# Tensor-based Space-Time Multiplexing (TSTM) for MIMO-OFDM Systems: Receiver Algorithms and Performance Evaluation

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Resumo-Em um trabalho recente [3], os autores propuseram uma nova técnica de multiplexagem espaço-temporal para sistemas de comunicação sem-fio MIMO-OFDM. Esta técnica é denominada TSTM (do inglês, Tensor-based Space-Time Multiplexing), e combina multiplexagem ortogonal e espalhamento espaçotemporal de multiplas sequências de simbolos através de uma abordagem tensorial para a modelagem dos sinais transmitido e recebido. A partir de tal modelagem tensorial, é possivel realizar a detecção/separação cega dos sinais transmitidos utilizando o algoritmo ALS (do inglês, Alternating Least Squares). Neste artigo, o desempenho de receptores para a técnica TSTM é estudado. Focalizando uma configuração pratica de receptor, considera-se o uso do estimador de canal baseado em tons piloto (do inglês, Pilot Assisted Channel Estimation (PACE)) em conjunto com o receptor ALS. Uma adaptação do receptor ALS para canais variantes no tempo é proposta. Visando avaliar melhor os méritos da técnica TSTM, seu desempenho é comparado com aqueles obtidos com esquemas MIMO classicos, tais como a multiplexagem espacial (do inglês, Spatial Multiplexing (SM)) e a diversidade de transmissão ortogonal (do inglês, Orthogonal Transmit Diversity (OTD). A avaliação de desempenho é realizada através de resultados de simulação computacional.

*Palavras-Chave*—Algoritmo dos minimos quadrados alternados, detecção cega, sistemas MIMO-OFDM, multiplexagem espaço-temporal, modelagem tensorial.

Abstract—In a recent work [3], we have proposed a new spacetime multiplexing technique for Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) wireless communication systems. The so-called Tensor-based Space-Time-Multiplexing (TSTM) technique combines multistream orthogonal spatial multiplexing with space-time spreading and relies on a tensor modeling of the transmitted and received signals. Based on this model, a blind separation/detection of the transmitted data streams is achieved by means of the Alternating least Squares (ALS) algorithm. In this paper, we study the receiver performance of the TSTM technique. Focusing on a more practical MIMO-OFDM setting, we consider Pilot-Assisted Channel Estimation (PACE) in conjunction with the ALS-based receiver. A adaptation of the ALS receiver for time-varying channels is proposed. In order to further evaluate the merits of the TSTM transceivers, we situate its performance with respect to those of classical MIMO schemes such as Spatial Multiplexing (SM) and Orthogonal Transit Diversity (OTD) by means of computer simulation results.

*Keywords*—Alternating least squares algorithm, blind detection, MIMO-OFDM systems, space-time multiplexing, tensor modeling.

# I. INTRODUCTION

Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) wireless communication systems have been the focus of intensive research in the past few years [1]. In MIMO-OFDM, the transmit antennas can be employed to achieve high data rates via spatial multiplexing as well as to improve link reliability through space-time/spacefrequency or space-time-frequency coding [2].

In [3], the authors have introduced a new framework to space-time multiplexing for MIMO-OFDM that affords a variable degree of space-frequency spreading and multiplexing over a frequency-selective MIMO channel. The socalled Tensor-based Space-Time-Multiplexing (TSTM) technique combines multi-stream orthogonal spatial multiplexing with space-time spreading and relies on a tensor modeling of the space-time multiplexing process. With this approach, it is possible to trade-off multiplexing, space-frequency spreading and rate in a simple way by adjusting a precoding tensor structure. The TSTM technique allows the use of blind separation/detection of the transmitted data streams by means of the Alternating least Squares (ALS) algorithm. The TSTM technique is a generalization of the tensor-based space-time codes of [4] and [5] to frequency-selective channels.

