Knowledge-Aided Parameter Estimation Based on Conjugate Gradient Algorithms

Silvio Fernando Bernardes Pinto and Rodrigo C. de Lamare
Center for Telecommunications Studies (CETUC)
Pontifical Catholic University of Rio de Janeiro, RJ, Brazil.
Emails: silviof@cetuc.puc-rio.br, delamare@cetuc.puc-rio.br

***Abstract*—The performance of many parameter estimation algorithms used for direction finding and localization techniques depends on the accuracy of the signal covariance matrix estimate. For a small number of sensors, the commonly used sample covariance matrix estimation procedure may only provide a poor estimate of the unknown true covariance matrix. In scenarios with low signal-to-noise ratio, stationary and non-stationary signal sources, a more accurate estimate of the signal covariance matrix can be achieved by incorporating *a priori* knowledge about the direction of arrival (DOA) of dominant signals. In this paper, we combine the weighted sample covariance matrix and a weighted knowledge-aided (KA) covariance matrix. We present a KA-Conjugate Gradient (KA-CG) algorithm that processes the enhanced covariance matrix estimate. Simulation results show that the proposed KA-CG algorithm substantially improves the probability of resolution of unknown close sources in the system, especially at middle low signal-to-noise ratios (SNR), requiring a reasonable number of samples for this aim.**

I. Introduction

In array signal processing and wireless communications, parameter estimation is a key task in a broad range of important applications including direction finding, localization and channel estimation and many different approaches have been developed over the years [1], [14]. In spite of the numerous parameter estimation techniques developed over the last decades and their specific properties, advantages and drawbacks, their estimation accuracy depends on the $(M \times M)$ dimensional signal covariance matrix of the sensor array data vector $\boldsymbol{x}(i)$, which is defined for the ith snapshot as

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{x}(i)\boldsymbol{x}^H(i)\right], \qquad i = 1, \ldots, N, \tag{1}$$

where the superscript H and $\mathbb{E}[\cdot]$ denote the conjugate transpose and the statistical expectation respectively, and N is the number of available snapshots. In practice, the true signal covariance matrix in (1) is unknown, but can be estimated via the widely used sample-average formula given by

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}(i)\boldsymbol{x}^H(i). \tag{2}$$

Applying the covariance matrix estimate in (2), the estimation accuracy is essentially determined by the data record size N. Thus, in applications where the number of available sensors M is small, the increase in the number of snapshots become more significant.

In practical scenarios with low signal-to-noise ratio (SNR), stationary and non-stationary sources whose DOAs are to be estimated, the knowledge of the directions of strong consistent users can be effectively exploited in order to increase the estimation accuracy of non-stationary sources, which enter the system. The knowledge of previously estimated DOAs can be exploited in the form of a known covariance matrix $\boldsymbol{C}$. Knowledge-aided (KA) signal processing techniques, which make use of *a priori* knowledge of key parameters of interest such as the existence of strong interferers, cognitive users and geographical localization of users [18] have recently gained significant attention [2]-[7]. In KA techniques, the key issues are how to obtain *a priori* knowledge about the parameters of interest and how to exploit them. Prior work on KA algorithms has considered the design of space-time adaptive processing (STAP) techniques [2]-[4],[6], [7] and beamforming algorithms [5]. These methods have shown superior performance to conventional approaches that do not rely on KA techniques when the limited sample support is used in highly nonstationary environments. Despite the existence of KA methods combined with classical algorithms for parameter estimation, KA methods have not been combined with high-resolution source localization algorithms so far, nor has any combined approach to obtain prior knowledge and estimate the parameters been detailed to date.

In this paper, we propose a knowledge-aided parameter estimation technique, termed as KA-CG, that combines the (CG) algorithm [8], [9], [10] and *a priori* knowledge of the directions of arrivals of source signals. We present a strategy to improve the estimates of the signal covariance matrix by incorporating *a priori* knowledge about the directions of dominant signals. The proposed KA-CG algorithm is developed for complex-valued data and considers the general case, where $\boldsymbol{C}$ is rank deficient and the noise power is assumed to be unknown. We also develop a procedure to obtain prior knowledge based on the CG algorithm that is employed with the proposed KA-CG algorithm. A study of the proposed KA-CG algorithm is carried out and shows its superior performance to the conventional CG algorithm, in terms of resolution of closely spaced-sources with a reasonable number of samples and a moderate SNR. The paper is organized as follows. Section 2 describes the system model. Section 3 formulates the problem, whereas in Section 4 the KA-CG algorithm is presented. Section 5 presents and discusses the simulation results and Section 6 gives the concluding remarks of this work.

