6 to Fig.9 show the magnitude of the E_z field after the ABC for four different wavelengths. The constant m in the Figures represents the reduction factor of one cell in the grid. The size of the non-uniform cells is $\delta = \Delta / m$. The term λ / Δ is the normalization of wavelength by spatial lattice discretization and θ ° is the wave incident angle in degrees.

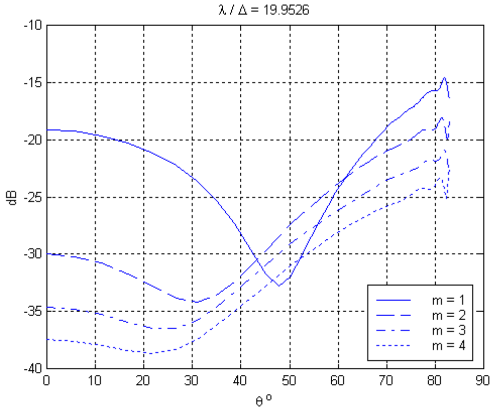

Fig. 6. Magnitude of the absorbing Ez field.

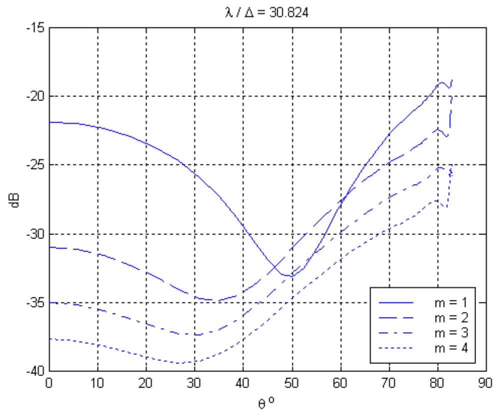

Fig. 7. Magnitude of the absorbing Ez field.

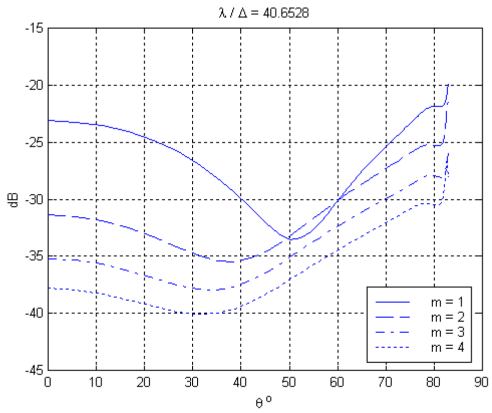

Fig. 8. Magnitude of the absorbing Ez field.

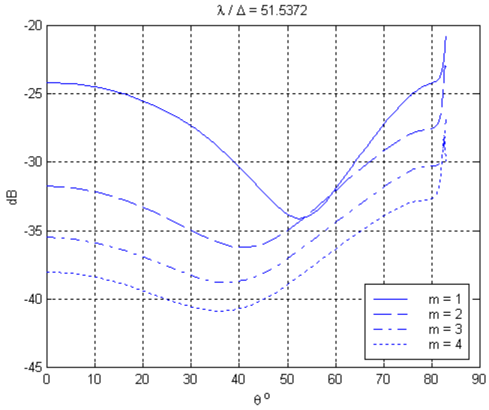

Fig. 9. Magnitude of the absorbing Ez field.

IV. Conclusion

This work presented a brief description of the Self-Teleportation of Fields Method and the Non-Uniform Grids Method, as well as its implementation using FDTD Method. By the results achieved, it is observed that the more refined the grid is, the more attenuation of incident wave will be, however this refinement increases the processing time as shown in (14).

References

[1] K. S. Yee, "Numerical solutions of initial boundary value problems involving Maxwell's equations in isotropic media", IEEE Trans. Antennas Propagat., vol. AP-14, pp. 302-307, May 1966.

[2] G. Mur. "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations." IEEE Trans. Electromagn. Compat., vol. EMC-22, pp. 377-382, Nov. 1981.

[3] J. P. Berenger. "A Perfectly Matched Layer for the Absorption of Electromagnetic Waves." J. Computational Physics, vol. 114, pp. 185-200, 1994.

[4] S. D. Gedney, "An Anisotropic Perfectly Matched Layer-Absorbing Medium for the Truncation of FDTD Lattices", IEEE Trans. Antennas and Propagat., vol. 44, no. 12, pp. 1630-1639, September 1996.

[5] R. Diaz, I. Scherbatko, "A New Radiation Boundary Condition for FDTD based on Self-Teleportation of fields", IEEE MTT-S Digest, 2003, pp. 2073-2076.

[6] R. E. Diaz, I. Scherbatko, "A New Multistack Radiation Boundary Condition for FDTD based on Self-Teleportation of fields", Journal of Computational Physics, 203 (2005) 176-190.

[7] A. Taflove, "Computational Electrodynamics: The Finite-Difference Time-Domain Method", Artech House inc.

[8] P. Monk, "Error estimates for Yee's method on non-uniform grids", IEEE Trans. Magnetics, vol. 30, 1994, pp. 3200-3203.

[9] J. N. Hwang, "A Compact 2-D FDTD Method for Modeling Microstrip Structures With Nonuniform Grids and Perfectly Matched Layer", IEEE Trans. Microwave Tech., vol. 53, no. 2, February 2005, pp. 653-659.

[10] S. S. Zivanovic, K. S. Yee, K. K. Mei, "A Subgridding Method for the Time-Domain Finite-Difference Method to Solve Maxwell's Equations", IEEE Trans. Microwave Tech., vol. 39, no. 3, March 1991, pp. 471-479.

[11] A. Vaccari, R. Pontalti, C. Malacarne, L. Cristoforetti, "A Robust and Efficient Subgridding Algorithm for Finite-Difference Time-Domain Simulations of Maxwell's Equations", Journal of Computational Physics, 194 (2004) 117-139.