BER Evaluation of OFDM Systems for Short Cyclic Prefix and Doppler Spread in Multipath Rayleigh Channels

Lucas Emerson dos Reis Garcia and Celso de Almeida

Abstract— **The performance in terms of bit error rate for OFDM systems for short cyclic prefix and Doppler spread is obtained. In order to evaluate the effects of the short cyclic prefix and Doppler spread on system performance, expressions for the bit error rate are obtained through mathematical fitting to the results obtained by simulation. We have considered several scenarios with different parameters as bandwidth, number of sub-carriers, channel dispersion time, duration of the cyclic prefix, mobile speed for BPSK modulation. In the scenarios examined in this paper, we have observed linear and nonlinear regions that depend on how much the cyclic prefix is shorter than channel dispersion. We have observed that Doppler spread decreases the performance. This paper is an important tool to evaluate the performance of OFDM systems with short cyclic prefix that can occur in big cells and Doppler spread.**

Keywords— **OFDM systems, Big Cells, Short Cyclic Prefix, BER Evaluation, Channel Dispersion, Linear and Non-linear Interference, Doppler Effects.**

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) technique uses cyclic prefix (CP) in order to eliminate the harmful effects of a multipath channel [1]. Using cyclic prefix duration (T_{cp}) less than the time dispersion of the channel (T_c), leads to inter-symbol interference (ISI) and interference between sub-carriers (ICI) [2] [3].

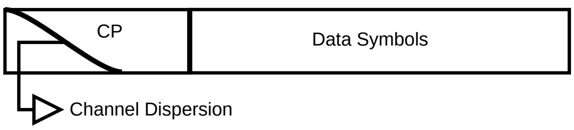

Fig. 1. Cyclic Prefix and Data Symbols in a Dispersion Channel.

However, in channels with fading and co-channel interference BER has floors, becoming practically immune to an increase in signal-to-noise ratio [4]. In the first part of the paper, we are interested to see what happen if the channel dispersion time is greater than CP in big cells. Thus, we propose expressions for the BER floors as a function of T_{cp}, T_c, bandwidth (B) and number of sub-carriers (L).

In the second part an expression for the BER when Doppler spreads is present in terms of the mobile speed (V_d) and the carrier frequency (f_c).

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This article is divided as follows. Section II describes digital implementation the OFDM systems. Section III presents BER expressions for short cyclic prefix. Section IV presents BER for Doppler spread and in section V the conclusions and final remarks are presented.

II. SYSTEM DESCRIPTION

A. Transmission and Reception

Consider the OFDM transmission and reception schemes shown in Fig. 2. Basically, it employs a IFFT block at the transmitter, and a FFT block at the receiver [5].

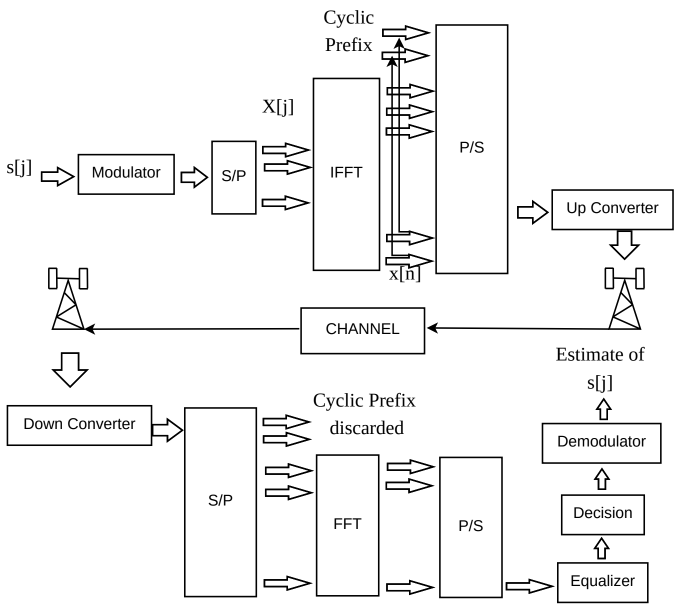

Fig. 2. Digital Implementation of an OFDM System.