II. System Model

Let us assume that P narrowband signals from far-field sources are impinging on a uniform linear array (ULA) of M ($M > P$) sensor elements with the unknown directions $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_P]^T$. The ith data snapshot of the $(M \times 1)$-dimensional array output vector can be modeled as

$$\boldsymbol{x}(i) = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{s}(i) + \boldsymbol{n}(i), \qquad i = 1, 2, \ldots, N, \tag{3}$$

where $\boldsymbol{s}(i) = [s_1(i), \ldots, s_P(i)]^T \in \mathbb{C}^{P\times 1}$ represents the zero-mean source data vector, $\boldsymbol{n}(i) \in \mathbb{C}^{M\times 1}$ is the vector of white circular complex Gaussian noise with zero mean and variance σ_n^2, and N denotes the number of available snapshots. The matrix $\boldsymbol{A}(\boldsymbol{\Theta}) = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_P)] \in \mathbb{C}^{M\times P}$ contains the array steering vectors $\boldsymbol{a}(\theta_j)$ corresponding to the nth source, which can be expressed as

$$\boldsymbol{a}(\theta_n) = [1, e^{j2\pi\frac{\Delta}{\lambda_c}\sin\theta_n}, \ldots, e^{j2\pi(M-1)\frac{\Delta}{\lambda_c}\sin\theta_n}]^T, \tag{4}$$

where $n = 1, \ldots, P$, Δ denotes the interelement spacing of the ULA and λ_c is the signal wavelength.

Using the fact that $\boldsymbol{s}(i)$ and $\boldsymbol{n}(i)$ are modeled as uncorrelated linearly independent variables, the $M \times M$ signal covariance matrix is calculated by

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{x}(i)\boldsymbol{x}^H(i)\right] = \boldsymbol{A}(\boldsymbol{\Theta})\boldsymbol{R}_{ss}\boldsymbol{A}^H(\boldsymbol{\theta}) + \sigma_n^2\boldsymbol{I}_M, \tag{5}$$

where $\boldsymbol{R}_{ss} = \mathbb{E}[\boldsymbol{s}(i)\boldsymbol{s}^H(i)]$, which is diagonal if the sources are uncorrelated and nondiagonal for partially correlated sources, and $\mathbb{E}[\boldsymbol{n}(i)\boldsymbol{n}^H(i)] = \sigma_n^2\boldsymbol{I}_M$ with $\boldsymbol{I}_M$ being the $M \times M$ identity matrix.

III. Problem Formulation

In order to obtain an enhanced covariance matrix estimate $\tilde{\boldsymbol{R}}$, we assume that the *a priori* knowledge matrix $\boldsymbol{C}$ is non-random, according to [12], and perform a linear combination of $\boldsymbol{C}$ and the sample covariance matrix $\hat{\boldsymbol{R}}$ by applying the weight factors α and β, which are formulated as

$$\tilde{\boldsymbol{R}} = \alpha\boldsymbol{C} + \beta\hat{\boldsymbol{R}}, \tag{6}$$

where the combination factors are constrained to $\alpha > 0$ and $\beta > 0$, and $\boldsymbol{C}$ is restricted to be positive semi-definitve to ensure that $\tilde{\boldsymbol{R}}$ is also positive semi-definite.

The aim is to find optimal estimates of the weight factors α and β, which efficiently combine $\boldsymbol{C}$ and $\hat{\boldsymbol{R}}$ depending on the scenario. One of the most common criteria is the minimization of the parameters in a mean squared error (MSE) sense, that is

$$\begin{aligned} \min_{\alpha,\beta} \quad & \text{MSE} = \mathbb{E}\left[\|\tilde{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right] \\ \text{s.t.} \quad & \tilde{\boldsymbol{R}} = \alpha\boldsymbol{C} + \beta\hat{\boldsymbol{R}}, \end{aligned} \tag{7}$$

