Aggregate interference from HAPS into FWA systems: A probabilistic approach

Alberth Tamo and Jose Mauro P. Fortes.

***Abstract*— This paper presents a probabilistic model to assess the interference produced by HAPS systems into FWA systems operating in the same frequency bands. To avoid some of the conservative assumptions considered in the traditional interference evaluation methods, in the proposed probabilistic approach the sidelobe antenna gains are characterized by random variables. Minimum operating distances between FWA receivers and HAPS systems are defined and evaluated using the proposed probabilistic model, for some specific interfering scenarios. Results are compared to those obtained with the usual deterministic approach.**

***Keywords*— High Altitude Platforms Stations; Probabilistic Modeling; Fixed Wireless Access; Point-to-Multipoint.**

I. Introduction

High altitude platform station (HAPS) systems are now a real option for wireless communications. They operate in the frequency bands allocated to the Fixed Service and can be found in specialized services like [1], [2] or projects involving global coverage [3], [4]. Once the technological barriers have been overcome, it is expected that HAPS can become the third largest infrastructure in telecommunications, together with terrestrial and satellite systems. HAPS have a larger service coverage as compared to terrestrial services, avoiding the environmental impact caused by base stations and the proportional increasing costs with the network growing. Compared with satellite networks this kind of systems operate with low propagation delays due to its lower heights, enabling the system growth and technology upgrade.

Among other Fixed Service systems, the Fixed Wireless Access (FWA) provides "last mile" broadband wireless access, specialized in services that require a point-to-point or point-to-multipoint high performances networks. HAPS and FWA systems can share the same frequency band if appropriate measures are taken to protect each of them from the harmful interference produced by the other. Available sharing studies between HAPS and FWA [5], [6], [7] have considered the usual deterministic approach for interference calculations that are based in some conservative assumptions as, for example having the antenna gains given by a reference radiation pattern.

This paper presents a probabilistic model to assess the interference produced by HAPS systems into FWA systems. In the proposed model the sidelobe antenna gains are modeled as random variables. This kind of approach, in which some of the technical parameters are modeled as random variables, is referred to as *statistical calculation of interference* and was initially used in connection with satellite communications [8], [9], [10].

In Section II the interference geometry and the associated interference equations are presented and the considered interfering scenarios are described. In Section III the proposed mathematical/probabilistic model is developed and an analytical expression is obtained for the probability distribution function of the interference to noise ratio. Numerical results are presented in Section IV and, finally, conclusions are drawn in Section V.

II. Mathematical model

This section describes the mathematical model for interference calculation involving multiple HAPS interfering on a FWA system. In the model, the sidelobe gains of transmitting antennas HAPS airship station (HAPS-AS) and HAPS ground stations (HAPS-GS) are modeled as statistically independent random variables. Under this assumption, the probability density function of the interfering power is determined for different distances between the victims stations, its FWA base station (FWA-BS) or FWA subscriber station (FWA-SS), and the nadir of the nearest HAPS-AS.

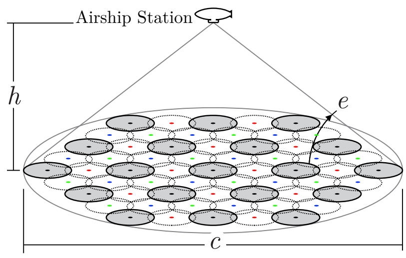

Fig. 1. HAPS geometry.

The HAPS-AS are assumed to be in fixed know position, at an altitude h above earth surface. A minimum elevation angle e delimits the coverage diameter c of the HAPS footprint. The coverage area of each HAPS is deployed with a multibeam antenna array having a frequency reuse equal to 4 ("four color code"), as illustrated in Figure 1.

The interference from HAPS into FWA systems is evaluated in four different scenarios. The first two involve the interference from HAPS-AS into a FWA-BS or into FWA-SS receivers, as shown in figures 2. The other two scenarios involve the interference from HAPS-GS transmitters into FWA receivers (FWA-BS or FWA-SS), as shown in Figure 3. These scenarios are considered in the following subsections.

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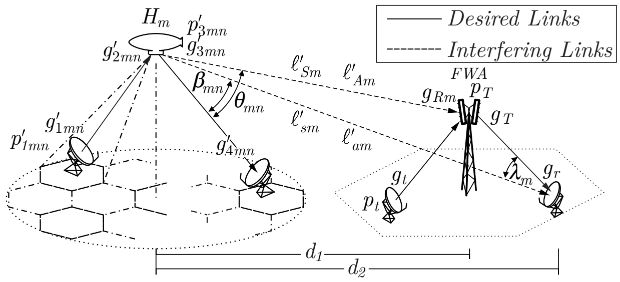

Fig. 2. Interference geometry from HAPS-AS into FWA system.

