

# Application of Photonic Material in Unilateral and Bilateral Finlines

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**Abstract** – The objective of this work is to use PBG (Photonic Band Gap) material in the unilateral and bilateral finlines. To analyze its efficiency, it's necessary to determine the phase, attenuation and effective dielectric constants. To analyze the behavior of the finline with this substrate, the full wave TTL (Transversal Transmission Line) method is used. New numerical results for the attenuation and effective dielectric constant of unilateral and bilateral finline with PBG substrate are presented. A good agreement in comparison to other works is obtained.

**Keywords** — TTL Method, PBG-Photonic Band Gap, finlines, Millimeter-Waves.

## I. INTRODUCTION

The unilateral finline consist of two conductors fins on the sides of a dielectric substrate, adapted in the E-plane of a rectangular millimeter-wave guide, as shown in Fig.1. According to the figure, 2a and 2b are the height and width of the wave guide, respectively,  $s$  and  $g$  are the thickness of the regions 1 and 2 respectively,  $\epsilon_r$  is the relative permittivity of the substrate material,  $w_1$  is the width of the slot and  $f$  is the infinitesimal thickness of the fin conductor. The bilateral finline has fins between the 1 and 2 regions.

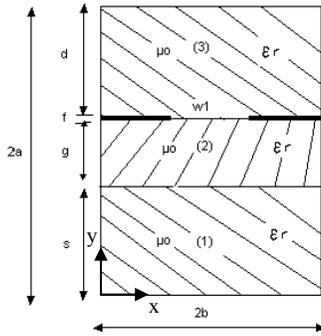


Fig 1. Traverse section of a unilateral finline.

The TTL method is used to determine the phase, attenuation and effective dielectric constants of unilateral and bilateral finlines using the PBG substrate, as the first time [1-8]. In this method the fields in the “y” direction to the real direction of propagation “z” and treat the general equations of the electric and magnetic fields as

functions of its components  $E_y$  and  $H_y$ , to obtain the electromagnetic field components inside of the millimeter waveguide. Using the boundary conditions, leading to the determination of the characteristic equation, Millimeter-Waves whose roots allow the obtention of the attenuation ( $\alpha$ ) and phase ( $\beta$ ) constants.

## II. FIELDS STRUCTURES

After using the Maxwell's equations in the spectral domain, the general equations of the electric and magnetic fields in the TTL method, are obtained as:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \tilde{E}_{yi} - j\omega\mu\Gamma \tilde{H}_{yi} \right] \quad (1.1)$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -\Gamma \frac{\partial}{\partial y} \tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi} \right] \quad (1.2)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \tilde{H}_{yi} + j\omega\epsilon\Gamma \tilde{E}_{yi} \right] \quad (1.3)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2} \left[ -\Gamma \frac{\partial}{\partial y} \tilde{H}_{yi} + \omega\epsilon\alpha_n \tilde{E}_{yi} \right] \quad (1.4)$$

Where:

$\gamma_i^2 = \alpha_n^2 - \Gamma^2 - k_i^2$ , is the propagation constant in “y” direction;  $\alpha_n$  is the spectral variable in “x” direction.

$k_i^2 = \omega^2 \mu \epsilon = k_0^2 \epsilon_{ri}^*$  is the wave number of  $i^{\text{th}}$  term of dielectric region;

$\epsilon_{ri}^* = \epsilon_{ri} - j \frac{\sigma_i}{\omega \epsilon}$  is the relative dielectric constant of the material with losses;

$\epsilon_i = \epsilon_{ri}^* \cdot \epsilon_0$  is the dielectric constant of the  $i^{\text{th}}$  region;

$\Gamma = \alpha + j\beta$  is the complex propagation constant;

$\omega = \omega_r + j\omega_i$  is the complex angular frequency.

The equations below are applied, being calculated the  $E_y$  and  $H_y$  fields through the solution of the Helmholtz equations in the spectral domain [1]-[2], the equations for the regions are:

For the region 1:

$$\begin{aligned}\overline{E}_y^1 &= A_1 e \cosh \gamma^1 y \\ \overline{H}_y^1 &= A_1 h \sinh \gamma^1 y\end{aligned}\quad (2.1)$$

For the region 2:

$$\begin{aligned}\overline{E}_y^2 &= A_2 e \sinh \gamma^2 y + B_2 e \cosh \gamma^2 y \\ \overline{H}_y^2 &= A_2 h \sinh \gamma^2 y + B_2 h \cosh \gamma^2 y\end{aligned}\quad (2.2)$$

For the region 3:

$$\begin{aligned}\overline{E}_y^3 &= A_3 e \cosh \gamma^3 (2a - y) \\ \overline{H}_y^3 &= A_3 h \cosh \gamma^3 (2a - y)\end{aligned}\quad (2.3)$$

