

# Degree of Polarization (DOP) Loss in the Statistics of Polarization Dependent Loss (PDL) in Recirculating Loop

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**Resumo** — Usamos o formalismo matricial de Mueller num modelo que leva em conta a perda do grau de polarização (DOP) na estatística da Perda Dependente da Polarização (PDL), válida para anéis de recirculação. Demonstramos que o grau de despolarização introduzido a cada volta da luz dentro do anel limita o valor máximo da PDL. Apresentamos evidências experimentais deste efeito, obtidas através de um Polarímetro Rápido.

**Palavras-Chave**—Anel de recirculação; perda dependente da polarização; grau de polarização.

**Abstract** — We use the Mueller matrix formalism to model the loss of Degree of Polarization (DOP) in the statistics of Polarization-Dependent Loss (PDL) in recirculating loops. We demonstrate that the degree of depolarization introduced at each round trip limits the maximum value of PDL. Evidence of this effect is reported showing preliminary results, obtained by a fast polarimeter.

**Keywords**— Optical transmission system, recirculating loop, polarization dependent loss, degree of polarization.

## I. INTRODUCTION

Recirculating loops may predict the accumulation and mutual interaction of degrading phenomena such as dispersion and nonlinear effects in optical transmission systems [1]-[3]. They provide useful information for designing equivalent straight-line systems with comparable parameters [4], although their statistics accounting for polarization related phenomena, such as polarization mode dispersion (PMD), polarization dependent loss (PDL), and polarization dependent gain (PDG), do not reproduce the straight-line statistics for the same parameters. That is due to differences between the periodical nature of round-trips and the in-line randomised distribution. The misalignment of polarization sensitive axes of optical components along the optical path is of particular concern for evaluating the global PDL effect because the total PDL of concatenated elements is not the single sum of PDL values, and depends on the relative orientation of each PDL axis along the link. Furthermore, as each PDL contribution may be subjected to fluctuations of environmental conditions, the statistical distribution of PDL in a recirculating loop requires a mechanism that will turn it equivalent to the distribution in

randomised straight-line systems, for which the expectation value of PDL grows as the square-root of the link length [5] - [10]. This is in contrast to the accumulated average PDL, which increases linearly with the number of recirculations through the loop.

For no negligible polarization phenomena, a loop-synchronous polarization scrambling provides a good approximation of the polarization effects distribution [11]-[15]. Asynchronous scheme may result in a less accurate measurement of bit error ratios (BER) [17], though it is sufficiently precise for simulating polarization effects in a CW recirculating loop. In a recent work, we reported the use of the Mueller method to measure the PDL in the loop, which incorporates a loop-asynchronous scrambling of polarization, for different round trips [18]. In our previous study, we observed a variation on the degree of polarization as a function of transmission length, probably induced by the accumulated amplified spontaneous emission within the loop. This result has led us to model the degree of depolarization induced at each round-trip, according to the Mueller matrix formalism, and to validate the results with experimental data.

In this paper, we highlight the main results of our previous investigation, by briefly describing such model, and demonstrating that depending on the degree of depolarization introduced at each round trip, the maximum value of PDL is limited to lower values when compared with completely polarized light. We then present some preliminary results of a further investigation, which uses the Jones matrix to estimate the loss of Degree of Polarization (DOP) per round trip, by means of a High Speed Polarization Analyzer (Polarimeter). The paper is organized as follows: in Sections 2 we describe our numerical analysis. In Section 3 we present the experimental results. Finally, Section 4 concludes our work.

## II. PDL AND DOP LOSS MODEL

Polarization dependent loss is the dependence of insertion loss on the state of polarization (SOP) of the input light, and corresponds to the maximum change in transmission of an optical component versus all possible input polarization states, defined as:

$$PDL_{dB} = 10 \log \left( \frac{P_{max}}{P_{min}} \right) \quad (1)$$

where  $P_{max}$  and  $P_{min}$  are the maximum and minimum power transmitted by the device under test (DUT), respectively, in response to the input SOP variation, as illustrated in Fig. 1,

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where the output power variation is the result of the variation on the SOP of the signal which is incident on the DUT.



Fig. 1. PDL measurement.

The two most important PDL-measurement techniques are the polarization scanning and the Mueller method. Some authors include a third one, the Jones calculus [20] but, essentially, Mueller- and Jones- are two different representations of the same system. In common, all have the objective of determining the values of  $P_{max}$  and  $P_{min}$ . The two matrix methods reduce the problem of finding those values to a problem of finding the function extremes and both rely upon the Stokes vector.