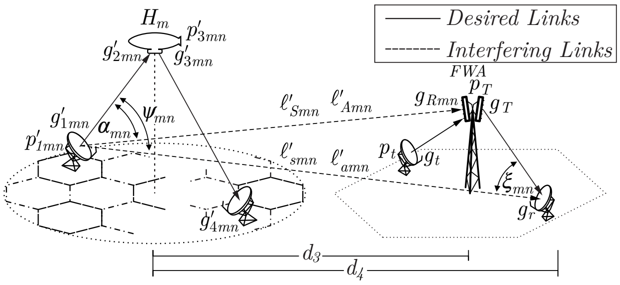

Fig. 3. Interference geometry from HAPS-GS into FWA system.

A. Interference from HAPS-AS into FWA-BS

Considering Figure 2, the aggregate interference power from M HAPS-AS into a FWA-BS receiver, is given by

$$i_1 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} \frac{p'_{3mn} g'_{3mn}(\theta_{mn}) g_{Rm}}{\ell'_{Sm} \ell'_{Am} \ell_{fR}} \quad (1)$$

with the index m denoting the interfering HAPS system and the indexes $n = 1, 2, ..., N_m$ indicating the interfering beams that cause co-channel interference of the m-th HAPS. In (1), p'_{3mn} denote the HAPS-AS transmitting powers density, $g'_{3mn}(\theta_{mn})$ are the HAPS-AS transmitting antenna gains in directions θ_{mn} degrees off the beam center, g_{Rm} is the FWA-BS receiving antenna gain in the direction of the m-th HAPS-AS and ℓ_{fR} is the FWA-BS receiving antenna feeder loss. Still in (1), ℓ'_{Sm} and ℓ'_{Am} represent, respectively, the free space loss and the atmospheric absorption corresponding to the path HAPS-AS to FWA-BS.

Note that (1) can be rewritten as

$$i_1 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} k_{1mn} x_{1mn} \quad (2)$$

with the HAPS-AS transmitting antenna gains modeled as random variables $x_{1mn} = g'_{3mn}(\theta_{mn})$ and

$$k_{1mn} = (p'_{3mn} g_{Rm})/(\ell'_{Sm} \ell'_{Am} \ell_{fR}) \quad (3)$$

B. Interference from HAPS-AS into FWA-SS

For scenarios that consider the aggregate interference from M HAPS-AS into a FWA-SS receiver (see Figure 2 again), the interference power is given by

$$i_2 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} \frac{p'_{3mn} g'_{3mn}(\beta_{mn}) g_r(\lambda_m)}{\ell'_{sm} \ell'_{am} \ell_{fr}} \quad (4)$$

where $g'_{3mn}(\beta_{mn})$ are the HAPS-AS transmitting antenna gains in directions β_{mn} degrees off the beam center, $g_r(\lambda_m)$ is the FWA-SS receiving antenna gain in directions λ_m degrees off the beam center and ℓ_{fr} is the FWA-SS receiving antenna feeder loss. In (4), ℓ'_{sm} and ℓ'_{am}, represent, respectively the free space loss and the atmospheric absorption corresponding to the path HAPS-AS to FWA-SS.

Note that (4) can be also written as

$$i_2 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} k_{2mn} x_{2mn} \quad (5)$$

with the HAPS-AS transmitting antenna gains modeled as random variables $x_{2mn} = g'_{3mn}(\beta_{mn})$ and

$$k_{2mn} = (p'_{3mn} g_r(\lambda_m))/(\ell'_{sm} \ell'_{am} \ell_{fr}) \quad (6)$$

C. Interference from HAPS-GS into FWA-BS

The third scenario considers the geometry in Figure 3. The aggregate interference power resulting from the all possible co-channel transmissions of HAPS-GS (one per co-channel beam) into a FWA-BS receiver is given by

$$i_3 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} \frac{p'_{1mn} g'_{1mn}(\alpha_{mn}) g_{Rmn}}{\ell'_{Smn} \ell'_{Amn} \ell_{fR}} \quad (7)$$

where p'_{1mn} denote the HAPS-GS transmitting powers density, $g'_{1mn}(\alpha_{mn})$ are the HAPS-GS transmitting antenna gains in directions α_{mn} degrees off the beam center, g_{Rmn} is the FWA-SS receiving antenna gain in the direction of the n-th HAPS-GS of the m-th HAPS. In (7), ℓ'_{Smn} and ℓ'_{Amn} represent, respectively, the free space loss and the atmospheric absorption corresponding to the paths HAPS-GS to FWA-BS.