where $\|\cdot\|_F$ denotes the Frobenius matrix norm. Note that the optimization problem is solved by minimizing the MSE with respect to the two parameters α and β, which as expected depend on each other and the unknown true covariance matrix $\boldsymbol{R}$. Another widely used criterion to reduce the complexity of the optimization problem, which can be considered a special case of the function (7), is the optimization described by (8):

$$\begin{aligned} \min_{\alpha} \quad & \text{MSE} = \mathbb{E}\left[\|\tilde{\boldsymbol{R}} - \boldsymbol{R}\|_F^2\right] \\ \text{s.t.} \quad & \tilde{\boldsymbol{R}} = \alpha\boldsymbol{C} + (1-\alpha)\hat{\boldsymbol{R}} \end{aligned} \tag{8}$$

with α being restricted to $\alpha \in (0, 1)$ to ensure the positive semi-definiteness of $\tilde{\boldsymbol{R}}$. Both types of optimization are briefly discussed and applied to the simulations in Section V.

IV. KNOWLEDGE-AIDED CONJUGATE GRADIENT ALGORITHM

Assuming the knowledge of the DOAs of k signals that are impinging on the array from the known directions $\overline{\theta} = [\theta_1, \ldots, \theta_k]^T$, the *a priori* covariance matrix $\boldsymbol{C}$ can be calculated by

$$\boldsymbol{C} = \sum_{l=1}^{k} \boldsymbol{a}(\theta_l)\boldsymbol{a}^H(\theta_l)\sigma_l^2, \tag{9}$$

where $\boldsymbol{a}(\theta_l)$ is the array steering vector of the lth known DOA and σ_l is the power of the lth signal.

Let α_0 and β_0 denote the optimal values α and β that satisfy (7) and (8). The estimates $\hat{\alpha_0}$ and $\hat{\beta_0}$, of α_0 and β_0, obtained from the available data, can be compactly expressed by means of two approaches, as follows:

A. KA-General Linear Combination

Where the estimates given in (10) and (11) are the two weight factors to be applied to (7)

$$\hat{\beta}_{\text{o}} = \frac{\hat{\gamma}}{\hat{\rho} + \hat{\gamma}}, \tag{10}$$

$$\hat{\alpha}_{\text{o}} = \hat{\nu}(1 - \hat{\beta}_{\text{o}}), \tag{11}$$

and $\hat{\gamma}$, $\hat{\nu}$ and $\hat{\rho}$ are defined as

$$\hat{\gamma} = \|\hat{\nu}\boldsymbol{C} - \hat{\boldsymbol{R}}\|_F^2, \tag{12}$$

$$\hat{\nu} = \frac{\text{Tr}\{\boldsymbol{C}^H\hat{\boldsymbol{R}}\}}{\|\boldsymbol{C}\|_F^2}, \tag{13}$$

$$\hat{\rho} = \frac{1}{N^2}\sum_{i=1}^{N}\|\boldsymbol{x}(i)\|_F^4 - \frac{1}{N}\|\hat{\boldsymbol{R}}\|_F^2. \tag{14}$$

B. KA-Convex Combination

Where the estimate given in (15) is the the sole weight factor to be applied to (8)

$$\hat{\alpha_0} = \frac{\hat{\rho}}{\hat{\rho} + \|\hat{\boldsymbol{R}} - \boldsymbol{C}\|_F^2}, \tag{15}$$

and $\hat{\rho}$ is defined as (14).

C. Summary of KA-CG

The aim of the proposed KA-CG algorithm is to exploit *a priori* knowledge in the form of the enhanced signal covariance matrix $\tilde{\boldsymbol{R}}$ in (6) and process it using a CG based algorithm. As can be seen in [12], one can calculate the *a priori* covariance matrix $\boldsymbol{C}$ in (9) by means of the steering vectors in (4) based on the known directions of impinging signals. The proposed alternative method considers the system model described in Section II and is composed of three stages. The first stage estimates the unknown DOAs, making use of the CG algorithm [8], [10]. The second stage encompasses two substeps. The first substep is to calculate the *a priori* covariance matrix $\boldsymbol{C}$, using the steering vectors of part of the preceding estimates. The second substep is to obtain an enhanced covariance matrix $\tilde{\boldsymbol{R}}$ (7) or (8), making use of $\boldsymbol{C}$, the covariance sample $\hat{\boldsymbol{R}}$ (2) and the weight factors according to the combination to be applied. For KA-General Linear Combination (KA-GLC), the factors are $\hat{\alpha}_{\mathrm{o}}$ (11) and $\hat{\beta}_{\mathrm{o}}$ and (10). In the case of KA-Convex Combination (KA-CC), $\hat{\alpha_0}$ is given by (15). Our proposed KA-CG algorithm makes use of the latter approach. In both cases, the covariance matrix to be applied to the first stage of the KA-CG algorithm is obtained by the sample average formula given in (2). In the last stage, the first stage is repeated after replacing the covariance matrix $\hat{\boldsymbol{R}}$ with $\tilde{\boldsymbol{R}}$ given in (7) or (8) to further enhance the estimates of the DOAs.

