Field Trials of the Average Fade Duration and Distribution of Duration of Fades at 836 MHz

Antonio Marcelo O. Ribeiro and Evandro Conforti, *Senior Member, IEEE*

Abstract — **Field measurements at 836.37 MHz in a suburban area were done. A spectrum analyzer was used as the measurement apparatus to acquire radio mobile envelope. The Nakagami average fade duration and Rice distribution of the duration of fades were verified in their ability to fit the empirical data calculated from the envelope recorded. Empirical data and theoretical formulas showed good match for most of data set.**

Keywords — **Fading channels, average fade duration, microwave measurements, Nakagami fading channels, probability.**

I. Introduction

The level crossing rate (LCR) and average fade duration (AFD) are two important second-order statistics that provide the dynamic behavior of the multipath fading channels. The LCR represents how often the envelope cross a certain level per unit duration in the negative (or positive) direction and the AFD corresponds to the average time the envelope remains under a certain level once this one has been crossed in the negative direction [1]. These two second-order statistics have important role in characterizing the statistics of burst error [2]. Also, they are useful for the analysis of handoff algorithms [3] and optimization of the interleaver size [4]. In particular, the AFD helps to determine how many bits of signal may be lost during the occurrence of a fading, that is, the average length of burst error. Therefore, the higher AFD is, the more likely long data blocks being affected by the channel fades.

Due to its flexibility in describing the different fading environments, the Nakagami distribution [5] has been widely accepted as a fading model since it was proposed. Here, a recently derived closed-form formula of AFD for the Nakagami channel model, [6] and [7], is verified in its ability of characterizing empirical AFDs, calculated from field trials in a suburban area at 836 MHz.

Another important second-order statistics is the distribution function of the duration of fades. When the envelope crosses downward through the level R, the probability that it will remain below R for more than τ seconds is denoted by $F_\tau(u,R)$, where $u=\tau/AFD$ [8]. Estimates of $F_\tau(u,R)$ were calculated by Rice [8] for the envelope of the narrow-band Gaussian noise. In this paper these estimated values will be compared to empirical data obtained from the field measurements at 836 MHz.

This work was partially supported by CAPES, FAPESP, and CNPq.

A. M. O. Ribeiro and E. Conforti are with Microwave and Optics Depart., Faculty of Electrical and Computer Eng., State University of Campinas, Sao Paulo, Brazil. E-mail: {aribeiro,conforti}@dmo.fee.unicamp.br.

II. Theoretical Statistics

A. Nakagami Distribution

The Nakagami fading model was derived from extensive field measurements in urban and suburban area [5]. Moreover, it has been found that the Nakagami distribution gives a very good fit for the fading conditions encountered in the mobile channel [9]. The Nakagami probability density function (PDF) of the envelope r is given by:

$$p(r)=\frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m}\exp\left(-\frac{mr^2}{\Omega}\right), \quad (1)$$

where

$$\Gamma(m)=\int_0^\infty x^{m-1}\exp(-x)dx \quad (2)$$

is the Gamma function, $\Omega=E[r^2]$ is the average power of the envelope, and $m=\Omega^2/\mathrm{var}[r^2]$ is the Nakagami parameter.

If the envelope is normalized to its root mean square (*rms*) value $r_{rms}=\sqrt{\Omega}$, that is, $\rho=r/\sqrt{\Omega}$, then applying the transformation $p(\rho)|d\rho|=p(r)|dr|$, it is straightforward to show that the Nagakami PDF normalized to the *rms* value $p(\rho)$ is:

$$p(\rho)=\frac{2m^m\rho^{2m-1}}{\Gamma(m)}\exp\left(-m\rho^2\right). \quad (3)$$

B. Nakagami Average Fade Duration

Theoretical formulas of LCR and AFD for the Nakagami mobile channel were recently derived in [6] and [7], considering the Clarke's model of isotropic scattering [10]. In particular, the Nakagami AFD, as derived in [6] and [7], is given by:

$$T(\rho)=\frac{\Gamma(m,m\rho^2)}{\sqrt{2\pi}f_m m^{m-(1/2)}\rho^{2m-1}\exp(-m\rho^2)}, \quad (4)$$

where $f_m = v/\lambda$ is the maximum Doppler shift, v is the velocity of the mobile receiver, λ is the wavelength of the carrier signal, and

$$\Gamma(a,b) = \int_0^b z^{a-1} \exp(-z) dz \quad (5)$$

is the incomplete Gamma function.