A binary random sequence $s[j]$ is modulated by a binary phase shift keying (BPSK) modulator, where each symbol is allocated to a sub-carrier, producing the sequence of length L:

$$X[j] = [X_0, X_1 ... X_{L-1}]^T \tag{1}$$

in which $X_0, X_1 ... X_{L-1}$ are the BPSK symbols allocated to L sub-carriers.

In order to generate a sequence of OFDM symbols in time domain, an Inverse Fast Fourier Transform (IFFT) is applied to $X[j]$ [6]:

$$x[n] = \mathcal{IFFT}\{X[j]\} = [x_0, x_1 ... x_{L-1}]^T \tag{2}$$

in which $x[n]$ is a OFDM symbol in time domain and $x_0, x_1 ... x_{L-1}$ are the data at each sub-carrier.

A cyclic prefix of duration T_{cp} which corresponds to a length of ν samples is supposed [7]. If $T_{cp} < T_c$, the cyclic prefix is insufficient and there will be interference and error floor. After including the cyclic prefix, the resulting signal is given by:

$$x_t[n] = [x_{L-\nu}, x_{L-\nu+1} ... x_{L-1}, x_0, x_1 ... x_{L-1}]^T \quad (3)$$

in which $x_{cp}[n] = [x_{L-\nu}, x_{L-\nu+1} ... x_{L-1}]$ is the CP.

This is followed by a parallel/serial conversion and *up-conversion* operation for transmission. The OFDM signal $x[n]$, is transmitted by a channel with *Rayleigh* fading and additive white gaussian noise (AWGN). The received signal is given by:

$$y[n] = x_t[n] \circledast h[n] + w_N[n] \quad (4)$$

in which $h[n]$ is the impulse response of the *Rayleigh* channel, $w_N[n]$ is the noise and $\circledast$ is the circular convolution [8].

If the cyclic prefix is sufficient, in the frequency domain we have that [9]:

$$Y[j] = X[j]H[j] + W_N[j] \quad (5)$$

where $H[j]$ is the channel transfer function, that is the Fourier transform of $h[n]$. This is the proof that if the CP is sufficient there is no interference [10].

At the receiver the *down-conversion* is made and, then, the signal is split in L parts by passing through a serial/parallel converter. Afterwards, the cyclic prefix is discarded and the fast Fourier transform (FFT) is applied, and the resulting signal is given by [11]:

$$Y(j) = \mathcal{FFT}\{y[n]\} \quad (6)$$

The signal passes again through a parallel/serial converter and, shortly thereafter, an estimate of the transmitted symbol is done using the *Zero Forcing* (ZF) equalizer [12] [13], given by:

$$\hat{X}[j] = \frac{Y[n]}{H[n]} \quad (7)$$

Finally the decision is made on the received symbol.

B. Smith Channel Model

When a user moves, the power spectrum density spreads. Gans has determined the power spectrum density $S(f)$ as [11]:

$$S(f) = \frac{1.5}{\pi f_{d,m} \sqrt{1 - \left(\frac{f - f_c}{f_{d,m}}\right)^2}} \quad (8)$$

in which $f_{d,m}$ is the maximum frequency Doppler and f_c is the carrier frequency. The power spectrum density is shown in Fig. 3.

Smith has used the power spectrum density model developed by Gans for generating the signal envelope in a *Rayleigh* fading channel [11]. Smith's method consists in generating two sets of random samples, zero mean and unit variance and multiplying each set by the Gans power spectrum density. Fig. 4 shows the algorithm.

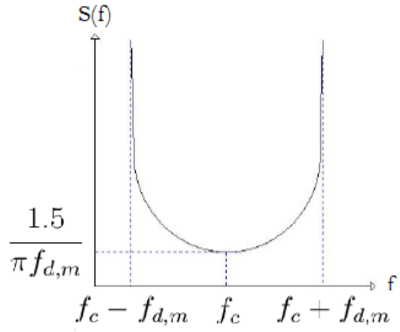

Fig. 3. Doppler Power Spectrum Density.