Substituting these solutions in the TTL method equations and applying the boundary conditions, the electromagnetic field components are determined for each region, using the boundary conditions,

$$\tilde{E}_{x3/y=t} = \tilde{E}_{x2/y=t} = \tilde{E}_{xt} \quad (3.1)$$

$$\tilde{E}_{z3/y=t} = \tilde{E}_{z2/y=t} = \tilde{E}_{zt} \quad (3.2)$$

$$\tilde{E}_{x1/y=s} = \tilde{E}_{x2/y=s} \quad (3.3)$$

$$\tilde{E}_{z1/y=s} = \tilde{E}_{z2/y=s} \quad (3.4)$$

$$\tilde{H}_{x1/y=s} = \tilde{H}_{x2/y=s} \quad (3.5)$$

$$\tilde{H}_{z1/y=s} = \tilde{H}_{z2/y=s} \quad (3.6)$$

Determination of the propagation's constants

$$\tilde{H}_{x2} - \tilde{H}_{x3/y=t} = J_{zt} \quad (4.1)$$

$$\tilde{H}_{z2} - \tilde{H}_{z3/y=t} = -J_{xt} \quad (4.2)$$

Where:  $J_{zt}$  and  $J_{xt}$  are current density.

A system equations is then obtained that in a matrix form is given as:

$$\begin{bmatrix} Y_{X^1 X^1} & Y_{X^1 Z^1} \\ Y_{Z^1 X^1} & Y_{Z^1 Z^1} \end{bmatrix} \begin{bmatrix} \tilde{E}_{xt} \\ \tilde{E}_{zt} \end{bmatrix} = \begin{bmatrix} J_{xt} \\ J_{zt} \end{bmatrix} \quad (4.3)$$

To eliminate the components of the current density, the moments method was applied with the expansion of  $\tilde{E}_{xt}$  and  $\tilde{E}_{zt}$  in terms of known base functions, as:

$$\tilde{E}_{xg} = A_{X0} f_{X0}(\alpha_n) + A_{X1} f_{X1}(\alpha_n) \quad (4.4)$$

$$\tilde{E}_{zg} = A_{Z0} f_{Z0}(\alpha_n) + A_{Z1} f_{Z1}(\alpha_n) \quad (4.5)$$

As a result, the equation is transformed in an homogeneous matricial equation, whose non-trivial solution corresponds to the characteristic equation, and it's roots supply the phase and attenuation constants.

$$\begin{bmatrix} K_{X^1 X^1} & K_{X^1 Z^1} & K_{X^1 X^2} & K_{X^1 Z^2} \\ K_{Z^1 X^1} & K_{Z^1 Z^1} & K_{Z^1 X^2} & K_{Z^1 Z^2} \\ K_{X^2 X^1} & K_{X^2 Z^1} & K_{X^2 X^2} & K_{X^2 Z^2} \\ K_{Z^2 X^1} & K_{Z^2 Z^1} & K_{Z^2 X^2} & K_{Z^2 Z^2} \end{bmatrix} \begin{bmatrix} A_{X0} \\ A_{X1} \\ A_{Z0} \\ A_{Z1} \end{bmatrix} = 0 \quad (5)$$

Where:

$$K_{X^1 X^1} = \sum_{-\infty}^{\infty} f_{X0}(\alpha_n) Y_{X^1 X^1} f_{X0}^*(\alpha_n) \quad (5.1)$$

$$K_{X^1 Z^1} = \sum_{-\infty}^{\infty} f_{X0}(\alpha_n) Y_{X^1 Z^1} f_{X1}^*(\alpha_n) \quad (5.2)$$

$$K_{Z^1 X^1} = \sum_{-\infty}^{\infty} f_{X1}(\alpha_n) Y_{X^1 X^1} f_{X0}^*(\alpha_n) \quad (5.3)$$

$$K_{Z^1 Z^1} = \sum_{-\infty}^{\infty} f_{X1}(\alpha_n) Y_{X^1 Z^1} f_{X1}^*(\alpha_n) \quad (5.4)$$

$$K_{X^1 X^2} = \sum_{-\infty}^{\infty} f_{X0}(\alpha_n) Y_{X^1 Z^1} f_{Z0}^*(\alpha_n) \quad (5.5)$$

The effective dielectric constant is obtained after numerical solutions of the matrix determinant by the relation:

$$\epsilon_{ef} = (\beta/ko)^2 \quad (6)$$

### III. PBG STRUCTURE

For a non-homogeneous structure submitted, the incident sign goes at the process of multiple spread. A solution can be obtained through a numerical process called homogenization [9-14]. The process is based in the theory related to the diffraction of an incident electromagnetic plane wave imposed by the presence of a air immersed cylinders in a homogeneous material [9].

