General Exact Level Crossing Rates and Average Fade Durations of Dual Diversity Combiners over Non-Identical Correlated Ricean Channels

José Cândido Silveira Santos Filho, Michel Daoud Yacoub, José Ricardo Mendes, and Gustavo Fraidenraich

***Resumo*— Este trabalho apresenta expressões gerais exatas para a taxa de cruzamento de nível e a duração média de desvanecimento de combinadores por seleção, ganho-igual e razão-máxima com dois ramos, operando sobre canais ricianos não-idênticos e correlacionados. Resultados numéricos são obtidos e analisados para um sistema de diversidade espacial com antenas omnidirecionais espaçadas horizontalmente na estação móvel. Observa-se que, quando as antenas são perpendiculares à direção do movimento, a duração média de desvanecimento é fracamente dependente do espaçamento entre antenas e do fator riciano, sendo quase idêntica àquela referente à condição de ramos independentes. Por outro lado, quando as antenas são paralelas à direção do movimento, a taxa de cruzamento de nível pode ser reduzida abaixo do valor obtido para sinais de desvanecimento independentes. Neste caso, em havendo componente de linha de visada, a duração média de desvanecimento também pode ser reduzida em relação à condição de independência.**

***Palavras-Chave*— Duração média de desvanecimento, correlação, métodos de combinação de diversidade, taxa de cruzamento de nível, canais ricianos de desvanecimento.**

***Abstract*— General exact expressions for the level crossing rate and average fade duration of dual-branch selection, equal-gain, and maximal-ratio combiners operating over non-identical correlated Ricean channels are derived. Sample numerical results are discussed by specializing the general expressions to a space-diversity system with horizontally spaced omnidirectional antennas at the mobile station. When the antennas are perpendicular to the direction of the vehicle motion, the average fade duration is loosely dependent on the antenna spacing and on the Ricean factor, being almost identical to that of the independent fading condition. When the antennas are parallel to the direction of the vehicle motion, the level crossing rate can be reduced below the value obtainable for independent fading signals. In this case, if a line-of-sight component exists, a further improvement in the average fade duration over the independent condition may be also achieved.**

***Keywords*— Average fade duration, correlation, diversity-combining methods, level crossing rate, Ricean fading channels.**

I. INTRODUCTION

Level crossing rate (LCR) and average fade duration (AFD) are widely used performance measures of wireless diversity systems. Although in practical systems the branch signals are often correlated to each other, no general exact expressions for the LCR and AFD of diversity-combining schemes operating over correlated fading channels have been published in the literature, apart from the bivariate balanced Rayleigh case [1]. Results on Ricean fading also exist [2], [3], but with a restrictive assumption, in which the processes described by the time derivatives of the branch envelopes are independent from each other as well as from the branch envelopes. In general, this is not true for correlated fading: although the envelope at the ith branch is indeed independent from its time derivative, the latter is usually correlated to the envelope and to the time derivative of the envelope at the jth branch, $i \neq j$, as correctly shown in [1]. Allowing for this general scenario, we provide *unrestricted*, *exact* expressions for the LCR and AFD of dual-branch selection (SC), equal-gain (EGC), and maximal-ratio combining (MRC) operating over *non-identical correlated* Ricean channels. For Rayleigh fading, our results specialize to those presented in [1].

The paper is organized as follows. In section II, the system model and preliminary concepts are introduced. Some key statistics involving the branch envelopes and their time derivatives are derived in sections III and IV. Relying upon these statistics, the general exact LCR and AFD expressions are presented in section V. Section VI discusses some numerical results by specializing the general expressions to a space-diversity system with horizontally spaced omnidirectional antennas at the mobile station. The main results of the paper are summarized in section VII.

The authors are with the Department of Communications, School of Electrical and Computer Engineering, University of Campinas, PO Box 6101, 13083-852 Campinas, SP, Brazil (Tel: +55 (19) 3788 5106, Fax: +55 (19) 3289 1395, E-mail:[candido,michel,ricardo,gf]@wisstek.org).

II. SYSTEM MODEL AND PRELIMINARIES

The received Ricean signal at the ith branch, $i \in \{1, 2\}$, can be represented in complex form Z_i as

$$Z_i = R_i e^{j\Theta_i} = X_i + jY_i \tag{1}$$

where the in-phase component $X_i = R_i \cos\Theta_i$ and the quadrature component $Y_i = R_i \sin\Theta_i$ are independent Gaussian random variables (RVs), $\mathrm{E}\{X_i\} = m_{X_i}$, $\mathrm{E}\{Y_i\} = m_{Y_i}$, and $\mathrm{Var}\{X_i\} = \mathrm{Var}\{Y_i\} = \sigma_i^2$. ($\mathrm{E}\{\cdot\}$ and $\mathrm{Var}\{\cdot\}$ denote mean and variance, respectively.) The envelope R_i follows the Rice distribution with Rice factor $k_i = (m_{X_i}^2 + m_{Y_i}^2)/(2\sigma_i^2)$, and $0 \leq \Theta_i \leq 2\pi$ is a random phase. Furthermore, X_1, Y_1, X_2, and Y_2 are jointly Gaussian RVs.

