Level Crossing Rates and Average Fade Durations for Diversity-Combining of Non-identical Correlated Hoyt Signals

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***Resumo*— Neste artigo, expressões exatas para a taxa de cruzamento de nível e tempo médio de desvanecimento para dois ramos utilizando combinação por seleção, ganho igual e máxima razão em ambiente com desvanecimento Hoyt são apresentadas. As expressões podem ser utilizadas para sistemas com diversidade em canais desbalanceados, não idênticos e correlacionados. A solução geral foi particularizada e validada para casos mais simples, onde já se conhecem os resultados. Como resultado intermediário, também foi encontrada a distribuição Hoyt bidimensional conjunta da fase e envoltória com parâmetros de desvanecimento arbitrários.**

***Palavras-Chave*— Tempo médio de desvanecimento, taxa de cruzamento de nível, combinação por seleção, combinação por ganho igual, combinação por razão máxima, desvanecimento Hoyt.**

***Abstract*— In this paper, exact expressions for the level crossing rate (LCR) and average fade duration (AFD) for two-branch selection, equal-gain and maximal-ratio combining systems in a Hoyt fading environment are presented. The expressions apply to unbalanced, non-identical, correlated diversity channels and have been validated by specializing the general results to some particular cases whose solutions are known. In passing, the joint bidimensional envelope-phase Hoyt distribution with arbitrary fading parameters is obtained.**

***Keywords*— Average fade duration, selection combining, equal-gain combining, maximal-ratio combining, Hoyt fading channels, level crossing rate.**

I. INTRODUCTION

Level crossing rate (LCR) and average fade duration (AFD) are widely-used performance measures of wireless diversity systems. However, although the branch signals may be correlated and non-identically distributed in practical systems [1]–[4], the literature on LCR and AFD of diversity techniques over non-identical correlated fading is rather scarce. Pioneering work on this issue was carried out by Adachi *et al.* [1] for dual branch selection (SC), equal-gain (EGC), and maximal ratio combining (MRC) over *balanced* correlated Rayleigh channels. The unbalanced correlated Rayleigh case was addressed in [2] for two-, three-, and four-branch MRC. More recently, [3] presented a unified treatment for the LCR and the AFD of M-branch SC over unbalanced correlated Rayleigh, Ricean, and Nakagami-m channels. In [4], the LCR and AFD for the MRC were derived for a correlated, unbalanced Nakagami environment.

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This paper fully generalizes the approach used in [1], and provides expressions for the LCR and AFD of dual-branch SC, EGC, and MRC operating over *non-identical, correlated* Hoyt (Nakagami-q) channels.

This work is organized as follows: Section II derives the Joint Bidimensional Envelope-Phase Hoyt distribution; Section III presents general expressions for LCR and AFD of the combining output; Section IV-A derives the matrices for the conditional joint distribution; Section IV-B computes the means and variances for each diversity system; Section V shows some numerical plots, and finally Section VI draws some conclusions.

II. THE JOINT BIDIMENSIONAL ENVELOPE-PHASE HOYT DISTRIBUTION

In a Hoyt fading environment, the received signal at the i-th antenna ($i = 1, 2$), can be represented in a complex form as

$$X_i(t) + jY_i(t) = R_i(t)\exp(j\,\Theta_i(t)) \tag{1}$$

where $X_i(t)$, $Y_i(t)$ are zero mean independent Gaussian processes with variance $\sigma^2_{X_i}$ and $\sigma^2_{Y_i}$, respectively. The variates $R_i(t)$ and $\Theta_i(t)$ follow the envelope and phase of the Hoyt distribution [5], respectively.

We now proceed to determine the joint distribution of $X_i \triangleq X_i(t)$, $Y_i \triangleq Y_i(t)$, $(i = 1, 2)$. Defining the vector $\mathbf{Z} = [X_1 Y_1 X_2 Y_2] = [R_1\cos(\Theta_1)\; R_1\sin(\Theta_1)\; R_2\cos(\Theta_2)\; R_2\sin(\Theta_2)]$, the joint Gaussian distribution $p_{\mathbf{Z}}(\mathbf{z})$ can be written as [6]

$$p_{\mathbf{Z}}(\mathbf{z}) = \frac{1}{(2\pi)^2(\det\mathbf{b})^{1/2}}\exp\left(-\frac{1}{2}\mathbf{z}\mathbf{b}^{-1}\mathbf{z}^T\right) \tag{2}$$

where:

(i) $[\cdot]^T$ denotes the transpose matrix, $\mathbf{b}$ is the covariance matrix given by

$$\mathbf{b} = \begin{bmatrix} \sigma^2_{X_1} & 0 & \sigma_{X_1}\sigma_{X_2}\,\mu_{12} & -\sigma_{X_1}\sigma_{Y_2}\,\eta_{12} \\ 0 & \sigma^2_{Y_1} & \sigma_{Y_1}\sigma_{X_2}\,\eta_{12} & \sigma_{Y_1}\sigma_{Y_2}\,\mu_{12} \\ \sigma_{X_1}\sigma_{X_2}\,\mu_{12} & \sigma_{Y_1}\sigma_{X_2}\,\eta_{12} & \sigma^2_{X_2} & 0 \\ -\sigma_{X_1}\sigma_{Y_2}\,\eta_{12} & \sigma_{Y_1}\sigma_{Y_2}\,\mu_{12} & 0 & \sigma^2_{Y_2} \end{bmatrix} \tag{3}$$