The CG method, which the first and the last stages of the KA-CG are based on, is used to minimize a cost function, or analogously, to solve a linear system of equations by approaching the optimal solution step by step via a line search along successive directions, which are sequentially determined at each direction [11]. As a result of the application of the CG algorithm to direction finding, we have a system of equations that is iteratively solved for $\mathbf{w}$ at each search angle:

$$\mathbf{R}\mathbf{w} = \mathbf{b}(\theta), \tag{16}$$

where $\mathbf{R}$ is the covariance matrix and $\mathbf{b}(\theta)$ is the initial vector defined as

$$\mathbf{b}(\theta) = \frac{\mathbf{R}\,\mathbf{a}(\theta)}{\|\mathbf{R}\,\mathbf{a}(\theta)\|} \tag{17}$$

where $\mathbf{a}(\theta)$ is the search vector.

The extended signal subspace of rank P is obtained by means of the CG algorithm summarized in the first stage of the Table I. The set of orthogonal residual vectors

$$\mathbf{G}_{cg,P+1}(\theta) = [\mathbf{g}_{cg,0}(\theta), \mathbf{g}_{cg,1}(\theta), \ldots, \mathbf{g}_{cg,P}(\theta)], \tag{18}$$

where $\mathbf{b}(\theta)$= $\mathbf{g}_0(\theta)$ generates the well-known extended Krylov subspace comprised of the true signal subspace of dimension P and the search vector itself. All the residual vectors are normalized except for the last one. If $\theta \in \{\theta_1, \ldots \theta_P\}$, the initial vector $\mathbf{b}(\theta)$ lies in the true signal subspace space spanned by the $[\mathbf{g}_{cg,0}(\theta), \mathbf{g}_{cg,1}(\theta), \ldots, \mathbf{g}_{cg,P-1}(\theta)]$ basis vectors of the extended Krylov subspace. Therefore, the rank of the generated signal subspace drops from P+1 to P and we have

$$\mathbf{g}_{cg,P}(\theta) = 0, \tag{19}$$

where $\mathbf{g}_{cg,P}$ is the last unnormalized residual vector.

In order to exploit this behavior, the proposed KA-CG algorithm makes use of the spectral function defined in [15]:

$$\mathcal{P}_{\mathcal{K}}(\theta^{(n)}) = \frac{1}{\|\mathbf{g}^H_{cg,P}(\theta^{(n)})\mathbf{G}_{cg,P+1}(\theta^{(n-1)})\|^2}, \tag{20}$$

where $\theta^{(n)}$denotes the search angle in the whole angle range $\{-90^o, \ldots, 90^o\}$ with $\theta^{(n)} = n\Delta^o - 90^o$, where Δ^o is the search step and $n = 0, 1, \ldots, 180^o/\Delta^o$. The matrix $\mathbf{G}_{cg,P+1}(\theta^{(n-1)})$ contains all residual vectors at the $(n-1)$th angle and $\mathbf{g}_{cg,P}(\theta^{(n)})$ is the last residual vector calculated at the current search step n. If $\theta^{(n)} \in \{\theta_1, \ldots, \theta_P\}$, $\mathbf{g}_{cg,P}(\theta^{(n)}) = 0$ and we can expect a peak in the spectrum. Taking into account that $\hat{\boldsymbol{R}}$ in (2) is only a sample average estimate, which is unknown in practical applications, $\mathbf{g}_{cg,P}(\theta^{(n)})$ and $\mathbf{G}_{cg,P+1}(\theta^{(n-1)})$ become approximations. Hence the spectral function in (20) can just provide very large values but they do not tend to infinity as for the original covariance matrix.