The output envelope R for each diversity-combining scheme is

$$R = \begin{cases} \max\{R_1, R_2\} & \text{SC} \\ \dfrac{R_1 + R_2}{\sqrt{2}} & \text{EGC} \\ \sqrt{R_1^2 + R_2^2} & \text{MRC} \end{cases} \tag{2}$$

and the LCR $N_R(r)$ and the AFD $T_R(r)$ of R at level r are given by

$$N_R(r) = \int_0^\infty \dot{r} f_{R,\dot{R}}(r, \dot{r}) d\dot{r} \tag{3}$$

$$T_R(r) = \frac{F_R(r)}{N_R(r)} \tag{4}$$

where $f_{R,\dot{R}}(\cdot,\cdot)$ is the joint probability density function (PDF) of R and its time derivative $\dot{R}$ and $F_R(\cdot)$ is the cumulative distribution function (CDF) of R.

III. Statistics of $\dot{R}$ conditioned on R_1, R_2, Θ_1, Θ_2

In this section, we derive the statistics of $\dot{R}$ conditioned on

$$\begin{aligned}\mathbf{Z} &\triangleq [X_1\ Y_1\ X_2\ Y_2]^T \text{ or, equivalenty,}\\ &\triangleq [R_1 \cos\Theta_1 \quad R_1 \sin\Theta_1 \quad R_2\cos\Theta_2 \quad R_2 \sin\Theta_2]^T\end{aligned}$$

as required. First, note from (1) that the time derivative $\dot{R}_i$ of R_i can be expressed in terms of the time derivatives $\dot{X}_i$ of X_i and $\dot{Y}_i$ of Y_i as

$$\dot{R}_i = \dot{X}_i \cos\Theta_i + \dot{Y}_i \sin\Theta_i \tag{5}$$

Thus, since $\dot{X}_1$, $\dot{Y}_1$, $\dot{X}_2$, and $\dot{Y}_2$ are jointly Gaussian RVs [4], $\dot{R}_1$ and $\dot{R}_2$ are also jointly Gaussian RVs conditioned on $\mathbf{Z}$. Using (5), it is easy to show that the conditional mean $\dot{m}_i(\mathbf{Z}) = \mathrm{E}\{\dot{R}_i|\mathbf{Z}\}$, variance $\dot{\sigma}_i^2(\mathbf{Z}) = \mathrm{Var}\{\dot{R}_i|\mathbf{Z}\}$, and covariance $\dot{\sigma}_{ij}^2(\mathbf{Z}) = \mathrm{Cov}\{\dot{R}_i, \dot{R}_j|\mathbf{Z}\}$, $i \neq j \in \{1, 2\}$, are obtained as

$$\dot{m}_i(\mathbf{Z}) = \mathrm{E}\{\dot{X}_i|\mathbf{Z}\}\cos\Theta_i + \mathrm{E}\{\dot{Y}_i|\mathbf{Z}\}\sin\Theta_i \tag{6a}$$

$$\begin{aligned}\dot{\sigma}_i^2(\mathbf{Z}) =& \mathrm{Var}\{\dot{X}_i|\mathbf{Z}\}\cos^2\Theta_i + \\ & \mathrm{Var}\{\dot{Y}_i|\mathbf{Z}\}\sin^2\Theta_i + \mathrm{Cov}\{\dot{X}_i, \dot{Y}_i|\mathbf{Z}\}\sin 2\Theta_i\end{aligned} \tag{6b}$$

$$\begin{aligned}\dot{\sigma}_{ij}^2(\mathbf{Z}) =& \mathrm{Cov}\{\dot{X}_i, \dot{X}_j|\mathbf{Z}\}\cos\Theta_i\cos\Theta_j + \\ & \mathrm{Cov}\{\dot{X}_i, \dot{Y}_j|\mathbf{Z}\}\cos\Theta_i \sin\Theta_j + \\ & \mathrm{Cov}\{\dot{Y}_i, \dot{X}_j|\mathbf{Z}\}\sin\Theta_i\cos\Theta_j + \\ & \mathrm{Cov}\{\dot{Y}_i, \dot{Y}_j|\mathbf{Z}\}\sin\Theta_i\sin\Theta_j\end{aligned} \tag{6c}$$