When using the Jones Matrix, the values are obtained from the DUT transfer matrix and their singular values [19] considering that only three independent polarizations have been launched into the DUT. The method requires the use of a polarimeter, but, in general, that reduces the measurement duration in just  $\sim 1$  s.

The Mueller matrix formalism exempts the use of polarimeter, and just one optical power meter is required. In this technique, four polarizations are needed and, consequently, four measurements must be performed without the DUT, for reference, and other four with the DUT. That procedure takes  $\sim 1$  s.

Stokes vector  $\vec{S} = (S_0, S_1, S_2, S_3)$  completely describes the power and SOP of an optical signal. In this expression,  $S_0$  is the total intensity,  $S_1$  describes the degree of linear polarization in the horizontal or vertical direction (horizontal, if  $S_1 > 0$ , and vertical, if  $S_1 < 0$ ),  $S_2$  is the degree of linear polarization at  $45^\circ$  ( $+45^\circ$ , if  $S_2 > 0$ , and  $-45^\circ$ , if  $S_2 < 0$ ). Finally,  $S_3$  is the degree of circular polarization (right, if  $S_3 > 0$ , and left, if  $S_3 < 0$ ).

In the case of completely polarized light, the relation given by Eq. (2) holds:

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad (2)$$

Another important parameter, the degree of polarization (DOP) of light, is defined by:

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (3)$$

Incident light has a Stokes vector  $\vec{S}_{in}$  and its interaction with the DUT is given by a  $4 \times 4$  real matrix known as Mueller matrix. In that case, the output Stokes vector is given by:

$$\vec{S}_{out} = M \vec{S}_{in} \quad (4)$$

For PDL analysis, only the output intensity is of interest. We can expand the matrix product of Eq. (4) to obtain the  $S_{0,out}$  element, given by:

$$S_{0,out} = m_{11} S_{0,in} + m_{12} S_{1,in} + m_{13} S_{2,in} + m_{14} S_{3,in} \quad (5)$$

To obtain the lightwave intensity at the output device, it is necessary to know the coefficients  $m_{11}$ ,  $m_{12}$ ,  $m_{13}$ , and  $m_{14}$  of the Mueller matrix,  $M$ . It is possible to show [21] that these elements may be obtained by applying four well-defined polarizations at the DUT input, as indicated in Fig. 2. In this figure,  $P_a$ ,  $P_b$ ,  $P_c$ , and  $P_d$  are the input powers and  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  the corresponding output optical powers. All these powers can be obtained from a power meter, in the two-step procedure illustrated.

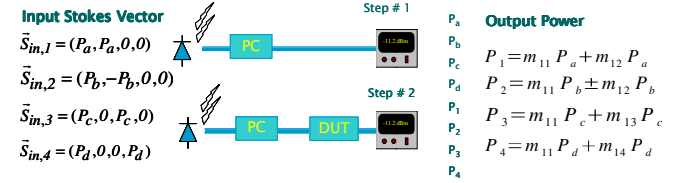


Fig. 2. Mueller Method for PDL measurement (PC = Polarization Controller).

In general, the polarization controller of Fig.2 consists of a three-plate device that can be rotated about a central axis (one polarizer plate (P), one quarter-wave plate (Q) and one is a half-wave plate (H)). The relative rotation of these plates generates all polarization states over the Poincaré sphere. In the case of interest the polarizer P is fixed at an angle ( $\alpha$ ), which is just an offset angle that maximizes the output power from the PC.

The  $M$  matrix elements result in:

$$\begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \end{pmatrix} = \begin{bmatrix} \frac{1}{2} \left( \frac{P_1}{P_a} + \frac{P_2}{P_b} \right) \\ \frac{1}{2} \left( \frac{P_1}{P_a} - \frac{P_2}{P_b} \right) \\ \frac{P_3}{P_c} - m_{11} \\ \frac{P_4}{P_d} - m_{11} \end{bmatrix} \quad (6)$$

The transmitted power  $T$  is given by:

$$T = \frac{S_{0,out}}{S_{0,in}} = \frac{m_{11} S_{0,in} + m_{12} S_{1,in} + m_{13} S_{2,in} + m_{14} S_{3,in}}{S_{0,in}} \quad (7)$$