C. Distribution function of the duration of fades

As previously stated, the distribution function of the duration of fades $F_\tau(u,R)$ was derived by Rice [8], considering an envelope of the narrow-band Gaussian noise. The function means the probability that $R(t) < R$ for an interval lasting longer than τ. Here, u is the variable τ normalized to the value of AFD corresponding to the level *R*. Estimates of $F_\tau(u,R)$ are given in Table I for $R = 0$ dB with respect to its *rms* value. Details how these values were calculated can be found in [8].

TABLE I
ESTIMATES OF $F_\tau(u,R)$, $R = 0$ dB WITH RESPECT TO ITS RMS VALUE

u	0	0.2	0.4	0.6	0.8	1.0	1.2
$F_\tau(u,R)$	1.0	0.969	0.843	0.624	0.464	0.360	0.279

1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8
0.216	0.168	0.130	0.101	0.078	0.061	0.047	0.037

III. EXPERIMENTAL RESULTS

A. Measurement Setup

The block diagram and equipment list of the measurement setup is shown in Fig. 1. In the transmitter, a continuous wave (CW) carrier at 836.37 MHz was radiated form a vertically polarized 12 dBi antenna located about 30 m in height. The mobile receiver, mounted in a vehicle, consisted of a vertically polarized 7 dBi omni antenna, a pre-amplifier, a spectrum analyzer, and a laptop computer. A RS232-GPIB converter was used as interface between the laptop and the spectrum analyzer. Although the envelope is usually acquired using specialized receivers and filters tuned in a specific frequency, a spectrum analyzer was chosen because the envelope data can be downloaded as a direct readout from the analyzer screen. Also the analyzer is easily tunable at any frequency within its range, and a simple software to control it and carry out the measurements can promptly be done.

B. Measurement Technique

The spectrum analyzer was centered in frequency at 836.37 MHz and set to *zero span*. With the vehicle in movement, an envelope section of 401 samples was acquired for each sweep of the spectrum analyzer. In this acquisition method the envelope was not continuously acquired. Instead, pieces (sections) of it were acquired in sequence for each run of the vehicle. A set of sections acquired per run formed an envelope record.

The local area of the measurements was a suburban area around the university campus far from transmitter about 1250 m. The envelope records were acquired at the velocities of 35 and 60 km/h. In particular, a data set of envelope records was collected at 35 and 60 km/h through several test runs at the local area. The goal here is to show the dependence of the AFD with the mobile receiver speed. Two mobility conditions are tested: speed limit in urban traffic (60 km/h) and low mobility condition (35 km/h).

In order to fulfill the Nyquist criterion the minimum sampling rate in which the envelope is acquired must satisfy $L_s = vT_s \le \lambda/4$ [11], where L_s is the distance between two consecutive samples and T_s is the sampling time. Since each section of envelope acquired by the spectrum analyzer has 401 samples and assuming S as the analyzer sweep time, then:

$$v\left(\frac{S}{400}\right) \le \frac{\lambda}{4}. \quad (6)$$

The carrier frequency $f_c = 836.37$ MHz results in $\lambda/4 = 0.09$ m. With $v = 35$ km/h $= 9.7$ m/s in (6) we have $S \le 3.7$ s. For $v = 60$ km/h $= 16.7$ m/s we obtain $S \le 2.2$ s. In our measurements, the analyzer sweep time was set to 400 ms and 200 ms for $v = 35$ km/h and $v = 60$ km/h, respectively.

