Adaptive Processing in Real Antenna Arrays

Eduardo B. Perri and Luiz C. Trintinalia

***Abstract*— Mutual coupling effects lead to resolution degradation of smart antenna arrays. To improve that, we employ a transformation of the real array into an equivalent ideal array, without coupling. The output signals of the ideal array are processed by the LMS algorithm, and we analyze the errors and the limitations originated by the transformation of the real array. The eigenvalues of the signal correlation matrix and the LMS convergence speed are investigated through simulation for different virtual array element spacing. By increasing the spacing between elements of the virtual array, it is possible to increase the speed of convergence.**

***Keywords*— adaptive process, direction-of-arrival estimation, intelligent antenna array, mutual coupling.**

***Resumo*— A existência de acoplamento mútuo provoca degradação de resolução em redes de antenas ineligentes. Para melhorar essa característica, é usada uma transformação da rede real numa rede ideal equivalente, sem acoplamento. Os sinais de saída da rede ideal são processados com o algoritmo LMS, sendo então analisados os erros e limitações originados na transformação da rede real. Através de simulações, são investigados os autovalores da matriz de correlação dos sinais e a velocidade de convergência do LMS para diferentes espaçamentos entre elementos da rede virtual. Aumentando-se o espaçamento entre elementos da rede virtual, é possível aumentar-se a velocidade de convergência.**

***Palavras-chave*— processo adaptativo, estimativa de direção de chegada, redes de antenas inteligentes, acoplamento mútuo.**

I. Introduction

In smart antenna array simulation and analysis, the antenna models used are generally considered ideal, disregarding the mutual coupling effects between their elements. Nevertheless, these effects are very significant in the performance of such arrays, leading to resolution degradation and decrease of precision of the adapted weights or the direction-of-arrival (DOA) estimation [5]. In a previous work [1] we have analyzed real adaptive arrays, taking into account the mutual coupling effects between their elements by employing a transformation of the real array into an equivalent ideal array, without coupling. The output of this ideal array was processed by the LMS adaptive algorithm to reduce the interference from signals coming from different directions, and the obtained weights were finally transported back to the real array. In the present work we investigate this subject a little further, analyzing the error and the limitations originated by the transformation of the real array into an ideal one.

II. Theory

We consider a simple linear equally spaced M-element antenna array, as in Fig. 1, operating in the receive mode (uplink). Each branch of the array has a weighting coefficient w_m. The weight vector whose components are the coefficients w_m is denoted by $\boldsymbol{w}$. The array response in a given direction is the steering vector, and a set of steering vectors, either measured or calculated over different angles, forms [1,2] the steering matrix $\boldsymbol{A}$. The process used here consists in calculating or measuring induced voltages in each array element by each incident signal over a certain angular sector, forming the steering matrix $\boldsymbol{A}(\phi)$, and then calculating the ideal steering matrix $\boldsymbol{A}_v(\phi)$ supposing that there is no mutual coupling. The ideal steering vector for incident direction ϕ_i is given by:

$$a_v(\phi_i) = e^{-jkx_i\cos\phi_i} \tag{1}$$

where k is the wave number and x_i is the distance from array element i to the reference element. The transformation $\boldsymbol{T}$ between the two steering matrices for all angles ϕ within the predefined sector is defined by:

$$\boldsymbol{TA}(\phi) = \boldsymbol{A}_v(\phi) \tag{2}$$

Using the Least Squares (LS) method, the solution to (2) is given by:

$$\boldsymbol{T} = \boldsymbol{A}_v\boldsymbol{A}^H\left(\boldsymbol{AA}^H\right)^{-1} \tag{3}$$

and the real weights can be obtained from the ideal ones by the expression:

$$\boldsymbol{w}^H = \boldsymbol{w}_v{}^H\boldsymbol{T} \tag{4}$$

$\boldsymbol{w}$ being the weight vector used to combine signals from the real array elements in order to obtain a desired radiation pattern, and $\boldsymbol{w}_v$ the corresponding virtual array weight vector. Here we use an evaluation of the interpolation error to measure the precision when finding the matrix $\boldsymbol{T}$, following the definition in [3] (L is the number of incident signals directions):

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$$\left\{ \frac{1}{ML} \sum_{i=0}^{M-1} \sum_{j=0}^{L-1} \left| \boldsymbol{A}_{v_{ij}} - [\boldsymbol{TA}]_{ij} \right|^{2} \right\}^{\frac{1}{2}} \quad (5)$$