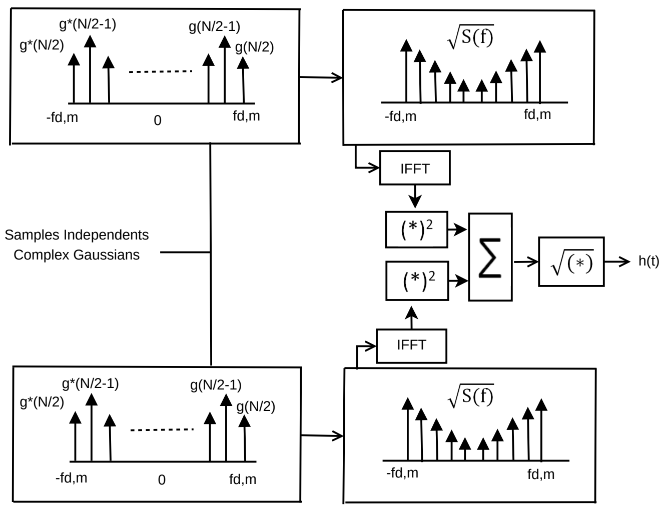

Fig. 4. Smith's Channel Model.

III. BER ANALYSIS FOR SHORT CP

The BER of OFDM systems using BPSK modulation in a Rayleigh channel, with sufficient CP is given by [14]:

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma_b}}{\overline{\gamma_b} - 1}}\right) \quad (9)$$

in which $\overline{\gamma_b} = \overline{\alpha^2} E_b/N_o$ (in dB) is the average SNR and α is the fading amplitude.

Through simulations using *Monte Carlo* method, it can be verified that BER varies with the CP duration, as is shown in Fig. 5.

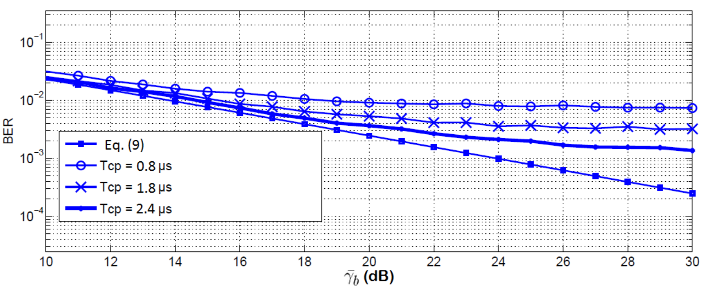

Fig. 5. BER versus $\overline{\gamma_b}$ for $B = 20$ MHz, $T_c = 3.3$ μs, $L = 1024$ sub-carriers, $T_{cp} = 0.8$ μs; 1.8 μs and 2.4 μs.

By varying T_{cp}, we have observed that the BER floor does present two behaviours: linear dependence when T_{cp} is very shorter than T_c and non-linear if T_{cp} is a bit shorter than T_c, as is show in Fig. 6.

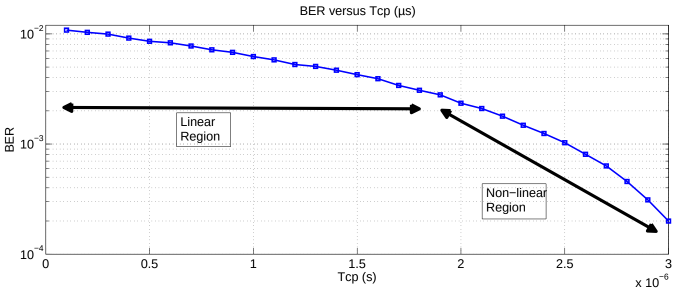

Fig. 6. BER floor versus T_{cp} for $B = 20\ MHz$, $T_c = 3.3\ \mu s$, $L = 1024$ sub-carriers, $\overline{\gamma_b} = 20\ dB$ and T_{cp} ranging from $0.1\ \mu s$ to $3.1\ \mu s$.