(ii) $\mu_{ij} = \mu_{ij}(0)$ and $\eta_{ij} = \eta_{ij}(0)$ are the correlation coefficients, defined as

$$\mu_{ij}(\tau) = \frac{\text{Cov}\left(X_i(t), X_j(t+\tau)\right)}{\sqrt{\text{Var}\left(X_i(t)\right)\text{Var}\left(X_j(t+\tau)\right)}} = \frac{\text{Cov}\left(Y_i(t), Y_j(t+\tau)\right)}{\sqrt{\text{Var}\left(Y_i(t)\right)\text{Var}\left(Y_j(t+\tau)\right)}} \quad (i \leqslant j) \tag{4}$$

and

$$\eta_{ij}(\tau) = -\frac{\text{Cov}(X_i(t), Y_j(t+\tau))}{\sqrt{\text{Var}\left(X_i(t)\right)\text{Var}\left(Y_j(t+\tau)\right)}} = \frac{\text{cov}(Y_i(t), X_j(t+\tau))}{\sqrt{\text{Var}\left(Y_i(t)\right)\text{Var}\left(X_j(t+\tau)\right)}} \quad (i \neq j) \tag{5}$$

and

$$\eta_{ii}(\tau) = \frac{\text{Cov}(X_i(t), Y_i(t+\tau))}{\sqrt{\text{Var}\left(X_i(t)\right)\text{Var}\left(Y_i(t+\tau)\right)}} \quad (i = 1, 2) \tag{6}$$

$\text{Var}(\cdot)$ and $\text{Cov}(\cdot)$ are the variance and covariance operators, respectively. For Gaussian processes, the following relations are valid: $\mu_{22} = \mu_{11}$, $\mu_{21} = \mu_{12}$, $\eta_{21} = -\eta_{12}$, and $\eta_{22} = \eta_{11}$ [7]. The joint density $p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2)$ can be written as $p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2) = |J|\, p_{\mathbf{Z}}(\mathbf{z})$, where $|J| = r_1 r_2$ is the Jacobian of the transformation. Accordingly, the joint bidimensional envelope-phase Hoyt distribution, as derived here, is given by

$$p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2) = \tag{7}$$

$$\frac{r_1\, r_2\, e^{-\frac{1}{2(1-\rho^2)}\left[r_1^2\left(\frac{\sin^2(\theta_1)}{\sigma_{Y_1}^2} + \frac{\cos^2(\theta_1)}{\sigma_{X_1}^2}\right)\right]}}{4\pi^2\,(1-\rho^2)\,\sigma_{X_1}\sigma_{Y_1}\sigma_{X_2}\sigma_{Y_2}}$$

$$\times e^{-\frac{r_2^2}{2(1-\rho^2)}\left(\frac{\sin^2(\theta_2)}{\sigma_{Y_2}^2} + \frac{\cos^2(\theta_2)}{\sigma_{X_2}^2}\right)} \tag{8}$$

$$e^{\frac{r_1\, r_2 \mu_{12}}{(1-\rho^2)}\left(\frac{\sin(\theta_1)\sin(\theta_2)}{\sigma_{Y_1}\sigma_{Y_2}} + \frac{\cos(\theta_1)\cos(\theta_2)}{\sigma_{X_1}\sigma_{X_2}}\right)} \tag{9}$$

$$e^{\frac{r_1\, r_2 \eta_{12}}{(1-\rho^2)}\left(\frac{\sin(\theta_1)\cos(\theta_2)}{\sigma_{Y_1}\sigma_{X_2}} + \frac{\cos(\theta_1)\sin(\theta_2)}{\sigma_{X_1}\sigma_{Y_2}}\right)} \tag{10}$$

where

$$\rho^2 = \mu_{12}^2 + \eta_{12}^2 \tag{11}$$

III. LCR AND AFD

The LCR $n_R(r)$ and AFD $T_R(r)$ of a random signal are defined, respectively, as the average number of upward (or downward) crossings per second at a given level and as the mean time the signal remains below this level after crossing it in the downward direction. The LCR and AFD of the combiner output $R = R(t)$ at level r are, respectively, given by [8]

$$n_R(r) = \int_0^\infty \dot{r} p_{R,\dot{R}}(r, \dot{r}) d\dot{r} \tag{12}$$

$$T_R(r) = \frac{P_R(r)}{n_R(r)} \tag{13}$$

where $p_{R,\dot{R}}(\cdot,\cdot)$ is the joint probability density function (JPDF) of R and its time derivative $\dot{R}$, and $P_R(\cdot)$ is the cumulative distribution function (CDF) of R. In the following, (12) and (13) shall be calculated for the dual-branch, correlated, non-identical Hoyt fading environment using the SC, EGC and MRC techniques.

A. Diversity Systems

The output envelope and its time derivative for the SC, EGC and MRC combining systems are given, respectively, by

$$R = \begin{cases} \max\{R_1, R_2\} & \text{SC} \\ \frac{R_1+R_2}{\sqrt{2}} & \text{EGC} \\ \sqrt{R_1^2 + R_2^2} & \text{MRC} \end{cases} \quad \dot{R} = \begin{cases} \dot{R}_1 \quad R_1 \geqslant R_2 & \multirow{2}{*}{SC} \\ \dot{R}_2 \quad R_1 < R_2 & \\ \frac{\dot{R}_1+\dot{R}_2}{\sqrt{2}} & \text{EGC} \\ \frac{R_1\dot{R}_1+R_2\dot{R}_2}{\sqrt{R_1^2+R_2^2}} & \text{MRC} \end{cases} \tag{14}$$

Given that $\dot{R}_i$ is zero-mean Gaussian [9], it is clear from (14) that the conditional density $p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2)$ is also Gaussian with means $m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)$ and variance $\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2)$. Of course, these quantities depend on the combining scheme and shall be determined latter. Now

$$p_{\dot{R},R_1,R_2,\Theta_1,\Theta_2}(\dot{r}, r_1, r_2, \theta_1, \theta_2) = p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2) p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2) \tag{15}$$