Note that (7) can be rewritten as

$$i_3 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} k_{3mn} x_{3mn} \quad (8)$$

with the HAPS-GS transmitting antenna gains modeled as random variables $x_{3mn} = g'_{1mn}(\alpha_{mn})$ and

$$k_{3mn} = (p'_{1mn} g_{Rmn})/(\ell'_{Smn} \ell'_{Amn} \ell_{fR}) \quad (9)$$

D. Interference from HAPS-GS into FWA-SS

Figure 3 can also be used to define the aggregate interference power from the all possible co-channel transmissions of HAPS-GS (one per co-channel beam) into a FWA-SS receiver. As a result,

$$i_4 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} \frac{p'_{1mn} g'_{1mn}(\psi_{mn}) g_r(\xi_{mn})}{\ell'_{smn} \ell'_{amn} \ell_{fr}} \quad (10)$$

where $g'_{1mn}(\psi_{mn})$ the HAPS-GS transmitting antenna gain in a direction of ψ_{mn} degrees off the beam center and $g_r(\xi_{mn})$ is the FWA-SS receiving antenna gains in directions ξ_{mn} degrees off the beam center.

Note that (10) can be also written as

$$i_4 = \sum_{m=1}^{M} \sum_{n=1}^{N_m} k_{4mn} x_{4mn} \quad (11)$$

with the HAPS-GS transmitting antenna gains modeled as random variables $x_{4mn} = g'_{1mn}(\psi_{mn})$ and

$$k_{4mn} = (p'_{1mn} g_r(\xi_{mn}))/(\ell'_{smn} \ell'_{amn} \ell_{fr}) \quad (12)$$

In summary, in all four scenarios, the aggregate interference power is written as a linear combination of antenna gains (x_{1mn}, x_{2mn}, x_{3mn} and x_{4mn}). Details on the probability density function assumed for these random variables and on the resulting probability density function for the aggregate interference powers are presented in the following section.

III. Probability density function of the aggregate interference

In the case of HAPS-GS transmitting antennas, the reference pattern in Recommendation ITU-R F.1245 [11] is used. For HAPS-AS transmitting beams, a radiation pattern similar to that in Recommendation ITU-R S.1528 [12] is considered. More specifically, we have used the elliptical radiation pattern given by

$$G(\phi)=\begin{cases} G_m - 3\,(\phi/\phi_b)^{\alpha} \text{ dBi}; & 0\leq\phi\leq a\phi_b \\ G_s - 25\log(\phi) \text{ dBi}; & a\phi_b<\phi\leq 48^\circ \\ -6-6\log(z) \text{ dBi}; & 48^\circ<\phi\leq 180^\circ \end{cases} \tag{13}$$

where G_m is the maximum antenna gain, ϕ_b is the antenna half beamwidth in the plane of interest (in degrees), z the ratio between the ellipsis major and minor axes (respectively ϕ_M and ϕ_m) and the constants $\alpha = 1.67 + \log(z)$, $a = 2\sqrt{1-1.2\log(z)} + \log(22.4\,z)$ and $G_s = G_m - 2 + 23\log(6.32z^{-2})$. These elliptical beams allowed for an optimal coverage of the HAPS service area. The major and minor ellipsis axes for these beams were calculated using the algorithm in [13].

In this paper, the sidelobe antenna gains, which are modeled as random variables, are assumed to have a gamma probability density function (this assumption is justified by several studies [14], [15]). The main-lobe antenna gains are assumed to be given by the reference radiation patterns indicated in the first paragraph. Under these considerations, the probability density function for the random variables x_{jmn}, in the j-th scenario, can be written as

$$p_{x_{jmn}}(X)=\begin{cases} \delta(X-G_{jmn}(\phi)) & \in\mathcal{S} \\ \Gamma(X,a_{jmn},b_{jmn}) & \notin\mathcal{S} \end{cases} \tag{14}$$

where $\mathcal{S}$ is angular region corresponding to the antenna main beam, $\delta(\cdot)$ is Dirac delta function and $G_{jmn}(\cdot)$ is the antenna gain given by the used reference radiation patterns. Still in (14), $\Gamma(\cdot)$ is the gamma probability density function with parameters a_{jmn} and b_{jmn} given by

$$\Gamma(X,a_{jmn},b_{jmn}) = \frac{X^{(a_{jmn}-1)}e^{-Xb_{jmn}}}{\gamma(a_{jmn})b_{jmn}^{-a_{jmn}}}\mathrm{u}(X) \tag{15}$$

The parameters a_{jmn} and b_{jmn} were determined considering a specific value for the standard deviation to mean ratio (SDMR=1.2) and the condition $P(x_{jmn} > G_{jmn}) = 0.1$.