TABLE I
PROPOSED KA-CONJUGATE GRADIENT ALGORITHM

First stage:

$\mathbf{w}_0 = 0,\ \mathbf{d}_1 = \mathbf{g}_{cg,0} = b,\ \rho_0 = \mathbf{g}^H_{cg,0}\mathbf{g}_{cg,0}$

for i=1 to P do:

$\mathbf{v}_i$	$= \mathbf{R}\,\mathbf{d}_i$
α_i	$= \rho_{i-1}$ / $\mathbf{d}_i^H\mathbf{v}_i$
$\mathbf{w}_i$	$= \mathbf{w}_{i-1} + \alpha_i\mathbf{d}_i$
$\mathbf{g}_{cg,i}$	$= \mathbf{g}_{cg,i-1} - \alpha_i\mathbf{v}_i$
ρ_i	$= \mathbf{g}^H_{cg,i}\mathbf{g}_{cg,i}$
β_i	$= \rho_i$ / $\rho_{i-1} = \|\mathbf{g}_{cg,i}\|^2$ / $\|\mathbf{g}_{cg,i-1}\|^2$
$\mathbf{d}_{i+1}$	$= \mathbf{g}_{cg,i} + \beta_i\mathbf{d}_i$

end for

form	$\mathbf{G}_{cg,P+1}(\theta)$ (18)
compute	$\mathcal{P}_{\mathcal{K}}(\theta^{(n)})$ (20)
find	$\hat{P}$ largest peaks of $\mathcal{P}_{\mathcal{K}}(\theta^{(n)})$ to obtain estimates $\hat{\theta}_l$ of the DOA

Second stage:

compute	$\boldsymbol{C}$ (9), for $\theta_l = \hat{\theta}_l$, $l < P$
compute	$\hat{\alpha_0}$ (15) for convex combination or $\hat{\beta}_0$ (10) and $\hat{\alpha_0}$ (11) for general linear combination
compute	$\tilde{\boldsymbol{R}}$ (7) or (8) according to the combination in use as previously mentioned

Last stage:

Repeat	the first stage to obtain enhanced estimates of DOA making use of $\tilde{\boldsymbol{R}}$ instead of $\mathbf{R}$

V. SIMULATIONS

In this section, we focus on the estimation performance of the proposed Knowledge-Aided Conjugate Gradient (KA-CG) algorithm for direction finding and localization techniques. Specifically, we evaluate the probability of resolution of two signals with closely-spaced angles. For this purpose, we compare the KA-CG, the KA-ESPRIT and the KA-MUSIC, where the *a priori* covariance matrices $\boldsymbol{C}$ (9) are based on estimates, to their original versions and also to their KAv versions, in which $\boldsymbol{C}$ is constructed with known DOAs. All experiments are based upon a scenario with $P = 2$ equal-power uncorrelated closely-spaced signals at $(89.05, 90.95)^{\circ}$ impinging on a ULA with $M = 12$ sensors equally spaced by half wavelength. The sample matrix (2) is computed with 180 snapshots and the simulated curves are obtained by averaging the results over 200 independent trials. We consider that in the the KA versions, $\boldsymbol{C}$ is calculated using the steering vector formed with one of the estimates obtained in the first stage, whereas in the KAv versions the steering vectors are formed supposing that the second DOA is already known. In order to assess the accuracy in terms of probability of resolution, we take into account the criterion [13],[15], in which two sources with DOA θ_1 and θ_2 are said to be resolved if their respective estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ are such that both $\left|\hat{\theta}_1 - \theta_1\right|$ and $\left|\hat{\theta}_2 - \theta_2\right|$ are less than $\left|\theta_1 - \theta_2\right|/2$.