Through mathematical fitting, we have found equations for the BER floor in the linear and non-linear regions. This method consists in varying one system parameter at a time with all other fixed and to analysis the mathematical behavior this parameter presents in BER expression.

The BER floor at the linear region is given by:

$$P_{b,L} = k_L \frac{B}{L}(T_c - T_{cp}) \tag{10}$$

that is, BER increases with B and $T_c - T_{cp}$, but decreases with the number of sub-carriers, in which $k_L = 0.2$, B is given in MHz and T_c and T_{cp} are in μs.

Similarly, for the non-linear region the BER floor is given by:

$$P_{b,NL} = k_{NL}\left(\frac{T_c}{T_{cp}}\right)^2 \frac{B}{L}(T_c - T_{cp}) \tag{11}$$

that is, BER increases with B, $T_c - T_{cp}$ and $(\frac{T_c}{T_{cp}})^2$, but decreases with L, in which $k_{NL} = 0.09$.

The overall system BER is composed by the BER without interference given by (9) and the BER floor given by (10) or (11):

$$P_{b(T,L)} = P_b + P_{b,L} \tag{12}$$

$$P_{b(T,NL)} = P_b + P_{b,NL} \tag{13}$$

Fig. 7 shows the BER floor as a function of T_{cp} using simulation and the theoretical equation (10) or (11). In this figure it can be seen a good agreement between both curves that is validates the equations found.

Fig. 8 and 9 shows the total BER as a function of $\overline{\gamma_b}$ using simulation and the theoretical equation found in 12 or 13 for the linear and non-linear regions, respectively. The similarity between both curves validates the equations found for linear and non-linear regions.

IV. Doppler BER Analysis

Fig. 10 shows the Doppler effects in OFDM systems as additional Gaussian noise. As the speed increases it can be seen that the BER increases proportionally [15].

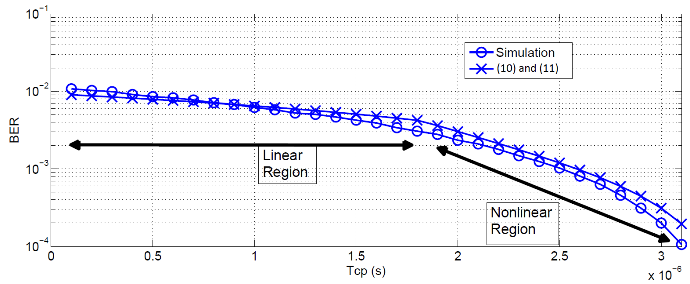

Fig. 7. BER Floor as a Function of T_{cp} for linear and non-linear regions. $L = 1024$ sub-carriers, $\overline{\gamma_b} = 20\ dB$, $B = 20\ MHz$, $T_c = 3.3\ \mu s$ and T_{cp} ranging from $0.1\ \mu s$ to $3.1\ \mu s$.

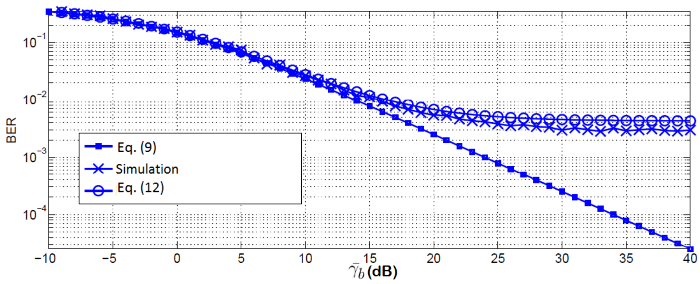

Fig. 8. Total BER versus $\overline{\gamma_b}$ for linear region, for $L = 512$ sub-carriers, $B = 10\ MHz$, $T_c = 3.3\ \mu s$ and $T_{cp} = 1.8\ \mu s$.