It is possible to show [20] that the maximum and minimum transmittance are given by:

$$T_{max} = m_{11} + \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \quad (8)$$

$$T_{min} = m_{11} - \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}$$

Finally, the PDL, expressed in dB, is calculated from:

$$PDL_{dB} = 10 \log \left( \frac{T_{max}}{T_{min}} \right) \quad (9)$$

The simulation of elements in a loop can be reduced to the

case of two basic elements in cascade: one is the polarization controller, represented by the  $M_{pc}$  matrix, the other is a generic element, corresponding to the PDL of one round trip in the loop, given by the  $M_{pdl}$  matrix. In the Jones matrix formalism these matrices are given by:

$$M_{pc} = \begin{pmatrix} \cos \theta e^{i\phi} & -\sin \theta e^{i\phi} \\ \sin \theta e^{-i\phi} & \cos \theta e^{-i\phi} \end{pmatrix}, \quad M_{pdl} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{s} \end{pmatrix} \quad (10)$$

As explained by C. Vinegoni et al. [5], the loop transmission,  $T_{loop}$ , is given by:

$$T_{loop} = (M_{pdl} \cdot M_{pc})^N \quad (11)$$

where  $N$  is the number of round trips (or number of cascaded elements). The  $M_{pdl}$  matrix gives the PDL of one round trip (one element) related to an  $s$  parameter, such that  $PDL_i(dB) = 10 \log(1/s)$ .

The PDL of  $N$  round trips is obtained from [19]:

$$PDL_N = 10 \log \left( \frac{\xi_2}{\xi_1} \right) \quad (12)$$

where  $\xi_i$  are the eigenvalues of the  $T_{loop}^\dagger T_{loop}$  matrix. To get a good statistics, we generated  $10^5$  SOPs choosing  $\theta$  to be in the interval  $[0, \pi/2]$  and  $\phi$ , in the interval  $[0, 2\pi]$ . To guarantee that the generated SOPs are uniformly distributed over the Poincaré sphere, the angles  $\theta$  and  $\phi$  must follow the density distribution functions given by:

$$\begin{aligned} p(\theta) &= \sin(2\theta), & \theta &\in \left[0, \frac{\pi}{2}\right] \\ p(\phi) &= \frac{1}{2\pi}, & \phi &\in [0, 2\pi] \end{aligned} \quad (13)$$

For each generated SOP, we calculate  $N \times$  PDLs, corresponding to the PDLs of each round trip. With these data, histograms are created as follows: (i) the sequences of  $10^5$  PDLs for each round trip are normalized by the mean value of the first round trip PDL ( $\langle PDL_i \rangle$ ); (ii) these new sequences are divided in intervals (bins) and the number (frequency) of PDLs within each interval is calculated; (iii) histograms are plotted (frequency  $\times$  (PDL/ $\langle PDL_i \rangle$ )).

As a reference, we reproduced the histograms obtained through the numeric simulation in Ref. [19]. To include a possible depolarization of the signal per round trip, Eq. 10 must be rewritten in the Mueller matrix formalism (the Jones formalism would be best suited only for completely polarized light [19]). The  $M_{pc}$  and  $M_{pdl}$  matrices are then given by:

$$M_{pc} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \cos \phi \sin 2\theta & \cos \phi \cos 2\theta & \sin \phi \\ 0 & -\sin \phi \sin 2\theta & -\sin \phi \cos 2\theta & \cos \phi \end{pmatrix}, \quad (14)$$

$$M_{pdl} = \begin{pmatrix} \frac{1+s}{2} & \frac{1-s}{2} & 0 & 0 \\ \frac{1-s}{2} & \frac{1+s}{2} & 0 & 0 \\ 0 & 0 & \sqrt{s} & 0 \\ 0 & 0 & 0 & \sqrt{s} \end{pmatrix}$$

The Mueller matrix  $M_{pdl}$  represents the PDL of an equivalent-device in the transmission system, as defined by Y. Fukada [22]. Based on Fukada's formalism and taking into account the degree of polarization (DOP), we have proposed a depolarization matrix,  $M_{dep}$ , given by:

$$M_{dep} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & DOP & 0 & 0 \\ 0 & 0 & DOP & 0 \\ 0 & 0 & 0 & DOP \end{pmatrix} \quad (15)$$

The transfer matrix for  $N$  round trips is thus given by:

$$T_{loop} = (M_{pdl} \cdot M_{dep} \cdot M_{pc})^N \quad (16)$$

The PDL for  $N$  round trips is obtained from (8) and (9).

