Adaptive Wavelet Image Compression Based on Local Optimization of Filter Banks

Henrique Mohallem Paiva and Roberto Kawakami Harrop Galvão

***Resumo* — Este artigo apresenta uma técnica de compressão de imagem na qual a implementação da transformada wavelet em banco de filtros é adaptada à imagem em consideração. Para isso, ajustam-se os filtros de forma a reduzir a energia da distorção da imagem para uma dada taxa de compressão. As restrições de reconstrução perfeita do banco de filtros são satisfeitas utilizando-se uma parametrização angular para os pesos dos filtros. Isto permite a utilização de algoritmos simples para otimização sem restrições. Resultados obtidos com duas figuras em escala de cinza ilustram a melhoria de qualidade na imagem reconstruída utilizando a técnica proposta.**

***Palavras-Chave* — Processamento de imagens, compressão de imagens, transformadas wavelet adaptativas, bancos de filtro em quadratura espelhada**

***Abstract* — This article presents an image compression technique in which the filter bank implementation of the wavelet transform is adapted to the image under consideration. For this purpose, the filters are adjusted in order to reduce the energy of the image distortion for a given compression rate. The perfect-reconstruction restrictions on the filter bank are enforced by employing an angular parameterization for the filter weights. In this manner, simple algorithms for unconstrained optimization can be used in the process. Results obtained with two grayscale pictures illustrate the quality improvement for the reconstructed image achieved by using the proposed technique.**

***Keywords* — Image processing, image compression, adaptive wavelet transform, quadrature-mirror filter banks.**

I. INTRODUCTION

The wavelet transform is a tool for joint space-scale analysis that has been widely used in several image compression applications [3], [7], such as fingerprint image compression [6], medical image compression [1] and mobile/portable applications [9]. In this context, it has been shown that performance improvements may be obtained by adapting the transform with respect to the image under analysis [5], [8].

In this article, the concept of wavelet adaptation is exploited to improve the performance of image compression algorithms based on standard wavelet thresholding. For this purpose, a technique based on a restriction-free parameterization of the wavelet filter bank is proposed to adjust the filter weights in order to reduce the energy of the reconstruction error.

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Research work supported by CNPq (research fellowship and PRONEX grant 015/98), FAPESP (grant 03/09433-5) and EMBRAER.

A. *Notation*

- $\mathbf{X}$: Matrix that characterizes the image to be compressed.
- $\mathbf{T}$: Matrix of wavelet coefficients of $\mathbf{X}$ for a given number of decomposition levels in the filter bank.
- $\mathbf{T_C}$: Matrix of wavelet coefficients after compression.
- $\mathbf{X_R}$: Image matrix reconstructed from $\mathbf{T_C}$.
- $\mathbf{E = (X - X_R)}$: Reconstruction error matrix.
- $J_Y = \text{trace}(\mathbf{YY}^T)$: Energy of a given matrix $\mathbf{Y}$, defined as the sum of the square of its elements.

II. THE PROPOSED COMPRESSION TECHNIQUE

Let $\mathbf{T}$ be the matrix of wavelets coefficients resulting from the decomposition of an image matrix $\mathbf{X}$ by a two-dimensional wavelet filter bank, as depicted in Figures 1 and 2. The standard hard-thresholding compression algorithm consists of keeping only the largest coefficients of $\mathbf{T}$ (in absolute value), the other ones being set to zero. The modification proposed in this article is aimed at adjusting filters H and G to reduce the energy of the image distortion caused by such a procedure.