In this paper, we provide further results on the TSTM technique for MIMO-OFDM systems. Our aim is to clarify its key properties and merits by focusing on a more practical MIMO-OFDM setting. We consider Pilot-Assisted Channel Estimation (PACE) [6] in conjunction with the ALS-based receiver. The idea is to evaluate to what extent performance gains are obtained when PACE is used to initialize the ALS estimates. A adaptation of the ALS receiver for time-varying channels is proposed. Moreover, we compare its performance with those of Spatial Multiplexing (SM) [7] and Orthogonal Transit Diversity (OTD) [8] schemes. Our performance study is completed by evaluating the average throughput performance of this technique for some transmitter configurations.

This paper is organized as follows. Section II present the MIMO-OFDM system model using the TSTM technique. In this section, the tensor modeling of the transmitted and the received signal is introduced. The precoding structure is also described in this section. In Section III, the receiver algorithms are detailed. An adaptation of the ALS-based receiver for time-varying channel is also presented in this section. Performance evaluation is carried out in Section IV from computer simulation results. The paper is concluded in Section V.

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Fig. 1. Proposed MIMO-OFDM system.



Fig. 2. TSTM visualization in a 3D coordinate system.

#### II. SYSTEM MODEL FOR TSTM

Figure 1 illustrates the considered MIMO-OFDM system. At the transmitter, an antenna array of  $M_T$  transmit antennas is divided into Q transmission groups of  $M' = M_T/Q$  antennas each. Each group transmits its own data stream. For each group, the input stream is demultiplexed into R sub-streams, which are spread over a  $M' \times P \times F$  space-time-frequency grid associated with M' transmit antennas, P OFDM symbols and F sub-carriers, and then linearly combined. These transformations are represented by the TSTM block in Figure 1 and will be detailed in Section II-B.

At the output of the TSTM block, the resulting signal and each transmit antenna is parsed into blocks of  $N_c$  symbols, and an Inverse Fast Fourier Transform (IFFT) is applied to each block followed by the insertion of a Cyclic Prefix (CP) before transmission. It is assumed that the Q transmission groups occupy the same frequency band and use the same set of subcarriers at the same time. The Q input data-streams can be either assigned to a single receiver/user (point-to-point MIMO) or they can be assigned to Q different receivers/users (point-tomultipoint MIMO). Although we do not distinguish between these two scenarios here, the proposed approach is valid for both cases. The receiver is equipped with  $M_R$  antennas. After baseband conversion, the CP is removed and Fast Fourier Transform (FFT) is applied at each receive antenna. Each timeslot is defined as the collection of P OFDM symbols.

## A. Matrix-based signal model

The discrete-time baseband equivalent model for the received signal is given by:

$$\mathbf{X}_n = \breve{\mathbf{H}} \mathbf{C}_n^T + \mathbf{V}_n, \tag{1}$$

where  $\mathbf{X}_n = [\mathbf{X}_{1,n}^T \cdots \mathbf{X}_{F,n}^T]^T \in \mathbb{C}^{FM_R \times P}$  is a matrix joining the received samples of F sub-carriers during the P OFDM symbols of the *n*-th time-slot, and

are the space-frequency MIMO channel and the code matrix transmitted at the *n*-th time-slot, n = 1, ..., N, both composed of Q blocks.  $\mathbf{V}_n \in \mathbb{C}^{FM_R \times P}$  is the additive noise matrix. In (1), we have absorbed the transmit power normalization factor  $\sqrt{\rho/M_T}$  into  $\mathbf{C}_n$  to simplify notation, where  $\rho$  is the Signal-to-Noise Ratio (SNR) at each receive antenna. The noise is assumed to be spatially and temporally white. The channel matrix is assumed to have i.i.d. entries following a zero-mean unit-variance complex-Gaussian distribution with  $E[\|\mathbf{\check{H}}\|_F^2] = M_T M_R F$ . We also have  $E[\|\mathbf{C}_n\|_F^2] = M_T P$ , n = 1, ..., N, and  $\|\cdot\|_F$  is the Frobenius norm.