Knowing $p_{\dot{R},R_1,R_2,\Theta_1,\Theta_2}(\dot{r}, r_1, r_2, \theta_1, \theta_2)$, as in (15), and the relations of $R_1, R_2, \dot{R}_1, \dot{R}_2$, as in (14), the joint density $p_{\dot{R},R}(\dot{r}, r)$ can be obtained to be used in (12). The kernel of the problem now turns out to be the estimation of

$$p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi}\,\sigma_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)} \times \exp\left(-\frac{(\dot{r} - m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2))^2}{2\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2)}\right) \tag{16}$$

More specifically, the tricky part of the problem is the determination of $m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)$ and $\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2)$ for each combining scheme. For the moment, assume that these quantities are known. Then, by means of [1, Eq. 8] for SC and of [9, Eqs. (12) and (17)] for EGC and MRC, respectively, the LCR can computed as in (17).

Now, with (16) in (18)

$$\vartheta(r_1, r_2) = \frac{\sigma_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)}{\sqrt{2\pi}} \exp\left(-\frac{m_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2)}{2\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2)}\right) + \frac{m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)}{2}\left(1 + \text{erf}\left(\frac{m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)}{\sqrt{2}\sigma_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)}\right)\right) \tag{19}$$

where $\text{erf}(\cdot)$ is the error function. The CDF $P_R(r)$ can be obtained as [10]

$$P_R(r) = \int_0^{\gamma_1}\int_0^{\gamma_2}\int_0^{2\pi}\int_0^{2\pi} \times p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r_2, \theta_1, \theta_2)\, d\theta_1 d\theta_2 dr_2 dr_1 \tag{20}$$

with

$$\begin{cases} \gamma_1 = \gamma_2 = r \quad \text{for SC} \\ \gamma_1 = \sqrt{2}r \quad \gamma_2 = \sqrt{2}r - r_1 \quad \text{for EGC} \\ \gamma_1 = r \quad \gamma_2 = \sqrt{r^2 - r_1^2} \quad \text{for MRC} \end{cases} \tag{21}$$

The AFD follows directly from (13), (17), and (20).

$$n_R(r) = \begin{cases} \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \vartheta(r_1, r) p_{R_1,R_2,\Theta_1,\Theta_2}(r_1, r, \theta_1, \theta_2)\, dr_1 d\theta_1 d\theta_2 \\ \quad + \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \vartheta(r, r_2) p_{R_1,R_2,\Theta_1,\Theta_2}(r, r_2, \theta_1, \theta_2)\, dr_2 d\theta_1 d\theta_2 & \text{SC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{\sqrt{2}r} \sqrt{2}\,\vartheta(r_1, \sqrt{2}r - r_1) p_{R_1,R_2,\Theta_1,\Theta_2}\left(r_1, \sqrt{2}r - r_1, \theta_1, \theta_2\right) dr_1 d\theta_1 d\theta_2 & \text{EGC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \frac{r}{\sqrt{r^2 - r_1^2}}\,\vartheta(r_1, \sqrt{r^2 - r_1^2}) p_{R_1,R_2,\Theta_1,\Theta_2}\left(r_1, \sqrt{r^2 - r_1^2}, \theta_1, \theta_2\right) dr_1 d\theta_1 d\theta_2 & \text{MRC} \end{cases} \tag{17}$$

in which

$$\vartheta(r_1, r_2) \triangleq \int_0^{\infty} \dot{r}\, p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2)\, d\dot{r} \tag{18}$$

IV. CONDITIONAL STATISTICS OF $\dot{R}$

The aim of this section is to find the mean $m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)$ and the variance $\sigma_{\dot{R}}(r_1, r_2, \theta_1, \theta_2)$ of the conditional Gaussian distribution $p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2)$ for each combining technique.

A. Preliminaries

From (1), it follows that $\dot{R}_i = \cos(\theta_i)\dot{X}_i + \sin(\theta_i)\dot{Y}_i$. Then the expressions in (22), (23), and (24) follow directly. Note that from (22) and (23) the mean and the variance of $\dot{R}_i$ given $\mathbf{Z}$ can be computed as a function of the mean, the variance, and the covariance of $\dot{X}_i$ and $\dot{Y}_i$ given $\mathbf{Z}$. In order to obtain these statistics, the following will be carried out: 1) Defining $\dot{\mathbf{Z}} = [\dot{X}_1\, \dot{Y}_1\, \dot{X}_2\, \dot{Y}_2]$, the multivariate Gaussian distribution $p_{\{\dot{\mathbf{Z}}\,\mathbf{Z}\}}(\{\dot{\mathbf{z}}\,\mathbf{z}\})$ [1] will be determined; 2) Using the result of [6], the mean matrix $\mathbf{M}$ and the covariance matrix $\boldsymbol{\Delta}$ of the conditional Gaussian distribution $p_{\dot{\mathbf{Z}}|\mathbf{Z}}(\dot{\mathbf{z}}|\mathbf{z})$ will be obtained; 3) Using these matrices, the mean, the variance and the covariance of $\dot{X}_i$ and $\dot{Y}_i$ given $\mathbf{Z}$ will be attained; 4) Finally, using (22) and (23), and (24) the mean and the variance of $\dot{R}$ given $\mathbf{Z}$ will be found.