The conditional statistics $\mathrm{E}\{\dot{X}_i|\mathbf{Z}\}$, $\mathrm{E}\{\dot{Y}_i|\mathbf{Z}\}$, $\mathrm{Var}\{\dot{X}_i|\mathbf{Z}\}$, $\mathrm{Var}\{\dot{Y}_i|\mathbf{Z}\}$, $\mathrm{Cov}\{\dot{X}_i, \dot{Y}_i|\mathbf{Z}\}$, $\mathrm{Cov}\{\dot{X}_i, \dot{X}_j|\mathbf{Z}\}$, $\mathrm{Cov}\{\dot{X}_i, \dot{Y}_j|\mathbf{Z}\}$, $\mathrm{Cov}\{\dot{Y}_i, \dot{X}_j|\mathbf{Z}\}$, and $\mathrm{Cov}\{\dot{Y}_i, \dot{Y}_j|\mathbf{Z}\}$ required in (6) are a crucial step for solving the problem addressed here and, to the best of the authors' knowledge, they have not been published yet. We derive them in the appendix. Now, knowing from (2) that

$$\dot{R} = \begin{cases} \dot{R}_1, \ R_1 \geq R_2, \ \text{and} \ \dot{R}_2, \ R_1 < R_2 & \text{SC} \\ \dfrac{\dot{R}_1 + \dot{R}_2}{\sqrt{2}} & \text{EGC} \\ \dfrac{R_1\dot{R}_1 + R_2\dot{R}_2}{\sqrt{R_1^2 + R_2^2}} & \text{MRC} \end{cases} \tag{7}$$

$\dot{R}$ is also a Gaussian RV conditioned on $\mathbf{Z}$. The corresponding PDF is

$$f_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\dot{\sigma}^2(\mathbf{z})}} \exp\left[-\frac{(\dot{r} - \dot{m}(\mathbf{z}))^2}{2\dot{\sigma}^2(\mathbf{z})}\right] \tag{8}$$

where $\mathbf{z} = [r_1\cos\theta_1 \quad r_1 \sin\theta_1 \quad r_2 \cos\theta_2 \quad r_2 \sin\theta_2]^T$, with the conditional mean $\dot{m}(\mathbf{Z}) = \mathrm{E}\{\dot{R}|\mathbf{Z}\}$ and variance $\dot{\sigma}^2(\mathbf{Z}) = \mathrm{Var}\{\dot{R}|\mathbf{Z}\}$ obtained by means of (7) in terms of the conditional branch statistics (6) as

$$\dot{m}(\mathbf{Z}) = \begin{cases} \dot{m}_1(\mathbf{Z}), \ R_1 \geq R_2, \ \text{and} \ \dot{m}_2(\mathbf{Z}), \ R_1 < R_2 & \text{SC} \\ \dfrac{\dot{m}_1(\mathbf{Z}) + \dot{m}_2(\mathbf{Z})}{\sqrt{2}} & \text{EGC} \\ \dfrac{R_1\dot{m}_1(\mathbf{Z}) + R_2\dot{m}_2(\mathbf{Z})}{\sqrt{R_1^2 + R_2^2}} & \text{MRC} \end{cases} \tag{9a}$$

$$\dot{\sigma}^2(\mathbf{Z}) = \begin{cases} \dot{\sigma}_1^2(\mathbf{Z}), \ R_1 \geq R_2, \ \text{and} \ \dot{\sigma}_2^2(\mathbf{Z}), \ R_1 < R_2 & \text{SC} \\ \dfrac{\dot{\sigma}_1^2(\mathbf{Z}) + \dot{\sigma}_2^2(\mathbf{Z}) + 2\dot{\sigma}_{12}^2(\mathbf{Z})}{2} & \text{EGC} \\ \dfrac{R_1^2\dot{\sigma}_1^2(\mathbf{Z}) + R_2^2\dot{\sigma}_2^2(\mathbf{Z}) + 2R_1R_2\dot{\sigma}_{12}^2(\mathbf{Z})}{R_1^2 + R_2^2} & \text{MRC} \end{cases} \tag{9b}$$

IV. Joint Statistics of R_1, R_2, Θ_1, Θ_2

As mentioned before, the RVs X_1, Y_1, X_2, and Y_2 follow a multivariate Gaussian PDF [5] given by

$$f_{X_1,Y_1,X_2,Y_2}(x_1, y_1, x_2, y_2) = \frac{1}{(2\pi)^2\sqrt{\det \mathbf{b}}} \exp\left(-\frac{(\mathbf{z} - \mathbf{m})\mathbf{b}^{-1}(\mathbf{z} - \mathbf{m})^T}{2}\right) \tag{10}$$

$\mathbf{z} = [x_1 \ y_1 \ x_2 \ y_2]^T$, with the mean vector $\mathbf{m}$ and the covariance matrix $\mathbf{b}$ presented in (17d) and (17b), respectively (see appendix). By means of a simple transformation of variables, the joint PDF of R_1, Θ_1, R_2, and Θ_2 is obtained as

$$f_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2) = \frac{r_1 r_2}{(2\pi)^2\sqrt{\det \mathbf{b}}} \exp\left(-\frac{(\mathbf{z} - \mathbf{m})\mathbf{b}^{-1}(\mathbf{z} - \mathbf{m})^T}{2}\right) \tag{11}$$

$\mathbf{z} = [r_1\cos\theta_1 \quad r_1\sin\theta_1 \quad r_2\cos\theta_2 \quad r_2\sin\theta_2]^T$.