#### B. Tensor-based signal model (TSTM)

We interpret the space-time coder at each sub-carrier and each group, as a third-order tensor  $\mathcal{W}^{(q)} \in \mathbb{C}^{P \times M' \times R}$ . The tensor coder has three dimensions: the first one equal to the number of transmit antennas, the second one corresponds to the code length while the third-one is equal to the number of demultiplexed data sub-steams. The coded signal is represented by the tensor  $C^{(q)} \in \mathbb{C}^{P \times M' \times N}$  collecting N codewords. Figure 2 illustrates the decomposition of the received signal in tensor form. It can be seen that the received signal tensor is a contribution of Q tensor signal components, each one of which being decomposed in terms of three factors, which are the associated channel matrix, multiplexing tensor and symbol matrix.

Let

$$\mathbf{s}_{n}^{(q)} = [s_{1,n}^{(q)} \cdots s_{R,n}^{(q)}]^{T} \in \mathbb{C}^{R},$$
(3)

be a symbol matrix concatenating R data sub-streams, and

$$\mathbf{S}^{(q)} = [\mathbf{s}_1^{(q)} \cdots \mathbf{s}_N^{(q)}]^T \in \mathbb{C}^{N \times R}.$$
(4)

The entries of  $\mathbf{S}^{(q)}$  are chosen from an arbitrary *J*-Phase Shift-Keying (PSK) or *J*-Quadrature Amplitude Modulation (QAM) constellation and satisfy the power constraint  $E[Tr(\mathbf{S}^{(q)}\mathbf{S}^{(q)H})] = NR$ . The TSTM precoding is defined a as transformation involving two tensor spaces, i.e., as a one-to-one mapping:

$$\mathcal{W}^{(q)}: \mathbf{S}^{(q)} \to \mathcal{C}^{(q)}, \quad q = 1, \dots, Q.$$

In scalar notation, the coding process can be written as:

$$c_{p,m',r} = \sum_{\substack{r=1\\r \neq 1}}^{R} w_{p,m',r}^{(q)} s_{n,r}^{(q)} = c_{p,m',n}^{(q)}.$$
 (5)

 $c_{p,m',n}^{(q)}$ ,  $w_{p,m',r}^{(q)}$  and  $s_{n,r}^{(q)}$  are typical elements of the code tensor, multiplexing tensor and symbol matrix respectively.

The key step towards the derivation of the received signal model is based on the observation that (1) and (5) can be combined to yield a block-tensor ("constrained block-PARAFAC") model for the received signal [9]. It can be shown [?] that the received signal tensor can be written, in absence of noise, as:

$$x_{f,k,p,n} = \sum_{q=1}^{Q} \sum_{m'=1}^{M'} \check{h}_{f,k,m'}^{(q)} \sum_{r=1}^{R} w_{m',p,r}^{(q)} s_{n,r}^{(q)},$$

where  $x_{f,k,p,n}$  is the received signal sample at the *k*-th receive antenna, *f*-th sub-carrier, *p*-th OFDM symbol and *n*-th timeslot.  $\check{h}_{f,k,m'}^{(q)}$  is linked to  $\breve{\mathbf{H}}$  in (2) as follows:

$$\check{h}_{f,k,m'}^{(q)} = [\check{\mathbf{H}}]_{(f-1)M_R+k,(q-1)M'+m'}$$

Note that  $x_{f,k,p,n}$  is a  $F \times M_R \times P \times N$  fourth-order tensor. However, we are interested in working with an equivalent  $FM_R \times P \times N$  third-order tensor as shown in Figure 2. Let us define a "matrix unfolding" of  $x_{f,k,p,n}$  as:

$$[\mathbf{X}_n]_{(f-1)M_R+k,p} = x_{f,k,p,n}.$$

Recalling (4), we define:

$$\mathbf{S} = [\mathbf{S}^{(1)} \cdots \mathbf{S}^{(Q)}] \in \mathbb{C}^{N \times QR}$$

as a symbol matrix concatenating the symbol matrices of the Q transmission groups. Let

$$\mathbf{X}_1 = [\operatorname{vec}(\mathbf{X}_1) \cdots \operatorname{vec}(\mathbf{X}_N)] \in \mathbb{C}^{PFM_R \times N}$$

be a matrix that collects the received signal samples over N time-slots. The operator  $vec(\mathbf{A})$  stacks the columns of  $\mathbf{A} \in$ 