In order to determine $p_{\{\dot{\mathbf{Z}}\,\mathbf{Z}\}}(\{\dot{\mathbf{z}}\,\mathbf{z}\})$, the joint multidimensional Gaussian distribution given in (2) will be used with the covariance matrix given by

$$\begin{aligned} \boldsymbol{\Lambda} &= E\left[\{\dot{\mathbf{Z}}\,\mathbf{Z}\}^T\{\dot{\mathbf{Z}}\,\mathbf{Z}\}\right] - E\left[\{\dot{\mathbf{Z}}\,\mathbf{Z}\}^T\right] E\left[\{\dot{\mathbf{Z}}\,\mathbf{Z}\}\right] \\ &= \begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c}^T & \mathbf{b} \end{bmatrix} \end{aligned}$$

From [11, Eq. 9.106], the following relations are valid: $\mathrm{E}[\mathrm{P}(t)\dot{\mathrm{P}}(t+\tau)] = \frac{\mathrm{d}E[\mathrm{P}(t)\mathrm{P}(t+\tau)]}{\mathrm{d}\tau}$, $\mathrm{E}[\dot{\mathrm{P}}(t)\mathrm{P}(t+\tau)] = -\frac{\mathrm{d}E[\mathrm{P}(t)\mathrm{P}(t+\tau)]}{\mathrm{d}\tau}$, $\mathrm{E}[\dot{\mathrm{P}}(t)\dot{\mathrm{P}}(t+\tau)] = -\frac{\mathrm{d}^2E[\mathrm{P}(t)\mathrm{P}(t+\tau)]}{\mathrm{d}\tau^2}$. Now we define $\dot{\mu}_{ij} = \left.\frac{\mathrm{d}\mu_{ij}(\tau)}{\mathrm{d}\tau}\right|_{\tau=0}$, $\ddot{\mu}_{ij} = \left.\frac{\mathrm{d}^2\mu_{ij}(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=0}$, $\dot{\eta}_{ij} = \left.\frac{\mathrm{d}\eta_{ij}(\tau)}{\mathrm{d}\tau}\right|_{\tau=0}$, $\ddot{\eta}_{ij} = \left.\frac{\mathrm{d}^2\eta_{ij}(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=0}$, where $\mu_{ij}(\tau)$ and $\eta_{ij}(\tau)$ are given by (4), (5), and (6), respectively. Then

$$\mathbf{a} = \begin{bmatrix} -\ddot{\mu}_{11}\sigma_{X_1}^2 & 0 & -\ddot{\mu}_{12}\sigma_{X_1}\sigma_{X_2} & \ddot{\eta}_{12}\sigma_{X_1}\sigma_{Y_2} \\ 0 & -\ddot{\mu}_{11}\sigma_{Y_1}^2 & -\ddot{\eta}_{12}\sigma_{Y_1}\sigma_{X_2} & -\ddot{\mu}_{12}\sigma_{Y_1}\sigma_{Y_2} \\ -\ddot{\mu}_{12}\sigma_{X_1}\sigma_{X_2} & -\ddot{\eta}_{12}\sigma_{Y_1}\sigma_{X_2} & -\ddot{\mu}_{11}\sigma_{X_2}^2 & 0 \\ \ddot{\eta}_{12}\sigma_{X_1}\sigma_{Y_2} & -\ddot{\mu}_{12}\sigma_{Y_1}\sigma_{Y_2} & 0 & -\ddot{\mu}_{11}\sigma_{Y_2}^2 \end{bmatrix} \tag{25}$$

$$\mathbf{c} = \begin{bmatrix} 0 & -\dot{\eta}_{11}\sigma_{X_1}\sigma_{Y_1} & -\dot{\mu}_{12}\sigma_{X_1}\sigma_{X_2} & \dot{\eta}_{12}\sigma_{X_1}\sigma_{Y_2} \\ \dot{\eta}_{11}\sigma_{X_1}\sigma_{Y_1} & 0 & -\dot{\eta}_{12}\sigma_{Y_1}\sigma_{X_2} & -\dot{\mu}_{12}\sigma_{Y_1}\sigma_{Y_2} \\ \dot{\mu}_{12}\sigma_{X_1}\sigma_{X_2} & \dot{\eta}_{12}\sigma_{Y_1}\sigma_{X_2} & 0 & -\dot{\eta}_{11}\sigma_{X_2}\sigma_{Y_2} \\ -\dot{\eta}_{12}\sigma_{X_1}\sigma_{Y_2} & \dot{\mu}_{12}\sigma_{Y_1}\sigma_{Y_2} & \dot{\eta}_{11}\sigma_{X_2}\sigma_{Y_2} & 0 \end{bmatrix} \tag{26}$$

and the matrix $\mathbf{b}$ is given in (3). Note that the diagonal elements in the matrix $\mathbf{c}$ are all null, because for a stationary process the correlation coefficient between the process and its time derivative is always null at $\tau = 0$ ($\dot{\mu}_{11} = 0$) [11].

Using the results from [6, pp. 495-496], the conditional distribution of $\dot{\mathbf{Z}}$ given $\mathbf{Z}$ [2], $p_{\dot{\mathbf{Z}}|\mathbf{Z}}(\dot{\mathbf{z}}|\mathbf{z})$, is Gaussian distributed with the mean matrix $\mathbf{M}$ and the covariance matrix $\boldsymbol{\Delta}$, respectively, given by (27) and (28). Using this and after a tedious procedure, the matrices obtained are given by