V. Second-Order Statistics

Making the appropriate transformation of variables to obtain $f_{R,\dot{R}}(\cdot,\cdot)$ from (8) and (11), as in [1, Eq. (8)] for SC and in [6, Eqs. (12) and (17)] for EGC and MRC, the general exact LCR of dual diversity over non-identical correlated Ricean channels can be evaluated from (3)—integrating with respect to $\dot{r}$ first—as in (12), where $\mathbf{z} = [r_1\cos\theta_1 \quad r_1\sin\theta_1 \quad r_2\cos\theta_2 \quad r_2\sin\theta_2]^T$. The CDF of R in terms of (11) is trivial for SC, and can be obtained directly from [6, Eqs. (10) and (16)] for EGC and MRC as in (13). Using (12) and (13) into (4), the AFD is attained.

VI. Numerical Results

The expressions derived above are general and can be applied to any type of diversity (space, frequency, polarization, time etc.). In this section, sample numerical results are discussed assuming a space-diversity system with horizontally spaced omnidirectional antennas at the mobile station, as sketched in Fig. 1. In this figure, d, v, and $0 \leq \alpha \leq \pi/2$ denote the antenna spacing, the vehicle speed, and the angle between the antenna axis and the direction of the vehicle motion indicated by the straight solid arrow, respectively.

$$N_R(r) = \begin{cases} \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} N_{R_1,R_2,\Theta_1,\Theta_2}(r,r_2,\theta_1,\theta_2) f_{R_1,R_2,\Theta_1,\Theta_2}(r,r_2,\theta_1,\theta_2) dr_2 d\theta_1 d\theta_2 & \\ + \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} N_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r,\theta_1,\theta_2) f_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r,\theta_1,\theta_2) dr_1 d\theta_1 d\theta_2 & \text{SC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{\sqrt{2}r} \sqrt{2} N_{R_1,R_2,\Theta_1,\Theta_2}(\sqrt{2}r - r_2,r_2,\theta_1,\theta_2) f_{R_1,R_2,\Theta_1,\Theta_2}(\sqrt{2}r - r_2,r_2,\theta_1,\theta_2) dr_2 d\theta_1 d\theta_2 & \text{EGC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \frac{r}{\sqrt{r^2-r_2^2}} N_{R_1,R_2,\Theta_1,\Theta_2}(\sqrt{r^2-r_2^2},r_2,\theta_1,\theta_2) f_{R_1,R_2,\Theta_1,\Theta_2}(\sqrt{r^2-r_2^2},r_2,\theta_1,\theta_2) dr_2 d\theta_1 d\theta_2 & \text{MRC} \end{cases} \tag{12a}$$

$$\begin{aligned} N_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) &\triangleq \int_0^{\infty} \dot{r} f_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1,r_2,\theta_1,\theta_2) d\dot{r} \\ &= \frac{\dot{\sigma}(\mathbf{z})}{\sqrt{2\pi}} \left\{ \exp\left[-\frac{1}{2}\left(\frac{\dot{m}(\mathbf{z})}{\dot{\sigma}(\mathbf{z})}\right)^2\right] + \sqrt{\frac{\pi}{2}}\frac{\dot{m}(\mathbf{z})}{\dot{\sigma}(\mathbf{z})}\operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\frac{\dot{m}(\mathbf{z})}{\dot{\sigma}(\mathbf{z})}\right)\right\} \end{aligned} \tag{12b}$$

$$F_R(r) = \begin{cases} \int_0^{2\pi}\int_0^{2\pi}\int_0^{r}\int_0^{r} f_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) dr_1 dr_2 d\theta_1 d\theta_2 & \text{SC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{\sqrt{2}r}\int_0^{\sqrt{2}r-r_2} f_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) dr_1 dr_2 d\theta_1 d\theta_2 & \text{EGC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{r}\int_0^{\sqrt{r^2-r_2^2}} f_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) dr_1 dr_2 d\theta_1 d\theta_2 & \text{MRC} \end{cases} \tag{13}$$

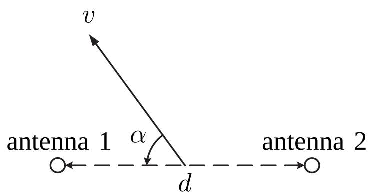

Fig. 1. Antenna configuration.

In this case, for isotropic scattering, it is known that [1]

$$\mu_a(\tau) = J_0(2\pi f_D \tau) \tag{14a}$$

$$\mu_c(\tau) = J_0\left(2\pi\sqrt{(f_D\tau)^2 + (d/\lambda)^2 + 2f_D\tau(d/\lambda)\cos\alpha}\right) \tag{14b}$$

$$\nu_a(\tau) = \nu_c(\tau) = 0 \tag{14c}$$

$$\mu_a = 1,\ \dot{\mu}_a = 0,\ \ddot{\mu}_a = -2(\pi f_D)^2 \tag{14d}$$

$$\mu_c = J_0\left(2\pi d/\lambda\right),\ \dot{\mu}_c = -2\pi f_D J_1\left(2\pi d/\lambda\right)\cos\alpha \tag{14e}$$

$$\ddot{\mu}_c = (2\pi f_D)^2\left[\frac{J_1\left(2\pi d/\lambda\right)}{2\pi d/\lambda}\cos 2\alpha - J_0\left(2\pi d/\lambda\right)\cos^2\alpha\right] \tag{14f}$$

$$\nu_a = \dot{\nu}_a = \ddot{\nu}_a = \nu_c = \dot{\nu}_c = \ddot{\nu}_c = 0 \tag{14g}$$

where $J_k(\cdot)$ is the Bessel Function of the first kind and kth order and $f_D = v/\lambda$ is the maximum Doppler shift in Hz for a carrier wavelength λ.