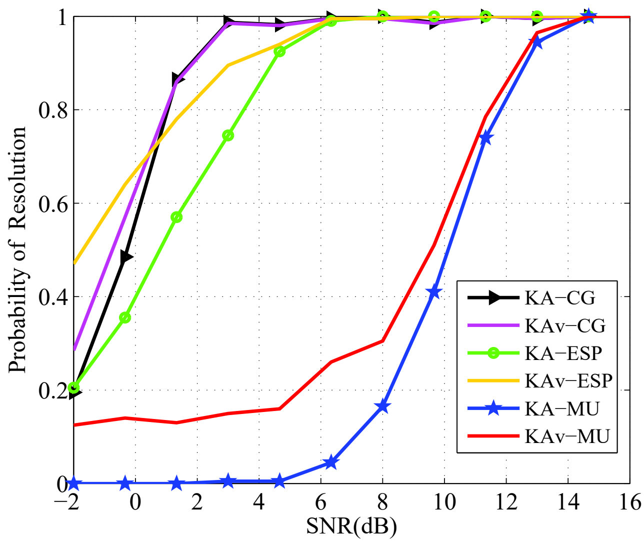

Fig. 1. Probability of resolution of the KA versions and KAv versions of CG, ESPRIT and MUSIC versus SNR with $M = 12$, $N = 180$, $P = 2$, $L = 200$ runs, unknown uncorrelated sources at $(89.05, 90.95)^{\circ}$

In the first experiment, we compare the probability of resolution of the KA-CG, KA-ESPRIT and KA-MUSIC, to their KAv versions, in which the *a priori* covariance matrices $\boldsymbol{C}$ (9) are obtained from the steering vector of the second DOA, which is supposed to be known. The results depicted in Fig.1 show the best performance of each KAv version, in which $\boldsymbol{C}$ is obtained by the configuration with one known

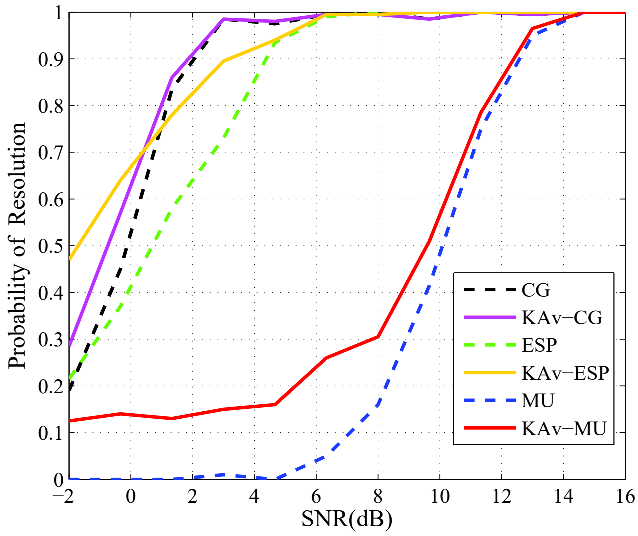

Fig. 2. Probability of resolution of the KA versions and original versions of CG, ESPRIT and MUSIC versus SNR with $M = 12$, $N = 180$, $P = 2$, $L = 200$ runs, unknown uncorrelated sources at $(89.05, 90.95)^{\circ}$

DOA, over its KA version, where $\boldsymbol{C}$ is calculated using one of the estimates.

Each KAv-version can be considered an upper bound of its KA version. Thus, it can be noticed that the small area limited by KA-CG and KAv-CG shows that the former already exploits its potential close to the effective optimal performance. The gap available to improvements is situated within [-1.9, 1.6] SNR(dB) where the probability of resolution is lower than 0.88. Differently from the previous KA-CG case, there is a larger area limited by KA-ESPRIT and KAv-ESPRIT that is available to enhancements. It can also be seen that their effective optimal performance (KAv-ESPRIT) is outperformed by both KAv-CG and KA-CG. The area limited by KA-MUSIC and KAv-MUSIC shows that most of the potential to be exploited is situated at the lower levels of the probability of resolution and that the potential of improvement of the KA-MUSIC is poor at higher ones.

In the last experiment, we compare the probability of resolution of KA-CG, KA-ESPRIT and KA-MUSIC to their original versions [8], [17], [16]. The results depicted in Fig.2 make clear the potential of the latter ones to be exploited in terms of probability of resolution.

VI. CONCLUSION

A novel knowledge-aided DOA estimation technique employing the classical CG algorithm has been proposed. Differently from the standard knowledge-aided DOA methods, based on known DOA, it only exploits estimates of part of the unknown uncorrelated closely-spaced sources and substantially improves the estimation accuracy of two unknown sources, which enter the system. The enhanced signal covariance matrix estimate $\tilde{\boldsymbol{R}}$ is obtained by adaptively combining the *a priori* covariance matrix $\boldsymbol{C}$, based upon steering vectors of the estimates, and the sample covariance matrix $\hat{\boldsymbol{R}}$, in a minimum mean squared error sense. The proposed KA-CG can be applied to scenarios in which the sources are close to each other and do not provide any restrictions on the rank of the *a priori* covariance matrix. Simulation results show that processing prior knowledge significantly increases the probability of resolution of unknown sources at low-to-medium SNR values, requiring a reasonable number of samples to this end.