$$\mathbf{M} = \begin{bmatrix} m_1 x_1 + m_2\frac{\sigma_{X_1}}{\sigma_{Y_1}}y_1 + m_3\frac{\sigma_{X_1}}{\sigma_{X_2}}x_2 + m_4\frac{\sigma_{X_1}}{\sigma_{Y_2}}y_2 \\ -m_2\frac{\sigma_{Y_1}}{\sigma_{X_1}}x_1 + m_1 y_1 - m_4\frac{\sigma_{Y_1}}{\sigma_{X_2}}x_2 + m_3\frac{\sigma_{Y_1}}{\sigma_{Y_2}}y_2 \\ -m_3\frac{\sigma_{X_2}}{\sigma_{X_1}}x_1 + m_4\frac{\sigma_{X_2}}{\sigma_{Y_1}}y_1 - m_1 x_2 + m_2\frac{\sigma_{X_2}}{\sigma_{Y_2}}y_2 \\ -m_4\frac{\sigma_{Y_2}}{\sigma_{X_1}}x_1 - m_3\frac{\sigma_{Y_2}}{\sigma_{Y_1}}y_1 - m_2\frac{\sigma_{Y_2}}{\sigma_{X_2}}x_2 - m_1 y_2 \end{bmatrix} \tag{29}$$

$$\boldsymbol{\Delta} = -\begin{bmatrix} \sigma_{X_1}^2\Delta_1 & 0 & \sigma_{X_1}\sigma_{X_2}\Delta_2 & -\sigma_{X_1}\sigma_{Y_2}\Delta_3 \\ 0 & \sigma_{Y_1}^2\Delta_1 & \sigma_{X_2}\sigma_{Y_1}\Delta_3 & \sigma_{Y_1}\sigma_{Y_2}\Delta_2 \\ \sigma_{X_1}\sigma_{X_2}\Delta_2 & \sigma_{X_2}\sigma_{Y_1}\Delta_3 & \sigma_{X_2}^2\Delta_1 & 0 \\ -\sigma_{X_1}\sigma_{Y_2}\Delta_3 & \sigma_{Y_1}\sigma_{Y_2}\Delta_2 & 0 & \sigma_{Y_2}^2\Delta_1 \end{bmatrix} \tag{30}$$

where $m_1 = \frac{\mu_{12}\,\dot{\mu}_{12} + \eta_{12}\,\dot{\eta}_{12}}{1-\rho^2}$, $m_2 = \frac{\eta_{12}\dot{\mu}_{12} - \mu_{12}\,\dot{\eta}_{12} - \dot{\eta}_{11}}{1-\rho^2}$, $m_3 = \frac{\dot{\eta}_{11}\eta_{12} - \dot{\mu}_{12}}{1-\rho^2}$, $m_4 = \frac{\dot{\eta}_{12} + \dot{\eta}_{11}\mu_{12}}{1-\rho^2}$, $\Delta_1 = \ddot{\mu}_{11} + \frac{\dot{\mu}_{12}^2 + \dot{\eta}_{12}^2 + \dot{\eta}_{11}^2 + 2\dot{\eta}_{11}(\mu_{12}\,\dot{\eta}_{12} - \dot{\mu}_{12}\,\eta_{12})}{1-\rho^2}$, $\Delta_2 = \ddot{\mu}_{12} + \frac{2\,\dot{\eta}_{12}(\eta_{12}\,\dot{\mu}_{12} - \dot{\eta}_{11}) - \mu_{12}\left(\dot{\mu}_{12}^2 - \dot{\eta}_{11}^2 - \dot{\eta}_{12}^2\right)}{1-\rho^2}$, $\Delta_3 = \ddot{\eta}_{12} + \frac{2\,\dot{\mu}_{12}(\dot{\eta}_{11} + \mu_{12}\,\dot{\eta}_{12}) + \eta_{12}\left(\dot{\eta}_{12}^2 - \dot{\eta}_{11}^2 - \dot{\mu}_{12}^2\right)}{1-\rho^2}$, and ρ is given in (11).

B. Mean and variance of $\dot{R}_i s$

Using (14), the conditional means and variances for each combining scheme can be obtained as:

1) Selection Combining:

- If $R_1 \geqslant R_2$

$$m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2) = m_{\dot{R}_1}(r_1, r_2, \theta_1, \theta_2) \tag{31}$$
$$\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2) = \sigma_{\dot{R}_1}^2(r_1, r_2, \theta_1, \theta_2) \tag{32}$$

- If $R_1 < R_2$

$$m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2) = m_{\dot{R}_2}(r_1, r_2, \theta_1, \theta_2) \tag{33}$$
$$\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2) = \sigma_{\dot{R}_2}^2(r_1, r_2, \theta_1, \theta_2) \tag{34}$$

[1] $\{\dot{\mathbf{Z}}\,\mathbf{Z}\} = \left[\dot{X}_1\dot{Y}_1\dot{X}_2\dot{Y}_2X_1Y_1X_2Y_2\right]$

[2] Note that to provide the information about the variates $\mathbf{Z} = [X_1Y_1X_2Y_2]$ is the same as to provide the information about the variates $[R_1\Theta_1R_2\Theta_2]$.