Figs. 2–4 show the LCR of SC, EGC, and MRC against the antenna spacing at level $r/\sqrt{\sigma_1^2+\sigma_2^2} = -20$dB, for $k_i = 0, 5$ and power imbalance factor $\delta \triangleq \sigma_2^2/\sigma_1^2 = 1, 9$. For simplicity, $m_{Y_1} = m_{Y_2} = 0$ is assumed. As expected, the introduction of a line-of-sight component reduces the LCR for all diversity schemes in comparison to the Rayleigh condition, by increasing the mean level of the input signals. For $k_i = 0$, as noted in [1], the LCR decreases as the antenna spacing increases, becoming oscillatory and convergent towards the independent-fading performance. Also note that the LCR increases with the antenna angle and with the power imbalance. A minimum LCR is achieved at $d/\lambda \approx 0.25$ for antennas arranged parallel to the direction of the vehicle motion ($\alpha = 0$). In contrast, for $k_i = 5$, the LCR is no more oscillatory and does not change monotonically with the antenna angle. Its minimum is achieved at a higher antenna spacing $d/\lambda \approx 0.6$ irrespective of α.

The corresponding AFD curves are shown in Figs. 5–7. Also here the performance is observed to improve as k_i increases. For $k_i = 0$, as noted in [1], the AFD increases as the antenna angle decreases and, when the antennas are perpendicular to direction of the vehicle motion ($\alpha = \pi/2$), the AFD is loosely dependent on the antenna spacing and almost identical to that of the independent-fading case, except for very small antenna spacings. On the other hand, note that for $k_i = 5$ the AFD can be reduced below the value obtained for independent fading signals when $\alpha = \pi/2$.

The effect of the power imbalance over the AFD for switched (SC) and addition (EGC and MRC) methods differ considerably. Though being deleterious for SC except for very small antenna spacings, the power imbalance is surprisingly observed to reduce the AFD for EGC and MRC when the antennas are parallel to the direction of the vehicle motion. Furthermore, when the antennas are perpendicular to the direction of the vehicle motion, the AFD curves are loosely dependent on the power imbalance for MRC and EGC but not for SC.

To the best of the authors' knowledge, the conclusions drawn from the above discussion are new.

VII. CONCLUSIONS

General exact expressions for the level crossing rate and average fade duration of dual-branch selection, equal-gain, and maximal-ratio combiners operating over non-identical correlated Ricean channels were derived. Sample numerical results were presented by specializing the general expressions to a space-diversity system with horizontally spaced omnidirectional antennas at the mobile station. When the antennas are perpendicular to the direction of the vehicle motion, the average fade duration is loosely dependent on the antenna