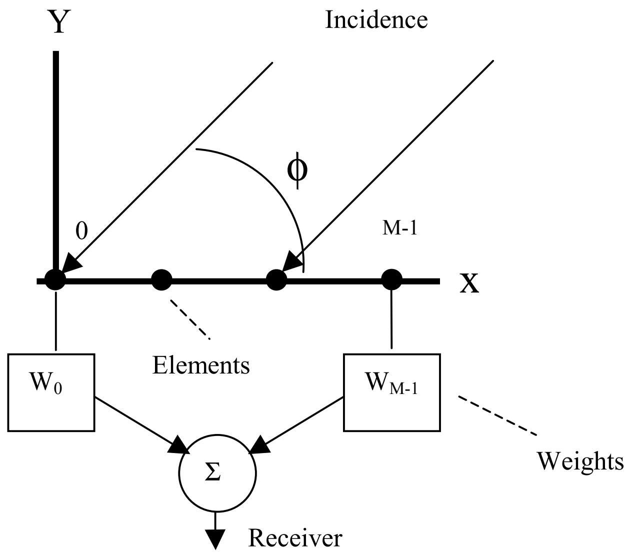

Fig. 1. A linear equally spaced array oriented along the x axis, receiving a plane wave from direction (θ,ϕ).

A. Effect of spacing between array elements

Matrices $\boldsymbol{A}$ and $\boldsymbol{A}_v$ can be identified to induced normalized voltages [2]. By making a detailed analysis of element (i,1) in matrix $\boldsymbol{A}_v$, we see that it can be expressed as a two-part sum, one coming from the corresponding element A_{i1} in matrix $\boldsymbol{A}$, and another coming from the other elements:

$$A_{v_{i1}} = T_{ii} A_{i1} + \sum_{k \neq i} T_{ik} A_{k1} \quad (6)$$

The second term on the right side of eq. (6) takes into account the differences in response from the two arrays (real and virtual), including the mutual effects and the progressive phase difference in the two arrays due to eventually different element spacings. If the spacing is the same in the real and in the virtual arrays, then the differences between the arrays come exclusively from the mutual effects, and in this case the matrix $\boldsymbol{T}$ will be symmetric, since its elements refer to mutual coupling coefficients which, considering the array reciprocity property [4], are also reciprocal. On the other hand, if mutual coupling effects are not considered, the coefficients T_{ik} for k not equal to i are null, and the matrix $\boldsymbol{T}$ equals the unity matrix $\boldsymbol{I}$. But, if spacing between elements in the real and virtual arrays is not equal, additional differences arise, eliminating the symmetry in matrix $\boldsymbol{T}$.

After the transformation to a virtual array, the obtained result differs from the corresponding ideal array only by a scale factor and by the error related to the LS process used in the referred transformation. So, there are two main aspects of the transformation $\boldsymbol{T}$ to consider: a) the modification in the correlation matrix $\boldsymbol{R}$ considering the virtual array geometry and the incident signals; and b) the modification in $\boldsymbol{R}$ from errors in the LS process that lead to the matrix $\boldsymbol{T}$, that is, errors coming from a distortion in $\boldsymbol{T}$.

B. Convergence of the LMS process

The LMS algorithm transient behavior [6] is associated with a geometrical series where an exponential envelope with time constant τ_k may be adjusted, considering one iteration cycle duration as being the unit of time. Time constant τ_k defines the time necessary to the k-th natural mode decrease to $1/e$ of its initial value, and the time needed by the exponential to reach a value equal to 0.02 of its initial value is approximately equal to four times the time constant τ_k . Being λ_{max} the largest eigenvalue of the signal correlation matrix and μ the step parameter, the algorithm is stable if and only if:

$$0 < \mu < \frac{2}{\lambda_{\max}} \quad (7)$$

III. SIMULATIONS AND RESULTS

The method here described was applied to a 5 parallel dipole array with spacing $\lambda/4$ in 900 MHz, λ being the wavelength. The desired signal is incident in the H plane from direction 80º, and interfering signals coming from 40º, 60º and 120º directions are also considered. The LMS algorithm was used to calculate the optimum weight vector for different virtual array spacings, with the purpose of observe and analyze the speed of convergence and the precision in the resulting radiation patterns. In order to avoid the occurrence of grating lobes [4], element spacing was restricted to a maximum of $\lambda/2$. Table 1 shows the minimum and maximum eigenvalues of the original (real) signal correlation matrix $\boldsymbol{R}$ and of the transformed matrix (ideal) $\boldsymbol{R}_v$ and also their respective spreads for different virtual array element spacing. It also shows the Least Squares solution error, the speed of convergence, and the interfering signal discrimination. From these values one can observe: a) a higher speed of convergence for a lower eigenvalue spread of the transformed signal correlation matrix; b) a higher Least Squares process error for larger virtual array element spacing. Fig. 2 shows detailed learning curves that correspond to some of the simulated cases and that confirm the observations made above.

In each of these cases, we have tried to obtain the maximum speed of convergence by maximizing the parameter μ in accordance to (7). To complete the analysis, we evaluated the interfering signal discriminations, and some radiation patterns are showed in Fig. 3. All simulations were performed using the method of the moments. We observe that the discrimination values indicated in Table 1 are in general higher than 40 dB for these cases, showing that the goal of discriminating the interfering signals with the employed array spacing was always reached.