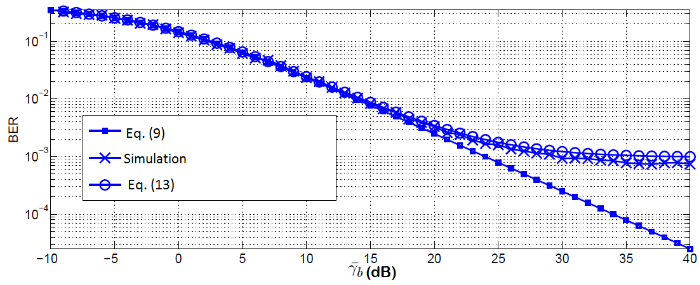

Fig. 9. Total BER versus $\overline{\gamma_b}$ with non-linear BER floor. $L = 1024$ sub-carriers, $B = 20\ MHz$, $T_c = 3.3\ \mu s$ and $T_{cp} = 2.6\ \mu s$.

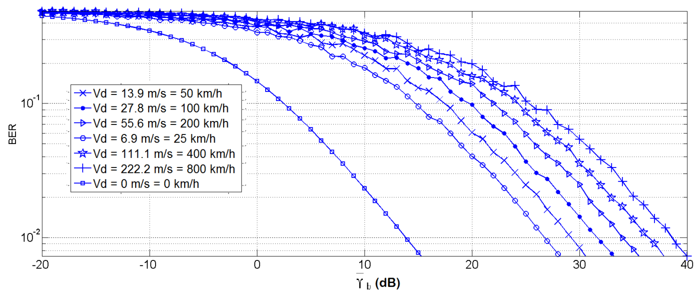

Fig. 10. BER versus $\overline{\gamma_b}$ for $B = 20\ MHz$, $T_{pc} = T_c$, $L = 1024$ subcarriers, $f_c = 2\ GHz$ and different speed (V).

The reason is that OFDM systems looses their orthogonality as the inter-carrier bandwidth becomes less than the maximum Doppler frequency ($f_{d,m}$).

Using mathematical fitting, we have that.

$$P_{b,DOP} = k_D \frac{T_c - T_{cp}}{T_c} V_D^{0.693\left(\frac{0.04T_{cp}}{BT_c^2}+1\right)} \tag{14}$$

in which $k_D = 0.005$, B is given in MHz, T_c and T_{cp} are in μs and V_D is the speed in m/s. Fig. 11 shows the BER as a function of $\overline{\gamma_b}$ using simulation and the theoretical equation found in (14). In this figure it can be seen a good agreement between both curves that validates (14).

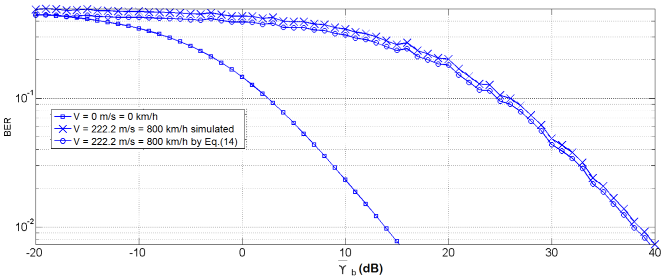

Fig. 11. BER versus $\overline{\gamma_b}$ with $B = 10\ MHz$, $T_{pc} = T_c$, $L = 512$ subcarriers, $f_c = 2.5\ GHz$ and $V_d = 222.2 m/s = 800\ km/h$.

V. Conclusions

Modeling the interference in OFDM systems when cyclic prefix is short or when Doppler spread is considered is not an easy task. For this reason, we have obtained expressions using mathematical fitting to the results obtained by simulation. In this paper theoretical expressions for the BER were presented for OFDM systems with short cyclic prefix and when the Doppler spread is considered for BPSK modulation in a multipath Rayleigh fading channel. For the scenarios examined in this work, it was demonstrated that despite the linear and non-linear regions, both expressions for the BER floor are very tight, the same can be said about the BER for Doppler spread. With these expressions it is possible to determine the BER for an OFDM system when the cyclic prefix is short and when Doppler spread is present.

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