 $\mathbb{C}^{I \times J}$  in a vector  $\mathbf{a} \in \mathbb{C}^{IJ \times 1}$ . It can be shown [5], [9] that  $\mathbf{X}_1$  admits a constrained block-PARAFAC representation given by:

$$\mathbf{X}_1 = (\mathbf{W} \diamond \mathbf{\check{H}} \mathbf{\Psi}) (\mathbf{S} \mathbf{\Phi})^T, \tag{6}$$

where  $\Psi = \mathbf{I}_{M_T} \otimes \mathbf{I}_R^T$  and  $\Phi = \mathbf{I}_Q \otimes \mathbf{I}_{M'}^T \otimes \mathbf{I}_R$  are constraint matrices of the tensor model of dimensions  $M_T \times RM'$  and  $QR \times RM'$  respectively. The operator  $\diamond$  stands for the Khatri-Rao product defined as:

$$\mathbf{A} \diamond \mathbf{B} = [\mathbf{a}_1 \otimes \mathbf{b}_1, \dots, \mathbf{a}_R \otimes \mathbf{b}_R]$$

with  $\mathbf{A} = [\mathbf{a}_1 \dots \mathbf{a}_R] \in \mathbb{C}^{I \times R}$ ,  $\mathbf{B} = [\mathbf{b}_1 \dots \mathbf{b}_R] \in \mathbb{C}^{J \times R}$ , and  $\otimes$  is the Kronecker product operator. Note that  $\Psi$  and  $\Phi$  are only depend on the transmit parameters M', R and Q which are known to the receiver.

The symmetry of the trilinear model allows us to rewrite  $X_1$  in two other "reshaped" forms [9]:

$$\mathbf{X}_{2} = (\mathbf{S} \mathbf{\Phi} \diamond \mathbf{W}) (\breve{\mathbf{H}} \mathbf{\Psi})^{T} \in \mathbb{C}^{NP \times FM_{R}},$$

$$\mathbf{X}_{3} = (\breve{\mathbf{H}} \mathbf{\Psi} \diamond \mathbf{S} \mathbf{\Phi}) \mathbf{W}^{T} \in \mathbb{C}^{FM_{R}N \times P}.$$

$$(7)$$

 $\mathbf{X}_{i=1,2,3}$  are different rearrangements of the received signal tensor shown in Figure 2. The thre received signal representations given in (6) and (7) will be exploited for blind detection at the receiver in Section III.

## C. Precoding structure

In [3], a design criterion and diversity analysis of the TSTM model was described. We have arrived at the condition that W must be full column rank in order to allow a maximum diversity gain, which requires  $P \ge RM_T$ . Assuming F > L, the diversity gain r is upper-bounded by:

$$r \le KLmin(P, M). \tag{8}$$

Concerning the chosen structure of the precoding matrix **W**, we first consider a joint Vandermonde-Hadamard design with:

$$\mathbf{W} = \sqrt{\frac{1}{R}} \left( \mathbf{\Omega}_1 \otimes \mathbf{\Omega}_2 \right), \in \mathbb{C}^{P \times RM_T}, \tag{9}$$

$$\Omega_1, \text{ Hadamard matrix } (Q \times Q),$$

$$\Omega_2, \text{ Vandermonde matrix } (P' \times RM'),$$

$$[\Omega_2]_{i,p'} = e^{j2\pi \frac{(i-1)}{RM'}(p'-1)},$$

$$i = (r-1)M' + m',$$
(10)

and P' = P/Q is an integer.  $\Omega_1$  and  $\Omega_2$  are called *between*group and within-group multiplexing matrices, respectively.  $\Omega_1$  controls the number of orthogonal transmission groups while  $\Omega_2$  controls the multiplexing and diversity within each group. This structure allows one to control of the multiplexingdiversity pattern within each transmission group, as well as of the number Q of transmission groups. The overall rate of the this TSTM scheme is given by:

$$Rate = \left(\frac{QR}{PF}\right) \cdot \log_2(\mu) \text{ bits/channel use,}$$
(11)

where  $\mu$  is the modulation order.