$$m_{\dot{R}_i}(r_1,r_2,\theta_1,\theta_2) \triangleq \mathrm{E}\left[\dot{R}_i|\mathbf{Z}\right] = \cos(\theta_i)\,\mathrm{E}\left[\dot{X}_i|\mathbf{Z}\right] + \sin(\theta_i)\,\mathrm{E}\left[\dot{Y}_i|\mathbf{Z}\right] \tag{22}$$

$$\sigma^2_{\dot{R}_i}(r_1,r_2,\theta_1,\theta_2) \triangleq \mathrm{Var}\left(\dot{R}_i|\mathbf{Z}\right) = \cos^2(\theta_i)\,\mathrm{Var}\left[\dot{X}_i|\mathbf{Z}\right] + \sin^2(\theta_i)\,\mathrm{Var}\left[\dot{Y}_i|\mathbf{Z}\right] + \sin(2\theta_i)\,\mathrm{Cov}\left[\dot{X}_i,\dot{Y}_i|\mathbf{Z}\right] \tag{23}$$

$$\begin{aligned}\sigma_{\dot{R}_1,\dot{R}_2}(r_1,r_2,\theta_1,\theta_2) &\triangleq \mathrm{Cov}\left(\dot{R}_1,\dot{R}_2|\mathbf{Z}\right) = \mathrm{E}\left[\dot{R}_1\,\dot{R}_2|\mathbf{Z}\right] - \mathrm{E}\left[\dot{R}_1|\mathbf{Z}\right]\mathrm{E}\left[\dot{R}_2|\mathbf{Z}\right] = \\ &= \cos(\theta_1)\left[\cos(\theta_2)\,\mathrm{Cov}\left(\dot{X}_1,\dot{X}_2|\mathbf{Z}\right) + \sin(\theta_2)\,\mathrm{Cov}\left(\dot{X}_1,\dot{Y}_2|\mathbf{Z}\right)\right] + \\ &\quad + \sin(\theta_1)\left[\cos(\theta_2)\,\mathrm{Cov}\left(\dot{Y}_1,\dot{X}_2|\mathbf{Z}\right) + \sin(\theta_2)\,\mathrm{Cov}\left(\dot{Y}_1,\dot{Y}_2|\mathbf{Z}\right)\right]\end{aligned} \tag{24}$$

$$\mathbf{M} = \begin{bmatrix} \mathrm{E}\left[\dot{X}_1|\mathbf{Z}\right] \\ \mathrm{E}\left[\dot{Y}_1|\mathbf{Z}\right] \\ \mathrm{E}\left[\dot{X}_2|\mathbf{Z}\right] \\ \mathrm{E}\left[\dot{Y}_2|\mathbf{Z}\right] \end{bmatrix} = \left(\mathbf{cb^{-1}}\right)\mathbf{Z} \tag{27}$$

$$\mathbf{\Delta} = \begin{bmatrix} \mathrm{Var}\left(\dot{X}_1|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{X}_1,\dot{Y}_1|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{X}_1,\dot{X}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{X}_1,\dot{Y}_2|\mathbf{Z}\right) \\ \mathrm{Cov}\left(\dot{X}_1,\dot{Y}_1|\mathbf{Z}\right) & \mathrm{Var}\left(\dot{Y}_1|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{Y}_1,\dot{X}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{Y}_1,\dot{Y}_2|\mathbf{Z}\right) \\ \mathrm{Cov}\left(\dot{X}_1,\dot{X}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{Y}_1,\dot{X}_2|\mathbf{Z}\right) & \mathrm{Var}\left(\dot{X}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{X}_2,\dot{Y}_2|\mathbf{Z}\right) \\ \mathrm{Cov}\left(\dot{X}_1,\dot{Y}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{Y}_1,\dot{Y}_2|\mathbf{Z}\right) & \mathrm{Cov}\left(\dot{X}_2,\dot{Y}_2|\mathbf{Z}\right) & \mathrm{Var}\left(\dot{Y}_2|\mathbf{Z}\right) \end{bmatrix} = \mathbf{a} - \mathbf{cb}^{-1}\mathbf{c}^T \tag{28}$$

2) Equal Gain Combining:

$$\begin{aligned} m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2) &= \frac{1}{\sqrt{2}}\left\{m_{\dot{R}_1}(r_1,r_2,\theta_1,\theta_2)\right. \\ &\quad \left.+\, m_{\dot{R}_2}(r_1,r_2,\theta_1,\theta_2)\right\} \end{aligned} \tag{35}$$

$$\begin{aligned} \sigma^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2) &= \frac{1}{2}\left\{\sigma^2_{\dot{R}_1}(r_1,r_2,\theta_1,\theta_2)\right. \\ &\quad +\sigma^2_{\dot{R}_2}(r_1,r_2,\theta_1,\theta_2) \\ &\quad \left.+\, 2\sigma_{\dot{R}_1,\dot{R}_2}(r_1,r_2,\theta_1,\theta_2)\right\} \end{aligned} \tag{36}$$

3) Maximal Ratio Combining:

$$\begin{aligned} m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2) &= \frac{1}{\sqrt{r_1^2+r_2^2}}\left\{r_1 m_{\dot{R}_1}(r_1,r_2,\theta_1,\theta_2)\right. \\ &\quad \left.+\, r_2 m_{\dot{R}_2}(r_1,r_2,\theta_1,\theta_2)\right\} \end{aligned} \tag{37}$$

$$\begin{aligned} \sigma^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2) &= \frac{1}{r_1^2+r_2^2}\left\{r_1^2\sigma^2_{\dot{R}_1}(r_1,r_2,\theta_1,\theta_2)\right. \\ &\quad r_2^2\sigma^2_{\dot{R}_2}(r_1,r_2,\theta_1,\theta_2) \\ &\quad \left.+\, 2\,r_1 r_2\sigma_{\dot{R}_1,\dot{R}_2}(r_1,r_2,\theta_1,\theta_2)\right\} \end{aligned} \tag{38}$$

C. Special Cases

For the Rayleigh case, $\sigma_{X_i} = \sigma_{Y_i} = \sigma$ $(i = 1, 2)$, then equations from (31) to (38) reduce in a *exact manner* to those of [1, Eqs. 26 and 27]. In the case of branch independence (e.g. large separation between the antennas) the mean and variance are not functions of R_1, R_2, Θ_1, and Θ_2 because $\mu_{12} = \dot{\mu}_{12} = \ddot{\mu}_{12} = \eta_{12} = \dot{\eta}_{11} = \dot{\eta}_{12} = \ddot{\eta}_{12} = 0$. Then the results coincide with those of [9] for EGC and MRC with the number of branches $M = 2$.