Let $\{h^{(k)}\}$ and $\{g^{(k)}\}$ be the weighting sequences of the lowpass and highpass filters H and G of the filter bank (Figure 1), each filter with $2k$ weights, such that the transfer functions of the filters are:

$$H(z) = \sum_{i=0}^{2k-1} h_i^{(k)} z^{-i} \, , \; G(z) = \sum_{i=0}^{2k-1} g_i^{(k)} z^{-i} \qquad (1)$$

If the filter bank is constrained to be orthonormal, nonlinear restrictions on $\{h^{(k)}\}$ and $\{g^{(k)}\}$ must be satisfied in the adjustment of the filters [11]. However, this problem can be circumvented by employing the parameterization proposed by Sherlock and Monro [4,10, 14]:

$$\begin{aligned}
&h_0^{(1)} = \mathrm{c}_0, \; h_1^{(1)} = \mathrm{s}_0 \\
&h_0^{(k+1)} = \mathrm{c}_k h_0^{(k)}, h_1^{(k+1)} = \mathrm{s}_k h_0^{(k)} \\
&\begin{cases} h_{2i}^{(k+1)} = \mathrm{c}_k h_{2i}^{(k)} - \mathrm{s}_k h_{2i-1}^{(k)} \\ h_{2i+1}^{(k+1)} = \mathrm{s}_k h_{2i}^{(k)} + \mathrm{c}_k h_{2i-1}^{(k)} \end{cases} , i = 1,2,\ldots,k-1 \\
&h_{2k}^{(k+1)} = -\mathrm{s}_k h_{2k-1}^{(k)}, h_{2k+1}^{(k+1)} = \mathrm{c}_k h_{2k-1}^{(k)}
\end{aligned} \qquad (2)$$

where s_k and c_k denote the sine and the cosine of an arbitrary angle θ_k, respectively.

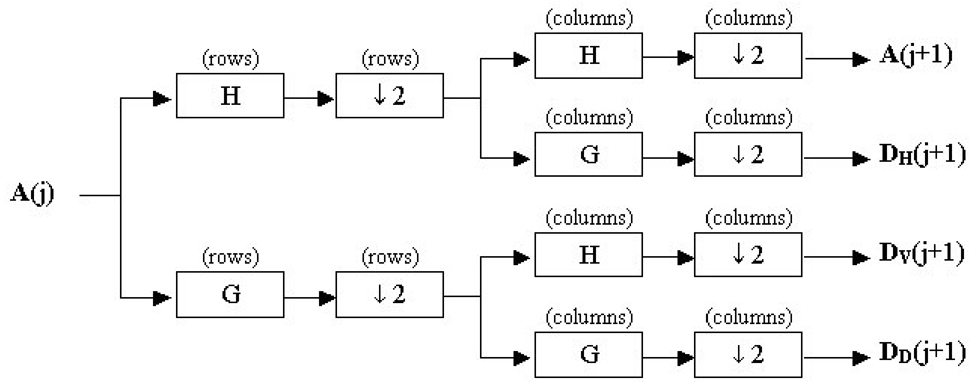

Figure 1. Two-dimensional wavelet filter bank. A(j), D_H(j), D_V(j) and D_D(j) denote the approximation and horizontal, vertical and diagonal details at resolution level j, respectively. Initialization is performed by setting A(0) = X, where X is the image matrix to be decomposed. H and G are lowpass and highpass filters, respectively. ↓2 is the downsampling operation. Superscripts (rows) and (columns) indicate that the operation is performed at each matrix row or column, respectively [11].

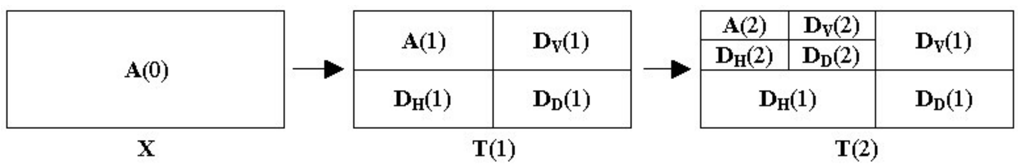

Figure 2. Wavelet decomposition of an image at first and second resolution levels. Matrix X characterizes the image to be decomposed and matrix T(j) stores the wavelet coefficients of X at resolution level j.