#### **III. RECEIVER ALGORITHMS**

In this section, we present some receiver algorithms that can be used for joint channel estimation and detection. First, we briefly review the classical ALS algorithm. Then, we describe the Pilot-Assisted Channel Estimation (PACE) method. A receiver adaptation for coping with time-varying channels is also proposed for channel tracking.

## A. Standard ALS algorithm

The standard TSTM receiver is a blind PARAFAC-based receiver for a joint channel/symbol estimation. It is based on the Alternating Least Squares (ALS) algorithm [4] is used for this purpose, which consists in fitting the constrained block-PARAFAC model (6) in a Least Squares (LS) sense to the received signal tensor, by alternating between the estimations of  $\mathbf{H}$  and  $\mathbf{S}$ . The cost function is given by:

$$J(\breve{\mathbf{H}}, \mathbf{S}) = \left\| \mathbf{X}_1 - (\mathbf{W} \diamond \breve{\mathbf{H}} \mathbf{\Psi}) (\mathbf{S} \mathbf{\Phi})^T \right\|_F^2$$

At the channel-blind receiver beginning of the algorithm, an initial estimate for  $\tilde{\mathbf{H}}_{i=0}$  is obtained either by using random initialization (channel-blind receiver) or by using a prior estimation based, e.g. on the use of pilot-assisted channel estimation. We consider both cases in our simulation results. At the *i*-th iteration, the update equations for  $\hat{\mathbf{S}}_i$  and  $\tilde{\mathbf{H}}_i$  are given by (see (6)-(7)):

$$\begin{aligned} \widehat{\mathbf{S}}_{i}^{T} &= \left[ (\mathbf{W} \diamond \widetilde{\breve{\mathbf{H}}}_{i-1} \Psi) \Phi^{T} \right]^{\dagger} \mathbf{X}_{1}, \\ \widehat{\breve{\mathbf{H}}}_{i}^{T} &= \left[ (\widehat{\mathbf{S}}_{i} \Phi \diamond \mathbf{W}) \Psi^{T} \right]^{\dagger} \mathbf{X}_{2} \end{aligned}$$

At the end of the *i*-th iteration, an overall error measurement between the estimated model and the received signal tensor is formed and compared to the error obtained in the previous estimation step. We declare that the convergence has been achieved if the difference between the actual error and the previous one falls below a certain prescribed threshold.

#### B. Pilot-Assisted Channel Estimation (PACE)

In practice, training sequence in the form of pilot symbols are available for channel estimation. In the context of the proposed TSTM receiver, Pilot-Assisted Channel Estimation (PACE) can be used for obtaining an initial (more accurate) initialization of the channel matrix for the ALS algorithm shown in (12). The PACE method consists in estimating the MIMO-OFDM channel by means of an LS method that uses an orthogonal training sequence structure [6]. Written using Khatri-Rao notation, PACE estimate is given by:

$$\breve{\mathbf{H}}_{tr} = \mathbf{\Gamma}(\mathbf{X}_{tr} \diamond \mathbf{\Gamma}^H) \mathbf{S}_{tr}^H,$$

where  $\mathbf{S}_{tr}^{H} \in \mathbb{C}^{M_T \times F}$  is the training sequence matrix,  $\Gamma \in \mathbb{C}^{F \times L}$  is the FFT matrix and  $\mathbf{X}_{tr} \in \mathbb{C}^{M_R \times F}$  is the received signal matrix associated with the training period. After the channel estimate, the transmitted data symbols can be recovered in the LS sense:

$$\widehat{\mathbf{S}}^T = \left[ (\mathbf{W} \diamond \widehat{\breve{\mathbf{H}}}_{tr} \Psi) \Phi^T 
ight]^{\dagger} \mathbf{X}_1.$$

In our simulation results, PACE will be used in two different manners: i) for direct comparison with the blind TSTM receiver and ii) for initialization of the TSTM receiver.