### III. EXPERIMENTAL RESULTS

Fig. 3 shows the recirculating loop assembled for the PDL measurements: a CW optical signal is generated, at 1545 nm, by a tuneable external cavity laser and boosted by an erbium-doped fiber amplifier (EDFA). PQH is the bulk polarization controller used to obtain the four SOPs of Fig. 2. The loop, controlled by acoustic optical switches - transmission switch (TS) and loop switch (LS), comprises a 25 km spool of dispersion-shifted fibre (zero dispersion at 1545 nm) and one in-line EDFA. The polarization controller within the loop (PC), the asynchronous scrambler, is used to generate randomised polarizations to span the Poincaré sphere. Two passband filters are inserted in the loop to remove the amplified spontaneous emission (ASE) mainly introduced by the in-line EDFA. The attenuator (ATT) equalizes the power of all round trips. The detection system consists of a photodetector coupled to a real time Scope or to a Fast Polarimeter, which measure the optical power that gets out of the loop through port 4 of the  $2 \times 2$  coupler (ACC) at each round trip, triggered by the loop switch. This system measures the power for each round trip in a very fast way ( $\sim 0.3$  s per round).

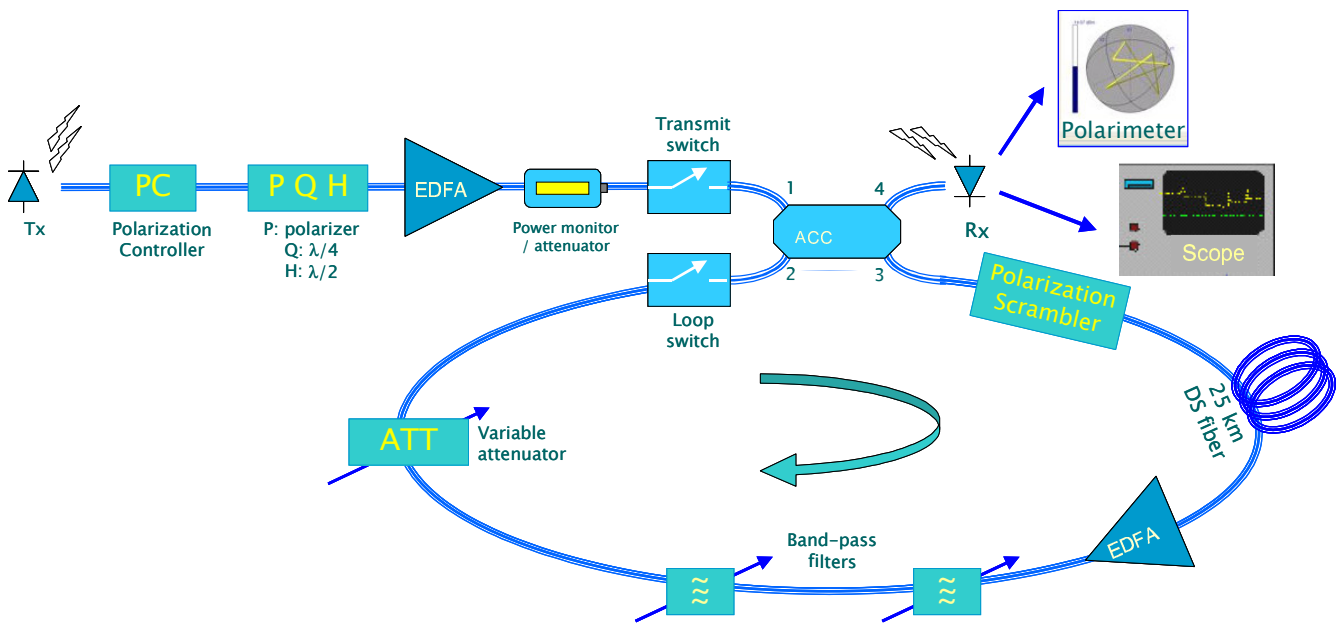


Fig. 3. Experimental setup.

We applied the Mueller matrix by measuring the optical powers as defined in Fig.2 for 10 round trips. The experimental histogram for the first round trip ( $\text{Frequency} \times PDL_i / \langle PDL_i \rangle$ ) corresponds to a Gaussian distribution due to measurement uncertainty. To properly include this experimental fluctuation in the numerical model, it has been convolved with a Gaussian distribution (mean value equal to zero and standard deviation of 0.3) as indicated in Fig. 4.