To ensure the orthonormality of the filter bank, the weights of filter G are obtained by *alternating flip* of the weights of filter H [11].

$$\begin{cases} g_{2i}^{(k)} = -h_{(2k-1)-2i}^{(k)} \\ g_{2i+1}^{(k)} = h_{(2k-1)-(2i+1)}^{(k)} \end{cases},\ i = 0,...,k-1 \qquad (3)$$

In such a formulation, any set of k independent angles θ_0, θ_1, ...,θ_{k-1} leads to a valid orthonormal filter bank system with filters of length 2k, and any such system can be expressed in terms of at least one such set of angles.

Moreover, for the filter bank to correspond to a regular wavelet transform, the DC gain of the highpass filter G must be zero [11], which is achieved by imposing [10]:

$$\sum_{i=0}^{k-1} \theta_i = \frac{\pi}{4} + n\pi,\ n \in \mathrm{Z} \qquad (4)$$

A possible approach to take equation (4) into account in the filter adjustment process consists of using θ_0, θ_1, ..., θ_{k-2} as the free optimization parameters and imposing [4]:

$$\theta_{k-1} = \frac{\pi}{4} - \sum_{i=0}^{k-2} \theta_i,\ n \in \mathrm{Z} \qquad (5)$$

Thus, it can be concluded that k-1 unconstrained angular parameters can be chosen to describe an orthonormal wavelet filter bank. As a result, filter adjustment can be carried out by using simple search algorithms for unconstrained optimization, such as the classical Simplex (flexible polyhedron) method [13,14], which is adopted in the present work. This local search algorithm is aimed at minimizing a given cost function (to be described below) on the region around a convenient starting point. Examples of adequate starting points are the sets of angles that characterize classical wavelet filters, such as those from the Daubechies or Symlet families [2], [11]. An algorithm for obtaining such an initial set of angles is described elsewhere [4].

The cost function adopted for the filter adjustment process in this article is the energy of the removed wavelet coefficients ($\mathbf{T} - \mathbf{T_C}$). The reason for such a choice will be now discussed.

The relationship between the orthonormal filter bank input **X** and output **T** can be represented, in matrix-vector form, as $\mathbf{T} = \mathbf{W}\ \mathbf{X}\ \mathbf{W}^T$ [12], where **W** is an orthogonal matrix, that is, $\mathbf{W}^T = \mathbf{W}^{-1}$. Thus, the inverse operation (signal reconstruction) can be written as $\mathbf{X} = \mathbf{W}^T\ \mathbf{T}\ \mathbf{W}$. Similarly, the relationship between $\mathbf{X_R}$ and $\mathbf{T_C}$ is given by $\mathbf{X_R} = \mathbf{W}^T \mathbf{T_C}\ \mathbf{W}$.

By defining the reconstruction error matrix as $\mathbf{E} = (\mathbf{X} - \mathbf{X_R})$, it follows that:

$$\mathbf{E} = (\mathbf{W}^T \mathbf{T}\ \mathbf{W} - \mathbf{W}^T \mathbf{T_C}\ \mathbf{W}) = \mathbf{W}^T (\mathbf{T} - \mathbf{T_C})\mathbf{W} \qquad (6)$$

The energy J_E of the reconstruction error can be calculated as

$$\begin{aligned} J_E &= \text{trace}(\mathbf{E}\,\mathbf{E}^T) = \\ &= \text{trace}([\mathbf{W}^T(\mathbf{T}-\mathbf{T_C})\mathbf{W}][\mathbf{W}^T(\mathbf{T}-\mathbf{T_C})\mathbf{W}]^T) = \\ &= \text{trace}(\mathbf{W}^T(\mathbf{T}-\mathbf{T_C})\mathbf{W}\mathbf{W}^T(\mathbf{T}-\mathbf{T_C})^T\mathbf{W}) = \\ &= \text{trace}(\mathbf{W}^T(\mathbf{T}-\mathbf{T_C})\,(\mathbf{T}-\mathbf{T_C})^T\mathbf{W}) \end{aligned} \qquad (7)$$