## C. Receiver Adaptation for Tracking Time-Varying Channels

Now, we suppose that the channel smoothly varies from slot-to-slot during the N time-slots, the variation rate depending on the Doppler shift. The following time-varying model is adopted:

$$\dot{\mathbf{H}}(n+1) = \dot{\mathbf{H}}(n) \exp(j2\pi f_D), \quad n = 1, \dots, N, \quad (12)$$

where  $\mathbf{H}(n) \in \mathbb{C}^{FM_R \times M_T}$  is the MIMO channel matrix at the *n*-th time-slot and  $f_D$  (normalized by the symbol period) may include the carrier frequency offset and the Doppler shift.

In order to cope with a time-varying channel, an adaptation of the standard ALS-based receiver for tracking timevariations of the channel is presented here. It consists in a *slotby-slot* version of ALS with the use of hard decision within each iteration of the algorithm. For the *n*-th time-slot, the received signal can be written in "vectorized" form as follows:

$$\begin{aligned} \mathbf{x}(n) &= \operatorname{vec}\left(\breve{\mathbf{H}}(n) \boldsymbol{\Psi} D_n(\mathbf{S} \boldsymbol{\Phi}) \mathbf{W}^T\right) \\ &= \left[\left(\mathbf{W} \diamond \breve{\mathbf{H}}(n) \boldsymbol{\Psi}\right) \boldsymbol{\Phi}^T\right] \mathbf{s}(n), \end{aligned}$$

where  $\mathbf{x}(n)$  and  $\mathbf{s}(n)$  are vectors containing, respectively, the received signal and transmitted symbols at the *n*-th time-slot. The cost function to be minimized at the *n*-th time-slot is:

$$J(\breve{\mathbf{H}}(n), \mathbf{s}(n)) = \left\| \mathbf{x}(n) - \left( \mathbf{W} \diamond \breve{\mathbf{H}}(n) \Psi \right) \mathbf{\Phi}^T \mathbf{s}(n) \right\|_F^2.$$
(13)

The tracking stage of the ALS algorithm starts with initial channel matrix  $\tilde{\mathbf{H}}_{old}$ , which can be randomly initialized or equal to previously obtained estimate (e.g. from the pilot-assisted PACE method). A first estimation of a symbol matrix block is obtained by using the first  $N_o$  time-slots:

$$\widehat{\mathbf{S}}_{old}^{T} = \left[ (\mathbf{W} \diamond \widehat{\vec{\mathbf{H}}}_{old} \Psi) \Phi^{T} \right]^{\dagger} \mathbf{X}_{1}(:, 1: No),$$

where  $\mathbf{X}_1(:, 1: No) \in \mathbb{C}^{PFM_R \times N_o}$  is a matrix collecting the first  $N_o$  received time-slots (i.e., the first  $N_o$  columns of  $\mathbf{X}_1$ ).  $N_o$  should be as small as possible so that the channel time-variation over  $N_o$  time-slots is minimized, which will facilitate the channel tracking capability of the algorithm. On the other hand  $N_o$  must be greater than or equal to QR, since a full column-rank  $\widehat{\mathbf{S}}$  is necessary for LS estimation. Therefore we choose  $N_o = QR$ .