V. Numerical Results

The expressions obtained for the LCR and AFD are general and can be applied to any type of diversity (space, frequency or time). In this section, we assume space diversity at the mobile station as [1]. For incoming multipath waves having equal amplitude and independent phases, the crosscorrelation functions are given by [7], [12]

$$\mu_{11}(\tau) = \frac{J_0(2\pi f_m\tau)}{1+\left(\Delta\omega\overline{T}\right)^2} \tag{39}$$

$$\mu_{12}(\tau) = \frac{J_0\left(2\pi\sqrt{(f_m\tau)^2 + (d/\lambda)^2 - 2\,(f_m\tau)\,(d/\lambda)\cos(\alpha)}\right)}{1+\left(\Delta\omega\overline{T}\right)^2} \tag{40}$$

$$\eta_{11}(\tau) = \Delta\omega\overline{T}\mu_{11}(\tau) \tag{41}$$

$$\eta_{12}(\tau) = \Delta\omega\overline{T}\mu_{12}(\tau) \tag{42}$$

where $J_0(\cdot)$ is the zero-order Bessel function, λ is the carrier wavelength, f_m is the maximum Doppler shift in Hz, d is the antenna spacing, $\Delta\omega$ is the angular frequency separation, $\overline{T}$ is the time delay spread, and $\alpha \in [0, 2\pi]$ is the angle between the antenna axis and the direction of the vehicle motion in radians.

For a nil frequency separation, then $\eta_{11}(\tau) = 0$ and $\eta_{12}(\tau) = 0$. The corresponding correlation coefficients can be calculated as

$$\mu_{11} = 1 \quad (43)$$
$$\mu_{12} = J_0(2\pi d/\lambda) \quad (44)$$
$$\dot{\mu}_{11} = 0 \quad (45)$$
$$\dot{\mu}_{12} = 2\pi f_m \cos(\alpha) J_1(2\pi d/\lambda) \quad (46)$$
$$\ddot{\mu}_{11} = -2(\pi f_m)^2 \quad (47)$$
$$\ddot{\mu}_{12} = (2\pi f_m)^2 \times \left\{ \frac{J_1(2\pi d/\lambda)}{2\pi d/\lambda} \cos(2\alpha) - \cos^2(\alpha) J_0(2\pi d/\lambda) \right\} \quad (48)$$

where $J_1(\cdot)$ is the first-order Bessel function. And this is the case explored here (as well as in [1]).

In the illustrations that follow we use the Hoyt parameter [5] $b_i \triangleq \frac{\sigma_{X_i}^2 - \sigma_{Y_i}^2}{\sigma_{X_i}^2 + \sigma_{Y_i}^2}$ and the individual power branches $\Omega_i = \sigma_{X_i}^2 + \sigma_{Y_i}^2$.

For the sake of simplicity, the branches are considered balanced and identical. Figs. 1 and 2 show the normalized LCR (left vertical axis), N_R/f_m, and AFD (right vertical axis), $T_R f_m$, for $\alpha = 0°$ and $\alpha = 90°$, respectively, as a function of the envelope level, for SC, EGC and MRC. The following arbitrary parameters have been used: $d/\lambda = 0.06$, $b_i = 0.5$.

Figs. 3 and 4 show the normalized LCR and AFD for two different antenna angles $\alpha = 0°$ and $\alpha = 90°$, respectively, as a function of the parameter d/λ, for the SC, EGC and MRC. An envelope level at $r/\sqrt{\frac{\Omega_1+\Omega_2}{2}} = -20$ dB, identical fading parameters $b_i = 0.5$, and balanced channels have been used. It can be seen that as the antenna spacing becomes larger, the LCR decreases, becoming oscillatory and convergent. Fig. 2 also shows that the MRC has the smaller LCR in both cases of antenna angles. It can be seen in Fig. 3 that the shape of the AFD curves for the SC, EGC and MRC are loosely dependent on the antenna spacing when $\alpha = \pi/2$.

VI. CONCLUSIONS

Exact formulas for level crossing rate and average fade duration of the dual branch SC, EGC and MRC techniques in a unbalanced, non-identical, and correlated Hoyt fading environment have been presented. In passing, this paper derives the joint Hoyt bidimensional envelope-phase distribution. The paper compares previous approximated results and the exact results obtained here showing that they differ considerably. These formulas have been validated by specializing the general results to some particular cases whose solutions are known.

REFERÊNCIAS

[1] F. Adachi, M. T. Feeney and J. D. Parsons. Effects of correlated fading on level crossing rates and average fade durations with predectection diversity reception. In *Proc. Inst. Elect. Eng.*, pages 11–17, Feb. 1988.

[2] Xiaofei Dong and Norman C. Beaulieu. Average Level Crossing Rate and Fade Duration of Maximal-Ratio Diversity in Unbalanced and Correlated Channels. In *WCNC - IEEE Wireless Communications and Networking Conference*, pages 762–767, Mar. 2002.

[3] M.-S. Lin Yang; Alouini. An exact analysis of the impact of fading correlation on the average level crossing rate and average fade duration of selection combining. In *Proc. IEEE Vehicular Technology Conference*, volume 1, pages 241–245, Apr. 2003.