It can be shown for both velocities of 35 and 60 km/h that the mobile receiver traveled a distance $L = Sv \le 3.9$ m during a single sweep of the spectrum analyzer. Also, from [11, eq. (4.93)] and for $f_c = 836.37$ MHz, the distance in which the envelope local mean is considered to be constant is $L_m = 9$ m. Since the distance traveled by the mobile receiver during single sweep time is less than L_m, the envelope within each section is stationary in relation to the local mean.

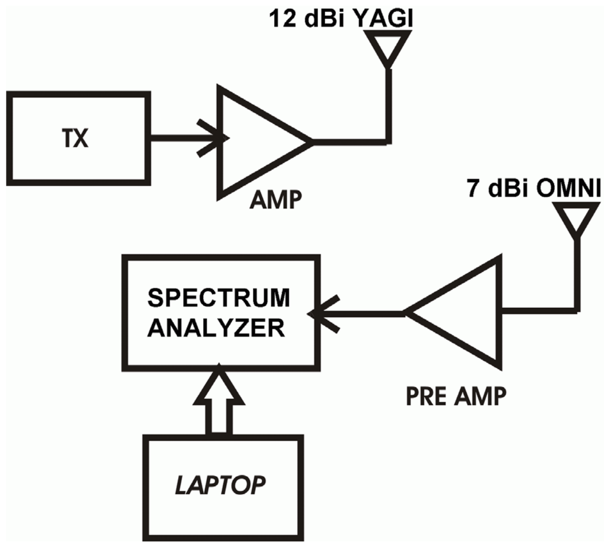
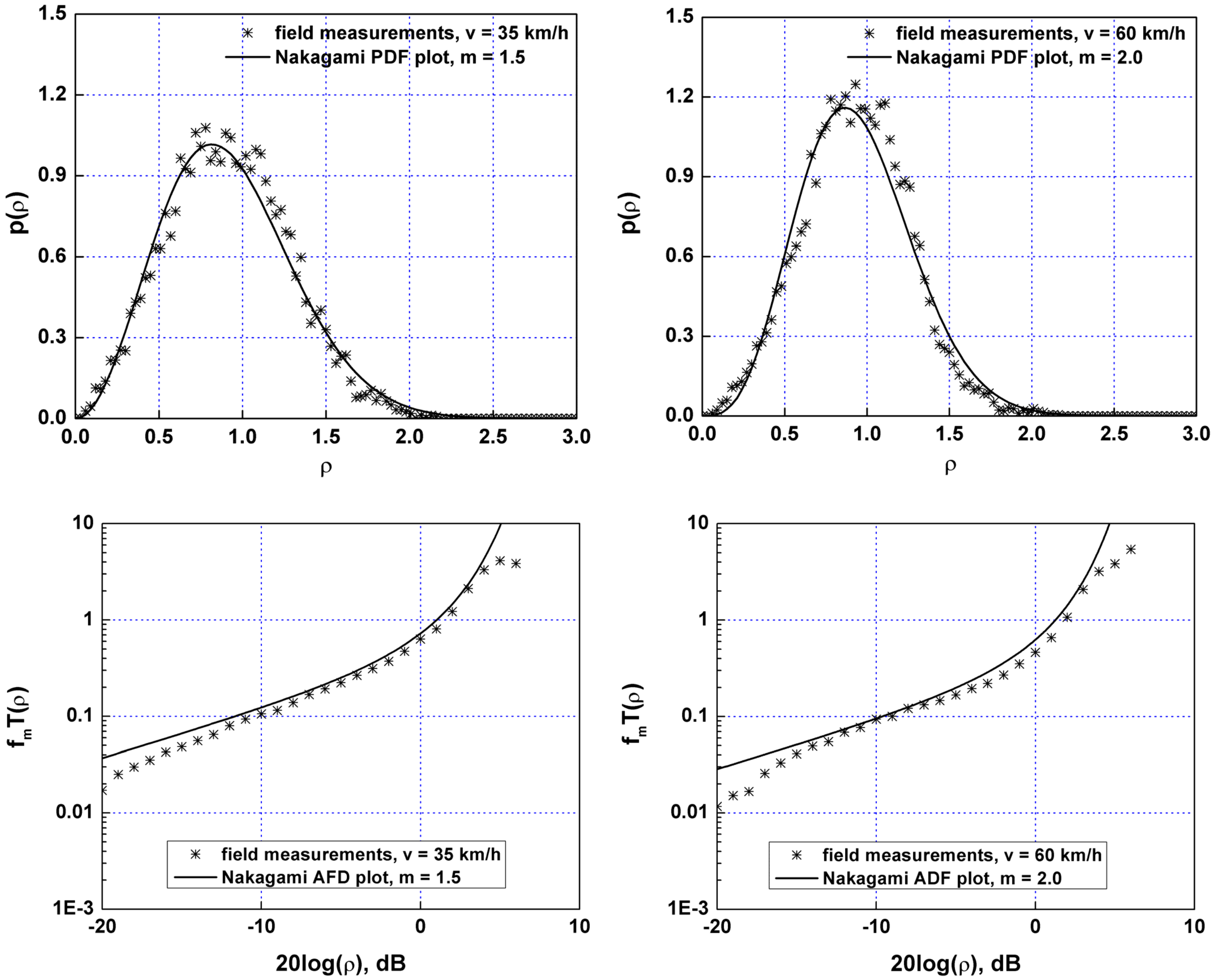