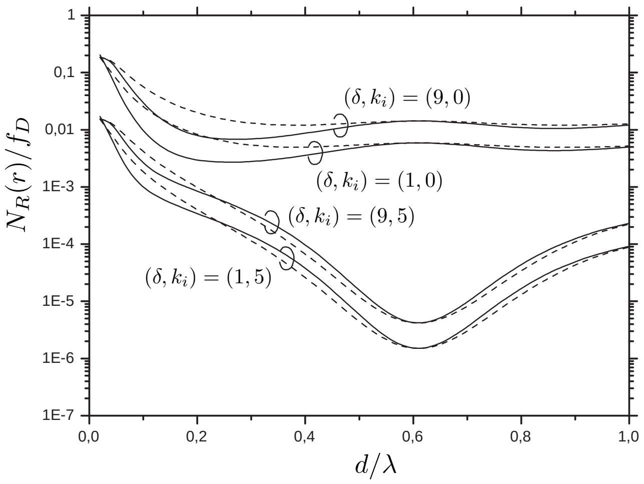

Fig. 2. LCR of SC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

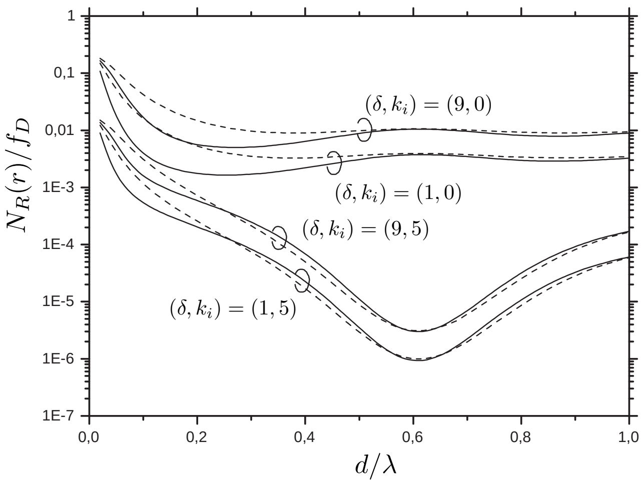

Fig. 3. LCR of EGC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

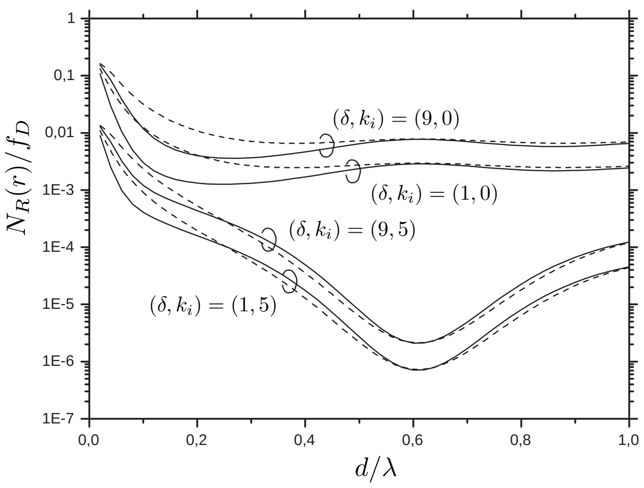

Fig. 4. LCR of MRC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

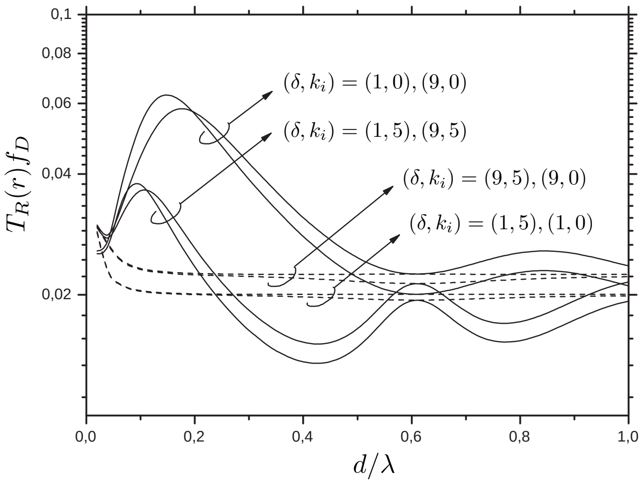

Fig. 5. AFD of SC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

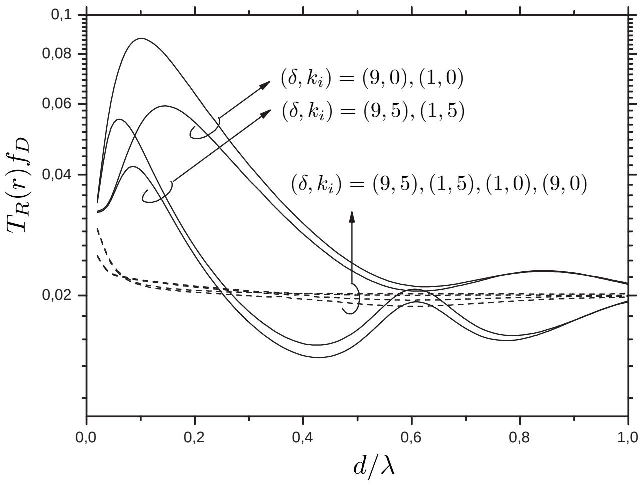

Fig. 6. AFD of EGC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

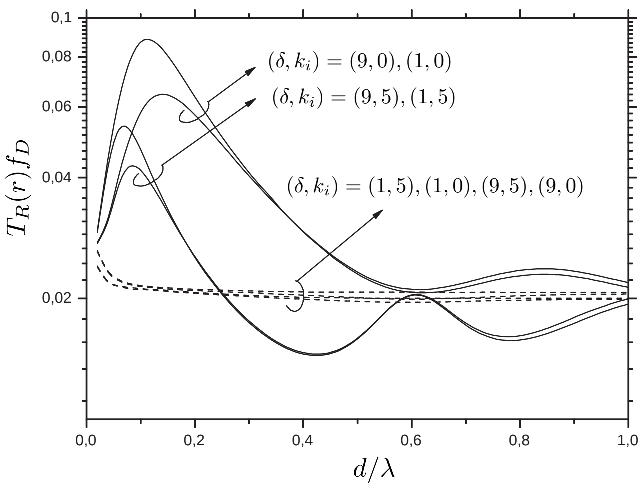

Fig. 7. AFD of MRC at $r/\sqrt{\sigma_1^2 + \sigma_2^2} = -20$dB (solid: $\alpha = 0$; dot: $\alpha = \pi/2$).

spacing and on the Ricean factor, being almost identical to that of the independent fading condition. When the antennas are parallel to the direction of the vehicle motion, the level crossing rate can be reduced below the value obtainable for independent fading signals. In this case, if a line-of-sight component exists, a further improvement in the average fade duration over the independent condition may be also achieved.