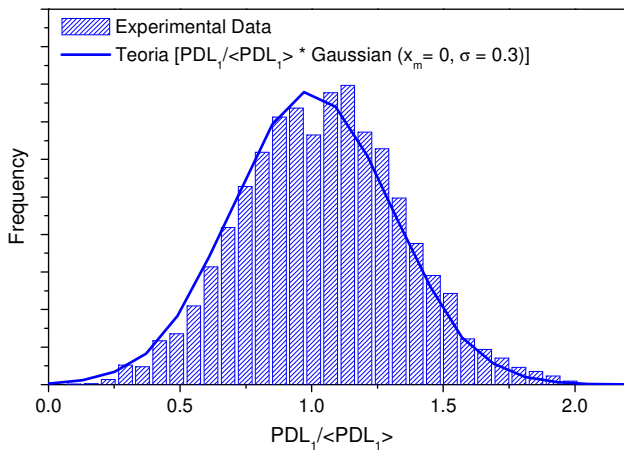


Fig. 4. Experimental distribution of PDL for the first round trip (normalized by the mean value  $\langle PDL_i \rangle$ ) and the numerical result convolved with a Gaussian, to simulate experimental uncertainty.

For the next round trips ( $N \geq 2$ ), the numeric results are convolved with the Gaussian seen in Fig. 4, and the results are shown in Fig. 5.

A final comparison is presented in Fig. 6. Considering the experimental data reported by C. Vinegoni et al. [5] as a reference of scrambling without lost of DOP, we notice a good

agreement for up to five round trips. From that point on, we believe that the difference is due to the depolarization introduced by accumulated ASE, which is, intrinsically a depolarized light. That difference has lead us to the include a degree of depolarization per round trip, which in this case has been estimated in 7 %, i.e.  $DOP \sim 0,93$ .

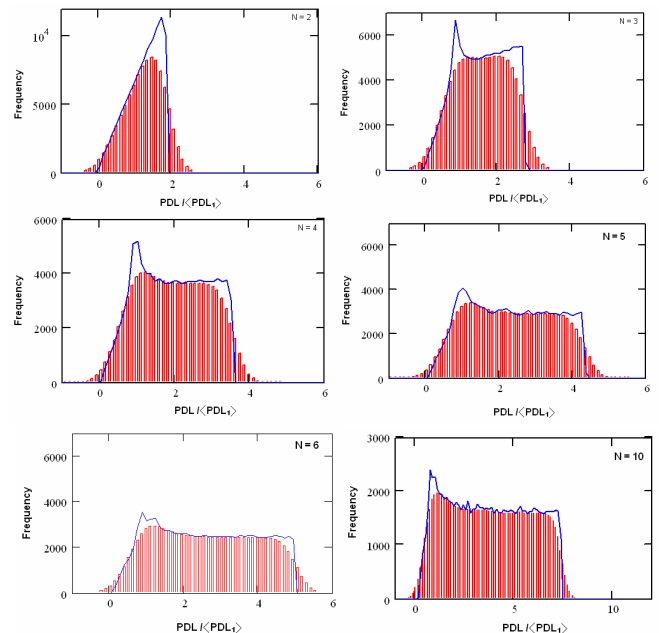


Fig. 5. Numerical results and the convolution between the numerical results and the Gaussian seen in Fig. 7.

Fig. 6 superimposes the experimental and the numerical results (convolved with the Gaussian of Fig. 4). We note that the maximum accumulated PDL is equivalent to about 7.5 times the mean PDL of one round trip ( $\langle PDL_i \rangle$ ). We also relate the experimental and simulated data with the mean PDL of the N-round-trips (normalized to  $\langle PDL_i \rangle$ ) in Fig. 7. In this figure,

the upper trace is the theoretical expected result for the recirculating loop system (mean accumulated PDL grows linearly with  $N$ ) and the lower trace is the expected result for straight line system (growth as  $\sqrt{N}$ ). Our experimental results and the comparison with the model that includes a partial depolarization of light per round-trip are placed in between these two traces.