REFERENCES

[1] H. Krim and M. Viberg, "Two decades of array signal processing research: parametric approach," *IEEE Signal Processing Magazine*, vol. 13, no. 4, pp. 67–94, July 1996.

[2] W. L. Melvin and J. R. Guerci, "Knowledge-aided signal processing: a new paradigm for radar and other advanced sensors," *IEEE Trans. Aero. Elec. Syst.*, vol. 42, no. 3, pp. 983– 996, 2006, 0018-9251.

[3] W. L. Melvin and G. A. Showman, "An approach to knowledge-aided covariance estimation," *IEEE Trans. Aero. Elec. Syst.*, vol. 42, no. 3, pp. 1021–1042, 2006.

[4] J. S. Bergin, C. M. Teixeira, P. M. Techau, and J. R. Guerci, "Improved clutter mitigation performance using knowledgeaided space-time adaptive processing," *IEEE Trans. Aero. Elec. Syst.*, vol. 42, no. 3, pp. 997–1009, 2006, 0018-9251.

[5] X. Zhu, J. Li and P. Stoica, "Knowledge-aided adaptive beamforming", *IET Signal Processing*, 2008, Vol. 2, No. 4, pp. 335-–345.

[6] R. Fa, R. C. de Lamare and V. H. Nascimento, "Knowledge-aided STAP algorithm using convex combination of inverse covariance matrices for heterogenous clutter," *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, pp.2742-2745, 14-19 March 2010.

[7] R. Fa and R. C. de Lamare, "Knowledge-aided reduced-rank STAP for MIMO radar based on joint iterative constrained optimization of adaptive filters with multiple constraints," *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, pp.2762-2765, 14-19 March 2010.

[8] H. Semira, H. Belkacemi and S.Marcos "High-Resolution Source Localization Algorithm Based on the Conjugate Gradient", *EURASIP Journal on Advances in Signal Processing*, vol. 2007, Article ID 73871, 9pp.

[9] H. Semira, H. Belkacemi and N.Doghmane "A novel Conjugate Gradient-Based Source Localization Algorithm", 9^{th} *International Symposium on Signal Processing and Its Applications*, pp.1-4, 2007.

[10] H. Semira, H. Belkacemi and N.Marcos "High-Resolution Direction Finding Using Krylov Subspace", 4^{th} *International Conference Sciences of Electronic Technologies of Information and Telecommunications*, March 25-29, 2007, 6pp.

[11] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd ed. Johns Hopkins University Press, Baltimore, 1996.

[12] P. Stoica, J. Li, X. Zhu, and J. R. Guerci, "On using a priori knowledge in space-time adaptive processing," *IEEE Trans. Sig. Proc.*, vol. 56, no. 6, pp. 2598-–2602, 2008.

[13] P.Stoica and A.B.Gershman, "Maximum-likelihood doa estimation by data-supported grid search", IEEE Signal Processing Letters, vol. 6, no. 10, pp. 273? 275, Oct 1999.

[14] H. L. Van Trees, *Detection, Estimation, and Modulation, Part IV, Optimum Array Processing*, John Wiley &Sons, 2002.

[15] R. Grover, D. A. Pados and M. J. Medley "Subspace Direction Finding With an Auxiliary-Vector Basis", *IEEE Trans on Signal Processing*,vol. 55, No.2 Feb 2007, pp 758-763.

[16] R. Schmidt, "Multiple emitter location and signal parameter estimation" *IEEE Trans on Antennas and Propagation*, vol.34, No.3, Mar 1986, pp 276-280.

[17] R. Roy and T. Kailath, "Estimation of signal parameters via rotational invariance techniques", *IEEE Trans. Acoust., Speech., Signal Processing*, vol. 37, July 1989, pp 984-995.

[18] Wang Ling and Shuzhi Gang Ge, "Localization Method of Multiple targets by Crossing Direction Finding", *IEEE 5th International Conference on Cybernetics and Intelligent Systems*, 17-19 September 2011, pp 190-195.