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APPENDIX

In this appendix, we derive the mean $\mathbf{M} = \mathrm{E}\{\dot{\mathbf{Z}}|\mathbf{Z}\}$ and covariance $\mathbf{\Lambda} = \mathrm{E}\{\dot{\mathbf{Z}}\dot{\mathbf{Z}}^T|\mathbf{Z}\} - \mathrm{E}\{\dot{\mathbf{Z}}|\mathbf{Z}\}\mathrm{E}\{\dot{\mathbf{Z}}^T|\mathbf{Z}\}$ matrices of $\dot{\mathbf{Z}} = [\dot{X}_1\ \dot{Y}_1\ \dot{X}_2\ \dot{Y}_2]^T$ conditioned on $\mathbf{Z} = [X_1\ Y_1\ X_2\ Y_2]^T$. To the best of the authors' knowledge, these results are new. Note that $\dot{X}_1$, $\dot{Y}_1$, $\dot{X}_2$, $\dot{Y}_2$, X_1, Y_1, X_2, and Y_2 are jointly correlated Gaussian variates and that $\mathrm{E}\{\dot{X}_i\} = \mathrm{E}\{\dot{Y}_i\} = 0$, $i \in \{1,2\}$. Correspondingly, from [7, pp. 495–496]

$$\begin{aligned} \mathbf{M} &= \mathbf{c}\mathbf{b}^{-1}(\mathbf{Z}-\mathbf{m}) \\ \mathbf{\Lambda} &= \mathbf{a} - \mathbf{c}\mathbf{b}^{-1}\mathbf{c}^T \end{aligned} \tag{15}$$

where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are the partitioned matrices of the covariance matrix of $[\dot{\mathbf{Z}}^T\ \mathbf{Z}^T]$

$$\begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c}^T & \mathbf{b} \end{bmatrix} = \mathrm{E}\left\{\begin{bmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{bmatrix}\begin{bmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{bmatrix}^T\right\} - \mathrm{E}\left\{\begin{bmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{bmatrix}\right\}\mathrm{E}\left\{\begin{bmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{bmatrix}^T\right\} \tag{16}$$

and $\mathbf{m} = \mathrm{E}\{\mathbf{Z}\}$ is the mean vector of $\mathbf{Z}$. In our case,

$$\mathbf{a} = \begin{bmatrix} -\sigma_1^2\ddot{\mu}_a & 0 & -\sigma_1\sigma_2\ddot{\mu}_c & -\sigma_1\sigma_2\ddot{\nu}_c \\ 0 & -\sigma_1^2\ddot{\mu}_a & \sigma_1\sigma_2\ddot{\nu}_c & -\sigma_1\sigma_2\ddot{\mu}_c \\ -\sigma_1\sigma_2\ddot{\mu}_c & \sigma_1\sigma_2\ddot{\nu}_c & -\sigma_2^2\ddot{\mu}_a & 0 \\ -\sigma_1\sigma_2\ddot{\nu}_c & -\sigma_1\sigma_2\ddot{\mu}_c & 0 & -\sigma_2^2\ddot{\mu}_a \end{bmatrix} \tag{17a}$$

$$\mathbf{b} = \begin{bmatrix} \sigma_1^2\mu_a & 0 & \sigma_1\sigma_2\mu_c & \sigma_1\sigma_2\nu_c \\ 0 & \sigma_1^2\mu_a & -\sigma_1\sigma_2\nu_c & \sigma_1\sigma_2\mu_c \\ \sigma_1\sigma_2\mu_c & -\sigma_1\sigma_2\nu_c & \sigma_2^2\mu_a & 0 \\ \sigma_1\sigma_2\nu_c & \sigma_1\sigma_2\mu_c & 0 & \sigma_2^2\mu_a \end{bmatrix} \tag{17b}$$

$$\mathbf{c} = \begin{bmatrix} 0 & \sigma_1^2\dot{\nu}_a & \sigma_1\sigma_2\dot{\mu}_c & \sigma_1\sigma_2\dot{\nu}_c \\ -\sigma_1^2\dot{\nu}_a & 0 & -\sigma_1\sigma_2\dot{\nu}_c & \sigma_1\sigma_2\dot{\mu}_c \\ -\sigma_1\sigma_2\dot{\mu}_c & \sigma_1\sigma_2\dot{\nu}_c & 0 & \sigma_2^2\dot{\nu}_a \\ -\sigma_1\sigma_2\dot{\nu}_c & -\sigma_1\sigma_2\dot{\mu}_c & -\sigma_2^2\dot{\nu}_a & 0 \end{bmatrix} \tag{17c}$$

$$\mathbf{m} = [m_{X_1}\ m_{Y_1}\ m_{X_2}\ m_{Y_2}]^T \tag{17d}$$

where

$$\mu_a(\tau) = \frac{\mathrm{Cov}\{X_i(t+\tau), X_i(t)\}}{\sigma_i^2} = \frac{\mathrm{Cov}\{Y_i(t+\tau), Y_i(t)\}}{\sigma_i^2} \tag{18a}$$