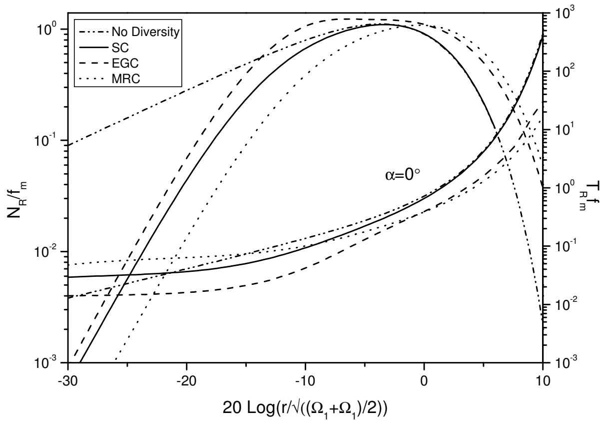

Fig. 1. Normalized LCR and AFD for $d/\lambda = 0.06$, $\alpha = 0°$, and identical Hoyt fading parameter $b_i = 0.5$.

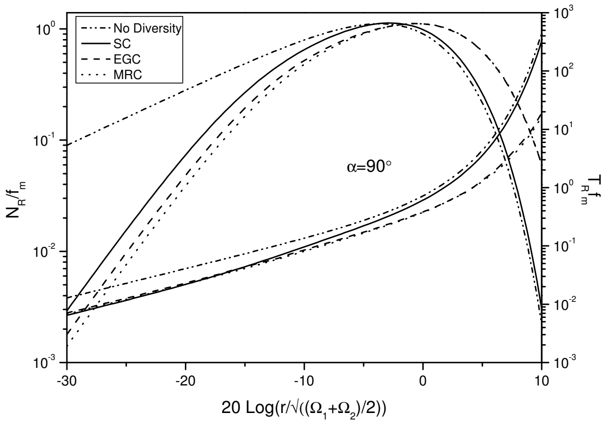

Fig. 2. Normalized LCR and AFD for $d/\lambda = 0.06$, $\alpha = 90°$, and identical Hoyt fading parameter $b_i = 0.5$.

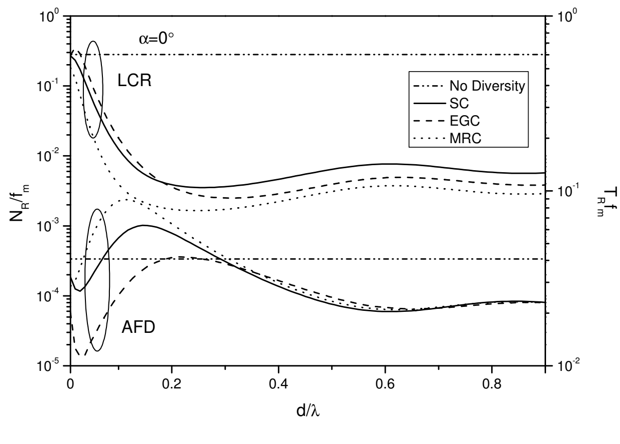

Fig. 3. Normalized LCR and AFD for $r/\sqrt{\frac{\Omega_1+\Omega_2}{2}} = -20\text{dB}$, $\alpha = 0°$, and identical Hoyt fading parameter $b_i = 0.5$.

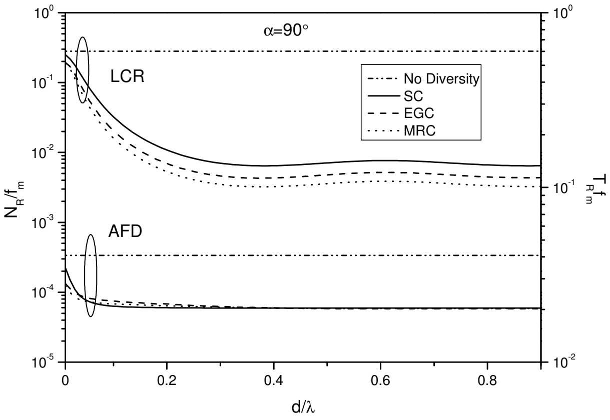

Fig. 4. Normalized LCR and AFD for $r/\sqrt{\frac{\Omega_1+\Omega_2}{2}} = -20\text{dB}$, $\alpha = 90°$, and identical Hoyt fading parameter $b_i = 0.5$.

[4] Dongdong Li and Vasant K. Prabhu. Average Level Crossing Rates and Average Fade Durations for Maximal-ratio Combining in Correlated Nakagami Channels. In *WCNC - IEEE Wireless Communications and Networking Conference*, pages 339–344, Mar. 2004.

[5] R. S. Hoyt. Probability Functions for the Modulus and Angle of the Normal Complex Variate. *Bell System Technical Journal*, 26:318–359, Apr. 1947.

[6] Schwartz M. Bennett W. R. and Stein S. *Communication Systems and Techniques*. New York, 1966.

[7] W. C. Jakes. *Microwave Mobile Communications*. New York: Wiley, 1974.

[8] S. O. Rice. Mathematical analysis of random noise. *Bell System Technical Journal*, 23:282–332, Jul. 1944.

[9] G. Fraidenraich, J. C. S. Santos Filho and M. D. Yacoub. Second-Order Statistics of Maximal-Ratio and Equal-Gain Combining in Hoyt Fading. *IEEE Commun. Lett.*, 9(1), Jan 2005.

[10] D. G. Brennan. Linear diversity combining techniques. In *IRE*, volume 47, pages 1075–1102, Jun. 1959.

[11] A. Papoulis. *Probability, Random Variables, and Stochastic Processes*. Fourth edition, 2002.

[12] W. C. Y. Lee. An extended correlation function of two random variables applied to mobile radio transmission. *Bell Syst. Tech. J.*, pages pp. 3423–3440, 1969.