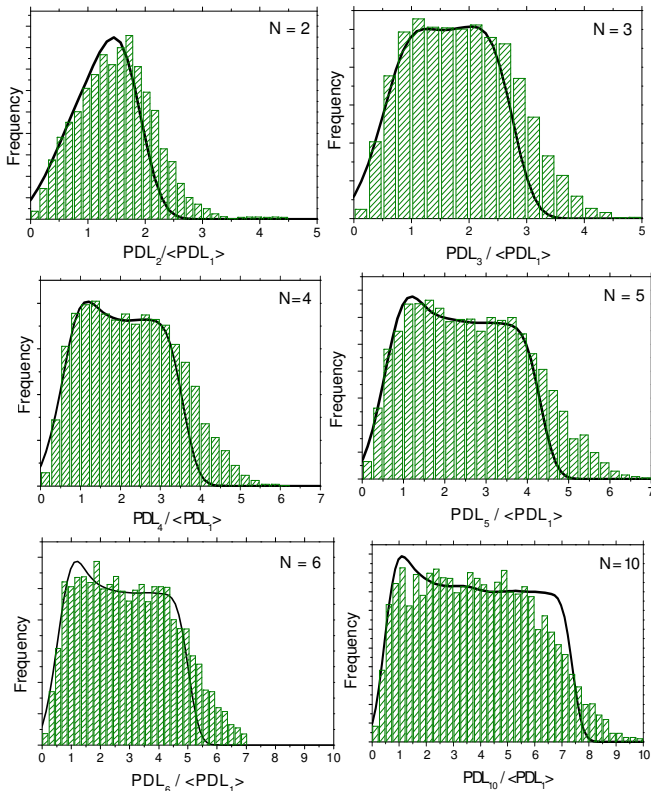


Fig. 6. Comparison between experimental data (bars) and simulation with convolution (solid lines). Simulations performed with 7% depolarization per round-trip assumption.

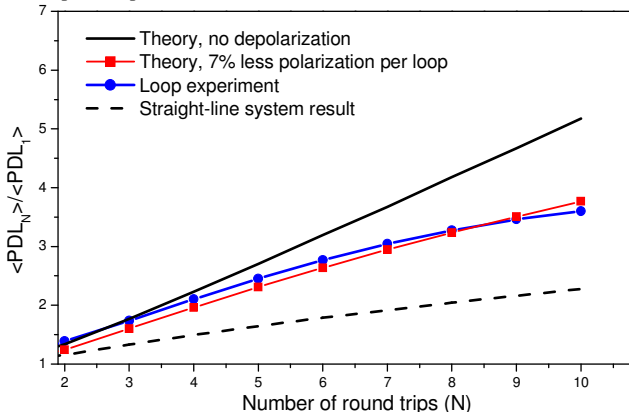


Fig.7. Mean accumulated PDL per round-trip behaviour.

Such small depolarization degree loss per round-trips is due to the double-filter ASE filtering. The accumulated PDL becomes smaller when just one filter is used, tending to saturate in a threshold of  $PDL / \langle PDL_1 \rangle = 4$ , though 10 round-trips have been analysed. In that case, the estimated depolarization per round-trip is 35%. The corresponding histograms are shown in Fig. 8 ( $N = 1$ ) and Fig. 9 ( $N \geq 2$ ).

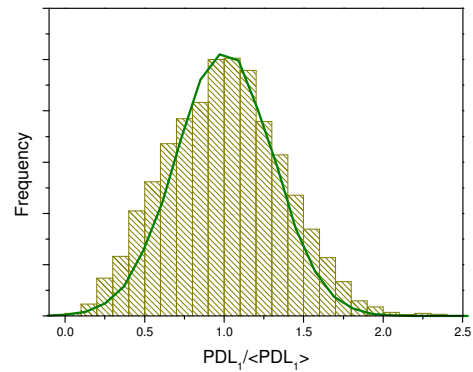


Fig. 8. First round-trip experimental distribution of PDL, normalized by its mean value ( $\langle PDL_1 \rangle$ ) (vertical bars), and numerical value convolved with a Gaussian, to simulate experimental errors in the measurement procedure (solid line).