Since $\mathbf{W}$ is an orthogonal matrix, then $\mathbf{W}^T\mathbf{Y}\mathbf{W}$ is a similarity transformation of a given matrix $\mathbf{Y}$, and thus the eigenvalues of $\mathbf{Y}$ and $\mathbf{W}^T\mathbf{Y}\mathbf{W}$ are the same. Since the trace is equal to the sum of the matrix eigenvalues it follows that trace($\mathbf{Y}$) = trace($\mathbf{W}^T\mathbf{Y}\,\mathbf{W}$). Thus, from (7), it follows that:

$$J_E = \text{trace}((\mathbf{T}-\mathbf{T_C})(\mathbf{T}-\mathbf{T_C})^T) \qquad (8)$$

which shows that the energy of the reconstruction error ($\mathbf{X} - \mathbf{X_R}$) is equal to the energy of the removed wavelet coefficients ($\mathbf{T} - \mathbf{T_C}$). Thus, reducing the energy of the removed wavelet coefficients is equivalent to reducing the energy of the reconstruction error.

Moreover, it should be noticed that, if the filter bank is constrained to be orthonormal, the total energy J_T will be constant, regardless of the filter weights, because

$$\begin{aligned} J_T &= \text{trace}(\mathbf{T}\mathbf{T}^T) = \\ &= \text{trace}(\mathbf{W}\mathbf{X}\mathbf{W}^T\,(\mathbf{W}\mathbf{X}\mathbf{W}^T)^T) = \\ &= \text{trace}(\mathbf{W}\mathbf{X}\mathbf{W}^T\mathbf{W}\mathbf{X}^T\mathbf{W}^T) = \\ &= \text{trace}(\mathbf{W}\mathbf{X}\mathbf{X}^T\mathbf{W}^T) = \\ &= \text{trace}(\mathbf{X}\mathbf{X}^T) = J_X \end{aligned} \qquad (9)$$

where constant J_X is the energy of the image matrix $\mathbf{X}$ under consideration.

In short, the adjustment of the filters is aimed at reducing the error energy J_E, or equivalently, the energy loss index $\upsilon = J_E/J_X$ for a given compression factor λ (ratio between the number of wavelet coefficients that are kept in the thresholding process and the total number of coefficients).

III. RESULTS AND DISCUSSION

The images presented in Figure 3 will be employed to illustrate the technique described above. All programs used in these examples were developed under the Matlab 6.1 platform with functions from the Wavelet and Optimization Toolboxes.

The compression factor used was $\lambda = 5\%$. In order to choose the most appropriate wavelet filters to use as starting point for the adjustment process, three different possibilities were tested, namely the Daubechies wavelet filters db4 ($k = 4$), db6 ($k = 6$) and db8 ($k = 8$) [2]. In each case, three to five decomposition levels were tested in the filter bank.

(a)

(b)

Figure 3. Images to be compressed. (a) 130 x 202 pixels (b) 160 x 300 pixels.

A. *Figure 3(a)*

Table 1 depicts the compression results obtained by processing Figure 3(a).

Table 1. Results obtained by processing Figure 3(a) using wavelet filters db4, db6 and db8, with three to five resolution levels, for a compression factor $\lambda = 5\%$.

Wavelet filter:	db4	db6	db8
Resolution Levels	Energy loss index υ (%)		
3	0.264	0.241	0.234
4	0.151	0.124	0.123
5	0.077	0.057	0.057

Table 1 shows that the best result (υ = 0.057 %) was obtained with db6 and five resolution levels. The reconstructed image for this case is presented in Figure 4(a). The db8 wavelet filters with five resolution levels provided the same result in terms of energy loss index, but the db6 wavelet filters were preferred because their smaller number of weights leads to a smaller computational effort.

The adjustment procedure was applied to the db6 filters by using the flexible polyhedron algorithm to adjust $k\text{-}1 = 5$ angular parameters. The resulting reconstructed image is depicted in Figure 4(b). Figure 5 presents a comparison of the weights of the lowpass filter H (Figure 1) before and after the adjustment.