$$\nu_a(\tau) = \frac{\mathrm{Cov}\{X_i(t+\tau), Y_i(t)\}}{\sigma_i^2} \tag{18b}$$

$$\mu_c(\tau) = \frac{\mathrm{Cov}\{X_1(t+\tau), X_2(t)\}}{\sigma_1\sigma_2} = \frac{\mathrm{Cov}\{Y_1(t+\tau), Y_2(t)\}}{\sigma_1\sigma_2} \tag{18c}$$

$$\nu_c(\tau) = \frac{\mathrm{Cov}\{X_1(t+\tau), Y_2(t)\}}{\sigma_1\sigma_2} = -\frac{\mathrm{Cov}\{Y_1(t+\tau), X_2(t)\}}{\sigma_1\sigma_2} \tag{18d}$$

$i \in \{1,2\}$, $\dot{\xi}(\tau) = \frac{d}{d\tau}\xi(\tau)$, $\ddot{\xi}(\tau) = \frac{d^2}{d\tau^2}\dot{\xi}(\tau)$, $\xi = \xi(0)$, $\dot{\xi} = \dot{\xi}(0)$, and $\ddot{\xi} = \ddot{\xi}(0)$ ($\xi = \mu_a, \nu_a, \mu_c, \nu_c$). Note that $\dot{\mu}_a = 0$ from stationarity and $\nu_a = \ddot{\nu}_a = 0$ from the Jake's model [4]. These are the null terms in (17c), (17b), and (17a), respectively.

Replacing (17) into (15), it follows that

$$\mathbf{M} = \begin{bmatrix} M_1 & -M_2 & -\frac{\sigma_1}{\sigma_2}M_3 & -\frac{\sigma_1}{\sigma_2}M_4 \\ M_2 & M_1 & \frac{\sigma_1}{\sigma_2}M_4 & -\frac{\sigma_1}{\sigma_2}M_3 \\ \frac{\sigma_2}{\sigma_1}M_3 & -\frac{\sigma_1}{\sigma_2}M_4 & -M_1 & -M_2 \\ \frac{\sigma_1}{\sigma_2}M_4 & \frac{\sigma_1}{\sigma_2}M_3 & M_2 & -M_1 \end{bmatrix}\begin{bmatrix} X_1 - m_{X_1} \\ Y_1 - m_{Y_1} \\ X_2 - m_{X_2} \\ Y_2 - m_{Y_2} \end{bmatrix} \tag{19a}$$

$$\mathbf{\Lambda} = \begin{bmatrix} \sigma_1^2\Lambda_1 & 0 & \sigma_1\sigma_2\Lambda_3 & \sigma_1\sigma_2\Lambda_4 \\ 0 & \sigma_1^2\Lambda_1 & -\sigma_1\sigma_2\Lambda_4 & \sigma_1\sigma_2\Lambda_3 \\ \sigma_1\sigma_2\Lambda_3 & -\sigma_1\sigma_2\Lambda_4 & \sigma_2^2\Lambda_1 & 0 \\ \sigma_1\sigma_2\Lambda_4 & \sigma_1\sigma_2\Lambda_3 & 0 & \sigma_2^2\Lambda_1 \end{bmatrix} \tag{19b}$$

$$\eta^{-1} \triangleq \mu_c^2 + \nu_c^2 - \mu_a^2 \tag{19c}$$

$$M_1 \triangleq \eta\left(\mu_c\dot{\mu}_c + \nu_c\dot{\nu}_c\right) \tag{19d}$$

$$M_2 \triangleq \eta\left(\nu_c\dot{\mu}_c + \mu_a\dot{\nu}_a - \mu_c\dot{\nu}_c\right) \tag{19e}$$

$$M_3 \triangleq \eta\left(\mu_a\dot{\mu}_c + \nu_c\dot{\nu}_a\right) \tag{19f}$$

$$M_4 \triangleq \eta\left(\mu_a\dot{\nu}_c - \mu_c\dot{\nu}_a\right) \tag{19g}$$

$$\Lambda_1 \triangleq \eta\left[2\dot{\nu}_a\left(\nu_c\dot{\mu}_c - \mu_c\dot{\nu}_c\right) + \mu_a\left(\dot{\mu}_c^2 + \dot{\nu}_a^2 + \dot{\nu}_c^2\right)\right] - \ddot{\mu}_a \tag{19h}$$

$$\Lambda_3 \triangleq \eta\left[2\dot{\nu}_c\left(\nu_c\dot{\mu}_c - \mu_a\dot{\nu}_a\right) + \mu_c\left(\dot{\mu}_c^2 - \dot{\nu}_a^2 - \dot{\nu}_c^2\right)\right] - \ddot{\mu}_c \tag{19i}$$

$$\Lambda_4 \triangleq \eta\left[2\dot{\mu}_c\left(\mu_c\dot{\nu}_c - \mu_a\dot{\nu}_a\right) + \nu_c\left(\dot{\nu}_c^2 - \dot{\nu}_a^2 - \dot{\mu}_c^2\right)\right] - \ddot{\nu}_c \tag{19j}$$