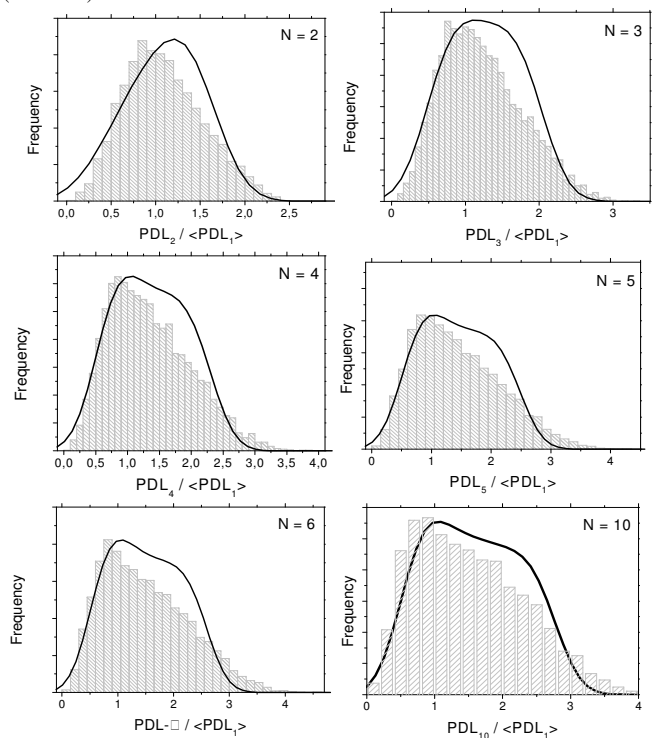


Fig.9. Experimental histograms (bars) and simulation (solid line) in the case that accumulated PDL is limited, probably due to depolarization introduced by the ASE of the in-line EDFA.

Finally, to evaluate the DOP loss in the loop, we are currently investigating the PDL statistics by measuring the real time SOPs with a High Speed Polarimeter, triggered by the loop switches. One should note that the operation of the Polarization Analyzer should be synchronized to the round trip duration. Fig. 10 shows some snapshots of different scrambling sets for the PC in the loop and the DOP evolution per round trip. It clearly represents another evidence of the mean DOP decrease with distance, in the loop. We have also noticed that, in some cases, the DOP is kept almost constant for all round trips but in other cases it can be reduced to less than 20% in the last round trip (10th). Further investigation to be carried out consists on doing the statistics of those DOP data, which will allow us to study the behavior of its mean value per each round trip to confirm such hypothesis.



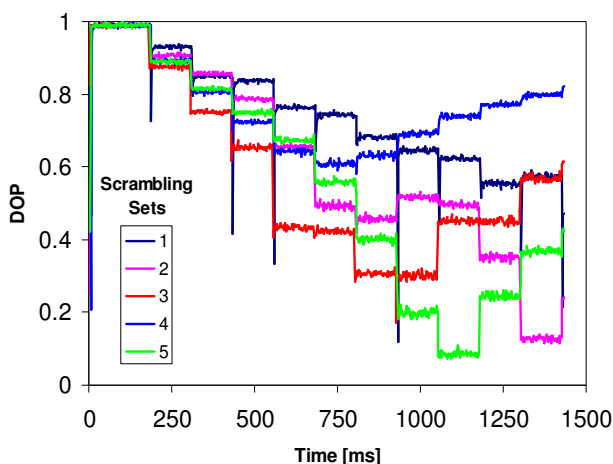


Fig. 10. Scrambling sets for the PC in the loop and evolution of DOP per round trip. The statistics of those sets are expected to indicate a DOP decrease after many roundtrips around the loop.

#### IV. CONCLUSION

We described the statistical distribution of the PDL in a recirculating loop, investigated in configurations with good and poor ASE filtering within the loop. We proposed to include a degree of polarization (DOP) matrix term in the transfer matrix of the loop, for taking into account any depolarizing source. We have derived a numerical model based on Mueller matrix formalism that demonstrates that, depending on the degree of depolarization introduced at each round trip, the maximum value of PDL becomes saturated in lower values than that obtained from recirculations of completely polarized light. Experimental evidence of this effect has been reported with good agreement with numerical prediction.

We presented preliminary measurements of the signal DOP loss per round trip, obtained by means of a fast polarimeter triggered to the loop. Numerical treatment of those experimental data will be presented at the Conference.