A comparison of Figure 4(a) and (b) shows that the adjustment procedure does improve the quality of the reconstructed image. In quantitative terms, the energy loss index υ decreases from 0.057 % to 0.050%.

(a)

(b)

Figure 4. Reconstructed image after a compression using (a) fixed and (b) adjusted db6 wavelet filter, five resolution levels and λ = 5%. The obtained energy loss indexes are (a) υ = 0.057 % and (b) 0.050%.

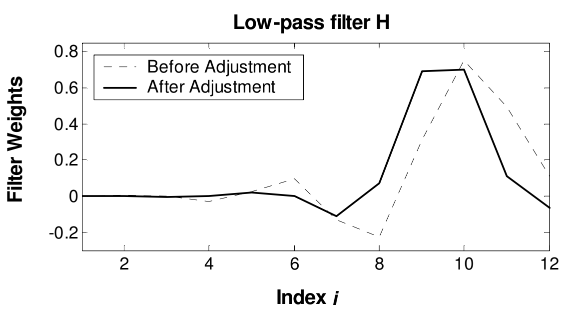

Figure 5. Filter weights $h_i^{(6)}$ of low-pass filter H as a function of index *i* before and after the adjustment.

B. *Figure 3(b)*

Table 2 depicts the compression results obtained by processing Figure 3(b).

Table 2. Results obtained by processing Figure 3(b) using wavelet filters db4, db6 and db8, with three to five resolution levels, for a compression factor λ = 5%.

Wavelet filter:	db4	db6	db8
Resolution Levels	Energy loss index υ (%)		
3	0.098	0.107	0.113
4	0.046	0.043	0.041
5	0.025	0.020	0.018

Table 2 shows that the best result (υ = 0.018 %) was obtained with db8 and five resolution levels. The reconstructed image for this case is presented in Figure 6(a).

The adjustment procedure was applied to the db8 filters by using the flexible polyhedron algorithm to adjust *k-1* = 7 angular parameters. The resulting reconstructed image is depicted in Figure 6(b). Figure 7 presents a comparison of the weights of the lowpass filter H (Figure 1) before and after the adjustment.

A comparison of Figure 6(a) and (b) shows that the adjustment procedure does improve the quality of the reconstructed image. In quantitative terms, the energy loss index υ decreases from 0.018 % to 0.009%.

(a)

(b)

Figure 6. Reconstructed image after a compression using (a) fixed and (b) adjusted db8 wavelet filter, five resolution levels and λ = 5%. The obtained energy loss indexes are (a) υ = 0.018 % and (b) 0.009%.

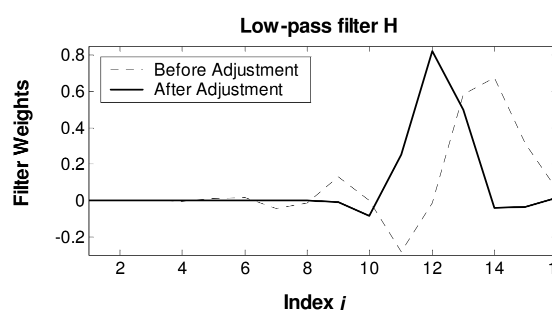

Figure 7. Filter weights $h_i^{(8)}$ of low-pass filter H as a function of index *i* before and after the adjustment.

IV. CONCLUSIONS

This article presented a wavelet-based compression method that is adapted to the image under analysis. For this purpose, an index related to the reconstruction error is employed as the cost function in a local optimization procedure. In order to circumvent the problems associated to the perfect-reconstruction restrictions on the filter bank implementation of the discrete wavelet transform, a trigonometric parameterization was employed, allowing the use of simple search algorithms for unconstrained optimization. The flexible polyhedron algorithm was chosen to accomplish this task. It should be noted that the solution found in this manner is not guaranteed to be a global minimum of the cost function. However, as shown in the examples used for illustration, the local search procedure does lead to perceptible improvements in the reconstructed image.

Future works could exploit the filter adaptation strategy described in this work in combination with techniques for optimizing the structure of wavelet packet trees [8,12].

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