At the n-th received time-slot, the corresponding symbol vector is estimated by minimizing (13) is the LS sense:

$$\hat{\mathbf{s}}(n) = \left[ \left( \mathbf{W} \diamond \breve{\mathbf{H}}_{old} \Psi \right) \Phi^T \right]^{\dagger} \mathbf{x}(n), \qquad (14)$$

where  $\mathbf{x}(n) = \mathbf{X}_1(:, N_o + n)$ . This is followed by a hard decision (FA projection):

$$\tilde{\hat{\mathbf{s}}} = \det(\hat{\mathbf{s}}(n)). \tag{15}$$

Then, a new (updated) symbol matrix  $\widehat{\mathbf{S}}_{new}$  is formed in the following manner:

$$\widehat{\mathbf{S}}_{new}(1:N_o-1,:) \leftarrow \widehat{\mathbf{S}}_{old}(2:N_o,:), = \widehat{\mathbf{S}}_{new}(N_o,:) = \widetilde{\mathbf{s}}^T, \quad (16)$$

where  $\widehat{\mathbf{s}}_{old}^T(N_o)$  represents the last row of  $\widehat{\mathbf{S}}_{old}$ . After updating  $\widehat{\mathbf{S}}_{new}$ , we form:

$$\widehat{\mathbf{Z}}_{(\widehat{\mathbf{S}}_{new})} = (\widehat{\mathbf{S}}_{new} \Phi \diamond \mathbf{W}) (\breve{\mathbf{H}}_{old} \Psi)^T,$$
(17)

and then a new LS estimate of the channel matrix is then obtained:

$$\widehat{\mathbf{H}}_{new}^{T} = \left[ (\widehat{\mathbf{S}}_{new} \mathbf{\Phi} \diamond \mathbf{W}) \mathbf{\Psi}^{T} \right]^{\dagger} \widehat{\mathbf{Z}}_{(\widehat{\mathbf{S}}_{new})}.$$
 (18)

Then, set  $n \leftarrow n + 1$ ,  $\widehat{\mathbf{S}}_{old} \leftarrow \widehat{\mathbf{S}}_{new}$ ,  $\mathbf{\check{H}}_{old} \leftarrow \mathbf{\check{H}}_{new}$ , and repeat estimation steps (14) to (18) up to  $n = N - N_o$ .

# **IV. PERFORMANCE EVALUATION**

This section, the performance evaluation of the TSTM technique under several configurations/scenarios is carried out. In all cases where OFDM is considered,  $N_c = 64$  subcarriers are assumed over a total bandwidth was equal to 1MHz, which means that the OFDM symbol duration is  $T = 128 \mu s$  without the cyclic prefix. We have assumed a two-ray equal-power delay profile, with a delay of  $20\mu s$  between the two rays. The results are shown in terms of the average Bit-Error-Rate (BER) versus SNR ratio. All the results are obtained from an average of 1000 Monte Carlo runs. At each Monte Carlo run, the MIMO channel coefficients are redrawn from a i.i.d. Gaussian generator while the transmitted symbols are redrawn from a pseudo-random  $\mu$ -PSK or  $\mu$ -QAM sequence. Each plotted BER curve is an average over the Q transmission groups. In each figure, the fixed simulation parameters (those valid for all the curves of that figure) are listed on the top of the figure.

## A. TSTM combined with PACE

We study the impact of the use of pilot symbols in the TSTM receiver. The combination of pilot symbols and ALS (PACE-ALS) is done in the following manner. At the first iteration of the ALS algorithm, an initial channel estimate is done using the PACE method. The ALS algorithm starts using this channel estimate to obtain a first estimate of the transmitted symbols. Then, it iterates a number of times between symbol and channel estimates in order to refine these estimates. The purpose of Figure 3 is to show the impact of the use of pilot symbols in the TSTM approach. It compares PACE-ALS with blind ALS for Q = 1 (single transmission group) and Q = 2 (two transmission groups). In a more challenging configuration, the temporal spreading factor is set to P = 2 (for maximum performance  $P \ge M_T R = 4$ is required). Note that the BER curves for the blind ALS receiver exhibit a floor at higher SNRs due to the lack of spatial degrees of freedom since the value of P is under the minimum required. The same comment is valid for the PACE-ALS receiver, although the BER performance is better than that of the blind ALS receiver.