From these results we can conclude that the convergence is slower with virtual array spacing smaller than in the real array, and that by increasing the virtual array spacing the speed of convergence may rise progressively. On the other hand, the error from the solution of the transformation matrix by the LS method has a minimum value when the chosen virtual array spacing is equal to the real array case (for linear arrays). This situation does not correspond, in general, to a higher speed of convergence, and this error is directly related to the pattern precision, since a LS process error means a deviation from the original data. From these results we can conclude that a magnitude of LS process error until around 0.5, as defined in (5), is acceptable under the point-of-view of desired radiation pattern and that, for more than one incident signal, arrays with larger element spacing and up to 0.5λ generally result in signal correlation matrix with smaller eigenvalue spread, allowing a higher speed of convergence with the LMS algorithm.

TABLE 1

RESULTS FROM ADAPTIVE PROCESSING IN A 5 PARALLEL DIPOLE ARRAY WITH SPACING $\lambda/4$ IN 900 MHZ, FOR INCIDENT SIGNAL FROM 80° AND INTERFERENCE SIGNALS FROM 40°, 60° AND 120°, AND FOR DIFFERENT EQUIVALENT IDEAL ARRAY SPACINGS.

Ideal array element spacing (wavelengths)		**0.20**	**0.25**	**0.30**	**0.40**	**0.50**
$\lambda_{max}(R_v)$		16.47	15.10	12.72	10.07	8.33
$\lambda_{min}(R_v)$		0.01	0.05	0.22	1.01	0.93
$\lambda_{max}(R_v)/\lambda_{min}(R_v)$		1511	299	56	10	9
μ		0.052	0.060	0.064	0.081	0.100
$\mu_{max} = 1/\lambda_{max}(R_v)$		0.060	0.066	0.078	0.099	0.120
Interpolation error		$0.2\text{x}10^{-2}$	$0.4\text{x}10^{-4}$	0.04	0.29	0.42
Steps to converge		6935	1307	282	47	47
Interference	**40°**	-	47	67	87	77
discrimination	**60°**	-	56	67	99	58
(dB)	**120°**	-	51	69	91	63

$\lambda_{max}(R) = 5.72$; $\lambda_{min}(R) = 0.04$; $\lambda_{max}(R)/\lambda_{min}(R) = 143$

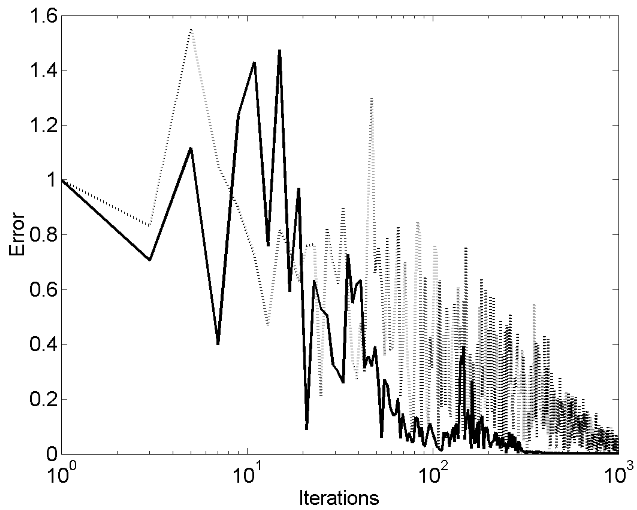

Fig. 2. Learning curves for real array with element spacing equal to 0.25λ and different virtual array spacings: ________ 0.5λ; .... 0.25λ .

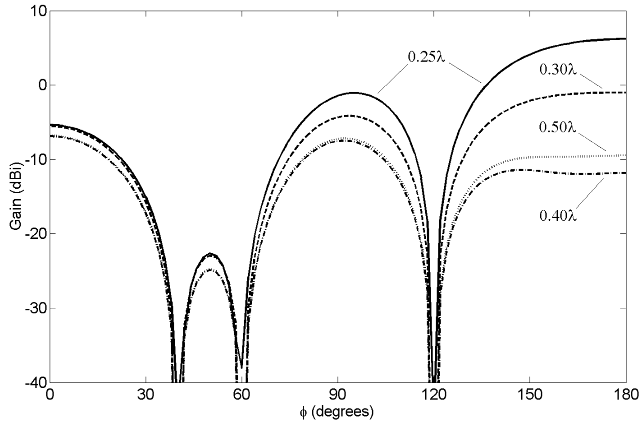

Fig. 3. Radiation patterns resulting from adaptive process for the cases described on Table 1. Element spacing values on the ideal equivalent array are as indicated.

IV. CONCLUSION

We have considered mutual coupling effects in adaptive arrays, recognizing that they are significant in resolution degradation and in the precision of adapted weights and direction-of-arrival (DOA) estimation. A transformation of the real array into an ideal one was used. The eigenvalues of the correlation matrix as well as the convergence speed in the LMS algorithm were investigated for different virtual array element spacing. From the results we concluded that by increasing the spacing between elements of the virtual array it is possible to increase the speed of convergence in the LMS algorithm.

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