In the Cartesian coordinates system of axes ( $O, x, y, z$ ), are shown in the Fig. 2. A cylinder is considered with relative permittivity  $\epsilon_1$ , with a traverse section in the plane  $xy$ , embedded in a medium of permittivity  $\epsilon_2$ . For this process the two-dimensional structure is sliced in layers whose thickness is equal at the cylinder diameter.

In each slice is realized the homogenization process.

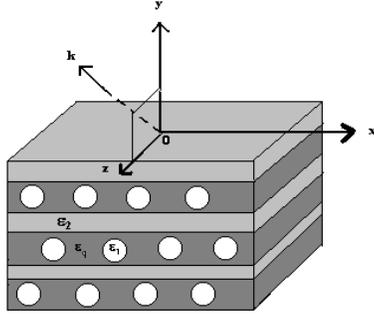


Fig. 2. Homogenized bidimensional crystal.

According to homogenization theory the effective permittivity depends on the polarization [1]. For the case of  $s$  and  $p$  polarization, respectively, we have:

$$\varepsilon_{eq} = \beta(\varepsilon_1 - \varepsilon_2) + \varepsilon_2 \quad (5)$$

$$\frac{1}{\varepsilon_{eq}} = \frac{1}{\varepsilon_1} \left\{ 1 - \frac{3\beta}{A_1 + \beta - A_2\beta^{10/3} + O(\beta^{14/3})} \right\} \quad (6)$$

where:

$$A_1 = \frac{2/\varepsilon_1 + 1/\varepsilon_2}{1/\varepsilon_1 - 1/\varepsilon_2} \quad (7)$$

$$A_2 = \frac{\alpha(1/\varepsilon_1 - 1/\varepsilon_2)}{4/3\varepsilon_1 + 1/\varepsilon_2} \quad (8)$$

and  $\beta$  is defined as the ratio between the area of the cylinders and the area of the cells,  $\alpha$  is an independent parameter whose value is equal to 0.523. The  $A_1$  and  $A_2$  variables in (7) and (8) were included only to simplify (6) equation.

#### IV. RESULTS

The computational program used to calculate the effective dielectric constant and attenuation constant for unilateral and bilateral finline, with PBG substrate, was developed in Fortran PowerStation and Matlab for Windows.

The Fig. 3. shows the attenuation constant ( $\alpha$ ) as function of the conductivity in the region 2, for two different frequencies for a unilateral finline with PBG substrate. When the conductivity increases the attenuation increases.

The Fig. 4. shows the effective dielectric constant as function of the slot width, in the region 2, for three different frequencies. Can be

noticed in Fig.4 that, when the slot width increases the effective dielectric constant decreases.

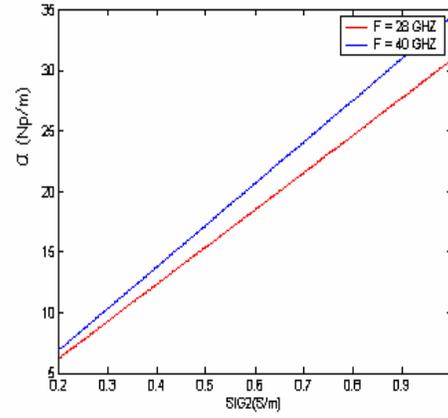


Fig. 3. Attenuation constant as function of the conductivity in the region 2.

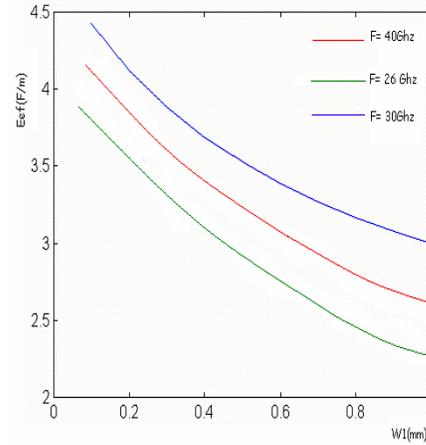


Fig. 4. Effective dielectric constant as function of the slot width.

The Fig. 5 shows the 3D results of the real resonance frequency as a function of the dielectric substrate thickness and of the normalized width.