Fig. 3. PACE versus blind ALS.



Fig. 4. Performance of TSTM versus SM.

## B. Comparison with SM and OTD

The BER performance of the TSTM technique with ALSbased receiver is compared with those of the Spatial Multiplexing (SM) and Orthogonal Transmit Diversity (OTD) schemes, respectively [7], [8]. The simulation of both SM and OTD consider perfect channel knowledge, which leads to the best performance they can achieve in an open-loop MIMO system. Contrarily to SM and OTD, our TSTM based receiver is simulated without considering any channel knowledge or training sequences, i.e., in a blind setting. The temporal spreading factor of the TSTM precoding structure is assumed to be P = QR. In terms of computational complexity, the proposed receiver is more complex than the classical SM receiver. However, the overall complexity of the SM receiver should also take into account the complexity associated with the channel estimation algorithm which is not considered here under the assumption of perfect channel knowledge. On the other hand, the overall complexity of the TSTM receiver depends on the number of iterations required for convergence of the ALS algorithm (typically within 15-30 iterations).



Fig. 5. Performance of TSTM versus OTD.

We first compare TSTM versus SM in a single-carrier setting  $(N_c = 1)$  with flat-fading channel. Figure 4 shows the BER vs. SNR performance for two different spectral efficiencies (2 and 4 bps/Hz) values and for  $M_R = 2$  and 3 receive antennas.  $M_T = 2$  is used in TSTM and SM. For TSTM, the temporal spreading factor of the precoding structure is P = 2. For achieving 2 and 4 bps/Hz, TSTM uses 4-PSK and 16-QAM, respectively. SM uses 2-PSK and 4-PSK, respectively. At 4bps/Hz, TSTM is worse than SM in low-to-medium SNR values. As SNR increases, TSTM tends to be better than SM. From the slope of the two BER curves, it can be noted that TSTM has a higher diversity gain than SM. We attribute such gain to the use of the precoding structure, which enforce orthogonality between the spatial channels of the parallel data streams. Note that TSTM  $M_R = 2$  has the same diversity gain of SM with  $M_R = 3$  (see the slope of the curves). The performance of TSTM and OTD is compared in Figure 5, over a two-path channel (L = 2) with independent (zero and one symbol-delayed) equal power taps. TSTM uses Q = 1 (no spatial multiplexing takes place). It is worth mentioning that the OTD codes the input symbols in the timedomain (across two consecutive OFDM symbols) and not in the frequency-domain (across subcarriers). Both achieve the same diversity gain, and the gap between TSTM ad OTD is approximately 7 dB for BER= $10^{-3}$ .

# C. Channel tracking

Now, we evaluate the impact of a time-varying channel on the performance of the PACE-ALS receiver. The adaptation of the ALS receiver presented in Section III-C is used. The channel time-variation follows (12) and the normalized Doppler shift is  $f_D = 0.025$ . Figure 6 shows the performance of the tracking algorithm against PACE-ALS with no tracking. A significant performance improvement is obtained by the channel tracking receiver over the other one.

#### V. CONCLUSION

This paper has presented further results and some improvements of the TSTM technique for MIMO-OFDM systems. We



Fig. 6. Performance of the TSTM receiver: PACE-ALS + tracking versus standard PACE-ALS (no tracking) for  $f_D = 0.025$ .

have focused on a more practical setting by considering the use of pilot-assisted channel estimation in conjunction with the ALS-based receiver. Our simulation results have demonstrated that significant performance gains can be obtained when pilot symbols are used. The benefits of the joint use of PACE and ALS are more visible in situations where the temporal spreading factor is small. In order to cope with time-varying channels, we have presented an adaptation of the ALS algorithm for performing channel tracking. Simulation results indicate that he use of the tracking method significantly improves the performance of the standard ALS with no tracking.

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