Fig. 1. Block diagram of the measurement setup. Equipment list: TX – Wavetek RF Generator 100 kHz - 1300 MHz, AMP – HP 8347A RF Amplifier 100 kHz - 3 GHz, PRE AMP – HP 8447E Amplifier 100 kHz - 1300 MHz, SPECTRUM ANALYZER – HP 8593E, and RS232-GPIB converter – National Instruments GPIB-232CT-A.



(a) (b)

Fig. 2. Empirical PDFs and AFDs compared to the corresponding Nakagami expressions. (a) Envelope record acquired at 35 km/h. (b) Envelope record acquired at 60 km/h.

C. Empirical AFD and Distribution of Duration of Fades

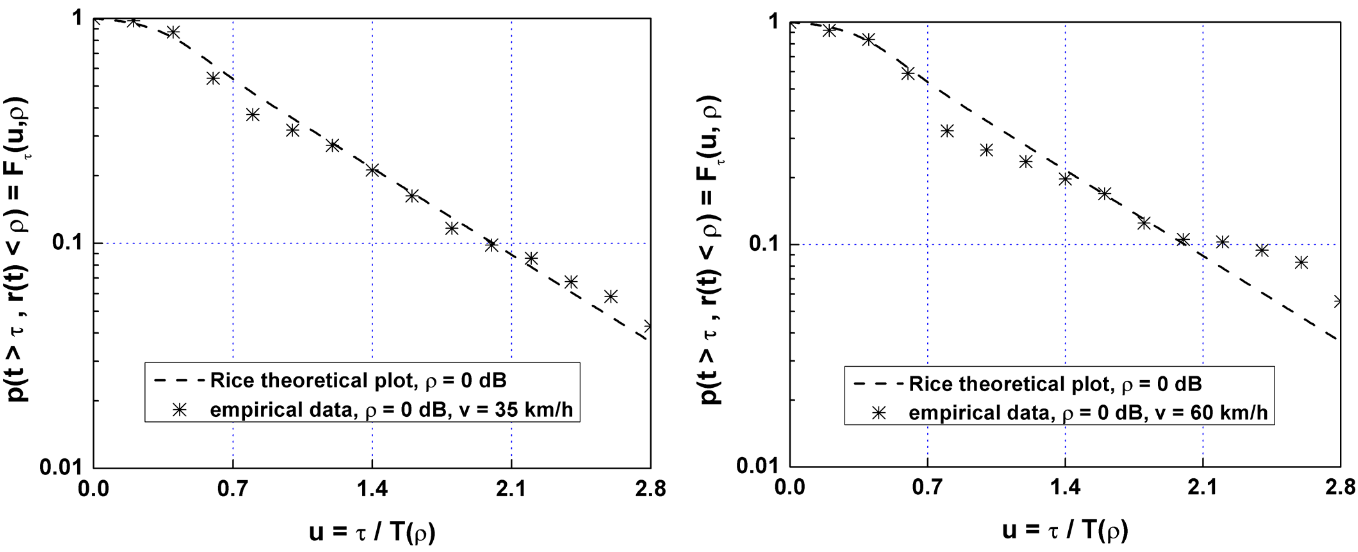
Before estimating the Nakagami parameter m from the collected data, all envelope sections were first normalized to their respective *rms* values since both Nakagami PDF and AFD are in function of $\rho = r/\sqrt{\Omega}$. After normalization, the maximum-likelihood (ML) estimator derived in [12] was used to estimate m for each envelope record:

$$\hat{m}_{ML} \approx \left[2\ln\left(\frac{\sum_{n=1}^{N} \alpha_n^2}{N\left(\prod_{n=1}^{N} \alpha_n^2\right)^{1/N}} \right) \right]^{-1}, \qquad (7)$$

where N is the number of samples of the signal and α_n is the nth-sample.

Empirical PDF and AFD were calculated from the envelope records and then compared to the corresponding Nakagami PDF and AFD, (3) and (4), respectively. Also empirical values of the distribution of the duration of fades were determined for $\rho = 0\,\text{dB}$ with respect to its *rms* value and compared to the data estimated by Rice, provided in Table I.

As examples of collected measurements, empirical curves (in asterisks) for the PDF and AFD statistics are shown both in Fig. 2(a) and Fig. 2(b), together with the corresponding Nakagami curves (in solid lines). In Fig. 2(a) the empirical curves were calculated from an envelope record of 36 sections of signal acquired at 35 km/h whereas in Fig. 2(b) a 34-section envelope record acquired at 60 km/h was used to calculate them. Note that although both Fig. 2(a) and (b) show good matches in terms of PDF, the same does not happen in terms of AFD. In particular, in Fig. 2(b) a good fit for PDF does not guarantee the same result for the corresponding AFD statistics. This fact that a close match for the first-order statistics (PDF) may result dissimilar second-



(a) (b)

Fig. 3. Probability that the envelope will remain below ρ for an interval lasting longer than τ seconds. $\rho = 0$ dB with respect to the *rms* value. (a) Envelope record acquired at 35 km/h. (b) Envelope record acquired at 60 km/h.

order statistics (LCR, AFD) is also reported in [7], where LCR and AFD field trials were carried out.

It can be observed in Fig. 2(a) and (b) that for values of $\rho > 5$ dB both empirical AFD statistics deviate from the corresponding Nakagami curves. This is due to a limitation of the measurement technique. Since the signal is acquired in sections, the maximum empirical AFD value is limited to the sweep time of the spectrum analyzer.

Fig. 3(a) and (b) show empirical values of the distribution of the duration of fades (in asterisks), together with the corresponding data (in solid line) estimated by Rice and retrieved from Table I. The same envelope records used to calculated the AFD statistics in Fig. 2(a) and (b) were respectively used here to generate the empirical values of the distribution of the duration of fades. In this case, a good fit is obtained in Fig. 3(a) at 35 km/h, while in Fig. 3(b) at 60 km/h the match is not so satisfactory, confirming the fact that a good fit for first-order statistics does not necessarily mean the same result for second-order statistics.

IV. CONCLUSIONS

In this paper radio mobile envelopes at 836.37 MHz were acquired by a spectrum analyzer as the measurement apparatus. Empirical AFD and distribution of the duration of fades were calculated from the measured data and compared to Nakagami AFD and Rice distribution of duration of fades, respectively. Good fit was obtained between the theoretical results and the empirical data for most of the envelope records. Also, it was noted that a good fit for first-order statistics (PDF) does not necessarily guarantee the same result for second-order statistics (LCR and AFD).

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