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#### REFERENCES

- [1] N. Bergano, "Undersea fiberoptic cable systems: high-tech telecommunications tempered by a century of ocean cable experience", in *Proceedings of Optical Amplifiers and their Applications, OAA'2002*, paper OMA1.
- [2] S. Baruh *et al.*, "Experimental Demonstration and Numerical Simulation of an Optical Recirculating Loop Operating at 10Gb/s", in *Proceedings of SBMO-IEEE-MTT International Microwave and Optoelectronic Conference, IMOC, 2003*, September, Foz do Iguaçu, Brazil.
- [3] I. Tomkos *et al.*, "Filter concatenation in metropolitan optical networks utilizing directly modulated lasers", *IEEE Photonics Technology Letters*, v. 13, pp. 1023 – 1025, 2001

- [4] M. Montoya *et al.*, "Automatically Controlled Recirculating Fiber Loop Operating at 10 Gb/s", in *Proceedings of MOMAG 2004 - 11o Simpósio Brasileiro de Microondas e Optoeletrônica*, paper199, 2004.
- [5] C. Vinegoni *et al.*, "The Statistics of Polarization-Dependent Loss in a Recirculating Loop", *Journal of Lightwave Technology*, v. 22, n. 4, pp. 968 – 976, 2004.
- [6] A. Mecozzi and M. Shtaif, "The statistics of polarization-dependent loss in optical communication systems", *IEEE Photonics Technology Letters*, v. 14, pp. 313 –315, 2002.
- [7] B. Marks *et al.*, "Polarization-state evolution in recirculating loops with polarization-dependent loss", *Optics Letters*, v. 27, n. 21, pp. 1881 – 1883, 2002.
- [8] Y. Yu Sun *et al.*, "Study of system performance in a 107-km dispersion managed recirculating loop due to polarization defects", *IEEE Photonics Technology Letters*, v. 13, pp. 966 –968, 2001.
- [9] P. Lu *et al.*, "System outage probability due to the combination effect of PMD and PDL", *J. Of Lightwave Technology*, v. 20, n. 10, pp. 1805 – 1808, 2002.
- [10] B. Httner *et al.*, "Polarization-induced distortions in optical fiber networks with PMD and PDL", *IEEE J. of Selected Topics in Quantum Electron.*, v. 6, n. 2, pp. 317 – 329, 2000.
- [11] H. Xu *et al.*, "Measurement of distributions of differential group delay in a recirculating loop with and without loop-synchronous scrambling", *IEEE Photonics Technology Letters*, v. 16, n. 7, pp. 1691 – 1693, 2004.
- [12] Y. Sun *et al.*, "Statistics of the system performance in a scrambled recirculating loop with PDL and PDG", *IEEE Photonics Technology Letters*, v. 15, n. 8, pp. 1067 – 1069, 2003.
- [13] Q. Yu *et al.*, "Loop-synchronous polarization scrambling technique for simulating polarization effects using recirculating fiber loops", *J. of Lightwave Technology*, v. 21, n. 7, pp. 1593 – 1600, 2003.
- [14] L. -S. Yan *et al.*, "Deleterious system effects due to low-frequency polarization scrambling in the presence of nonnegligible polarization-dependent loss", *IEEE Photonics Technology Letters*, v. 15, n. 3, pp. 464 – 466, 2003.
- [15] S. Lee *et al.*, "A short recirculating fiber loop testbed with accurate reproduction of Maxwellian PMD statistics", in *Proc. Optical Fiber Conference OFC 2001* paper WT2.
- [16] B. Bakhshi *et al.*, "1 Tbit/s (101 x 10 Gbit/s) transmission over transpacific distance using 28 nm -band EDFAs", in *Proceedings of Optical Fiber Conference, OFC 2001*, Postdeadline paper PD21.
- [17] J.-X. Cai *et al.*, "@.4 Tb/s (120 x 20 Gb/s) transmission over transoceanic distance using optimum FEC overhead and 48 % spectral efficiency", in *Proceedings of Optical Fiber Conference, OFC 2001*, Postdeadline paper PD20.
- [18] C. Florida *et al.*, "Inclusion of depolarization effects in polarization-dependent loss statistics of a recirculating loop", in *Proceedings of SPIE International Symposium, OPTO Ireland 2005*.
- [19] C. Hentschel and D. Derickson, *Fiber Optic Test and Measurement*, Prentice Hall, 1998.
- [20] B. Heffner, "Deterministic, analytically complete measurement of polarization-dependent transmission through optical devices", *IEEE Photonic Technology Letters*, v. 4, n. 5, pp. 451 – 454, 1992.
- [21] C. Hentsche and S. Schmidt, "PDLmeasurements using the Agilent 8169A Polarization controller", *Agilent Application Notes*.
- [22] Y. Fukada, "Probability density function of polarization dependent loss (PDL) in optical transmission system composed of passive devices and connecting fibers", *J. of Lightwave Technology*, v. 20, n. 6, pp. 953 – 964, 2002.