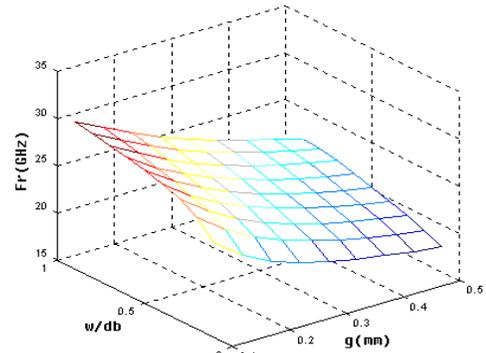


Fig. 5. 3D results of the real resonance frequency as a function of the dielectric substrate thickness and of the normalized width.

The effective dielectric constant as a function of the frequency is shown in the Fig.6, for different slot width of bilateral finline with PBG. In this figure the effective dielectric constant increase when the frequency increase.

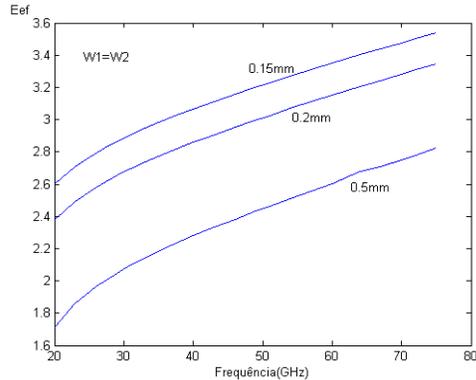


Fig.6. Effective dielectric constant as a function of the frequency.

## V. CONCLUSION

The full wave transverse transmission line (TTL) method was used to characterization of the unilateral and bilateral finlines considering Photonic Band Gap (PBG) substrate, at applications in millimeter waves. New numerical results for the attenuation and effective dielectric constant of unilateral and bilateral finline with PBG substrate were presented. A good agreement in comparison to other works was obtained when the substrate is a semiconductor [2], [8].

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## REFERENCES

- [1] J. P. Silva, S. A. P. Silva and H. C. C. Fernandes, "Coupling Analysis at the Coupler and Unilateral Edge-Coupled Fin Line", International Conference on Millimeter And Submillimeter Waves and Applications II, SPIE's 1998 International Symposium on Optical Science, Engineering and Instrumentation, San Diego, California, USA. Conf. Proc. pp. 53-54, Jul. 1998.
- [2] O. S. D. Costa, S. A.P. Silva and H. C. C. Fernandes, "3D Complex Propagation of Coupled Unilateral and Antipodal Arbitrary Finlines", Brazilian CBMAG'96-Congress of Electromagnetism, Ouro Preto-MG, pp. 159-162, 24-27 of Nov., 1996.
- [3] P. J. Meier, "Integrated fin-line millimeter components", IEEE Trans. on Microwave Theory and Tech., Vol. MTT-22, pp. 1209-1216, Dec. 1974.
- [4] B. Baht and S. K. Koul, "Analysis, design and applications of finlines", Artech House, 1987.
- [5] H.C.C.Fernandes and M. C. Silva, "Dynamic TTL Method Applied to the Fin-Line Resonators", Journal of Microwaves and Optoelectronics, Vol.2,N.3,pp.57-66,Jul.2001.
- [6] H.C.C.Fernandes and M. C. Silva, "Fin-Line Resonators by TTL Method", Special Issue on Signals, Systems and Electronics Technology, IEICE-Transactions, pp.599-601, Tóquio-Japão, Mar. 2002.
- [7] H.C.C.Fernandes, "TTL Method Applied to the Fin-Line Resonators", Journal of Electromagnetic Waves and Applications, USA, Vol. 15, 11p., 2001.
- [8] H.C.C.Fernandes; E. A. M. Souza and I. de S. Queiroz Jr., "Metallization thickness in bilateral and unilateral finlines", International Journal of Infrared and Millimeter Waves", Vol. 15 °.6, pp. 1001-1014, Jun.1994, Editora Plenum Press, USA.
- [9] E. Centeno and D. Felbacq, "Rigorous vector diffraction of electromagnetic waves by bidimensional photonic crystals", J. Optical Soc. American A/Vol. 17, No.2, pp.320-327, February 2000.
- [10] K. Guillouard, M. F. Wong, V. Fouad Hanna, J. Citerne, "Diakoptics using finite element analysis", MWSYM 96 Vol. 1, p. 363-366.
- [11] J. Tan, G. Pan, "A general functional analysis to dispersive structures", MWSYM 96 Vol. 2, p. 1027-1030.
- [12] A. R. Barros Rocha and H. C. C. Fernandes, "Analysis of Antennas with PBG Substrate", International Journal of Infrared and Millimeter Waves, Vol. 24,Págs:1171-1176, USA, Jul. 2003.
- [13] H. C. C. Fernandes and S. P. Santos, "Change in the directivity of planar array with PBG substrate", WSEAS Trans. on Communications, Athens, Greece, pp.433-437, Jul., 2004.