Considering (14), the aggregate interference power in (2), (5), (8) and (11) can be written in the form

$$i_j = K_j + y_j \;;\quad j=1,\ldots,4 \tag{16}$$

where

$$K_j = \sum_{\substack{m=1\\ m,n\in\mathcal{S}}}^{M}\sum_{n=1}^{N_m} k_{jmn}G_{jmn} \tag{17}$$

and the random variable y_j is given by

$$y_j = \sum_{\substack{m=1\\ m,n\notin\mathcal{S}}}^{M}\sum_{n=1}^{N_m} k_{jmn}x_{jmn} \tag{18}$$

meaning that y_j is a linear combination of statistically independent gamma distributed random variables. It is worth noting that, in (18), the products $y_{jmn} = k_{jmn}x_{jmn}$ are also gamma distributed random variables with probability density function given by $p_{y_{jmn}}(Y) = \Gamma(Y, a_{jmn}, b_{jmn}/k_{jmn})$. So, alternatively, y_j can be also seen as being the sum of statistically independent gamma distributed random variables. In [16], a convergent series is used to determine the probability density function of the sum of statistically independent gamma distributed random variables. Specifically,

$$p_{y_j}(Y) = \prod_{m,n\notin\mathcal{S}}\frac{b_{jmn}^{a_{jmn}}B^{-a_{jmn}}}{k_{jmn}^{a_{jmn}}}\sum_{k=0}^{\infty}\frac{Y^{\rho+k-1}\,e^{-BY}\alpha_k}{B^{-\rho-k}\,\Gamma(\rho+k)}\,\mathrm{u}(Y) \tag{19}$$

where

$$\rho = \textstyle\sum_{m,n\notin\mathcal{S}}(a_{jmn}), \tag{20}$$

$$B = \max_{m,n\notin\mathcal{S}}(b_{jmn}/k_{jmn}), \tag{21}$$

and α_k, with $\alpha_0 = 1$, is given by

$$\alpha_{k+1} = \frac{1}{k+1}\sum_{i=1}^{k+1} i\,\gamma_i\,\alpha_{k+1-i} \quad ; k = 0, 1, \ldots \tag{22}$$

in (22)

$$\gamma_k = \sum_{m,n\notin\mathcal{S}}(a_{jmn}/k)(1-b_{jmn}/(B\,k_{jmn}))^k \;; k=1,2,\ldots \tag{23}$$

The probability density function of the interfering power i_j is obtained considering (16), that is, $p_{i_j}(I) = p_{y_j}(I - K_j)$. Finally, it is easy to show that the probability distribution function of the interference to noise ratio, when expressed in dB, is given by

$$F_{(i_j/N)_{dB}}(\alpha) = \int_{-\infty}^{\alpha-N_{dB}}\frac{\ln 10}{10^{\beta/10}10}p_{i_j}(10^{-\beta/10})\,\mathrm{d}\beta \tag{24}$$

where N_{dB} is the thermal noise spectral level, in dB(W/MHz).

IV. Numerical Results

The mathematical model developed in sections II and III was applied to an interference scenario where a FWA system is affected by the interference produced by a HAPS constellation of 143 homogeneous HAPS uniformly deployed over a square area on the earth surface, as shown in Figure 4. In this framework, spherical earth surface geometry and only line-of-sight interference, were considered. As shown in Figure 4, d is defined as the distance between the FWA receiver (BS or SS) location and the nadir of the nearest interfering HAPS. It is assumed that d is positive when the FWA system moves away from the HAPS constellation and negative when moves toward it.

In the case of a FWA-SS victim, its receiving antenna azimuth and elevation angles are chosen based on a worst case condition (pointing to the nearest HAPS position). For a FWA-BS victim a receiving antenna with 0"$\circ$ elevation and omnidirectional in azimuth is assumed.

The technical parameters for HAPS systems operating in frequency bands 27.5-28.35 GHz and 31-31.3 GHz are specified in ITU Recommendation ITU-R F.1569 [17]. It indicates, for example, a minimum recommended elevation angle $e = 20^\circ$ at an altitudes h equal to 20 km, results in coverage diameter equal to 110. Other technical parameters taken from

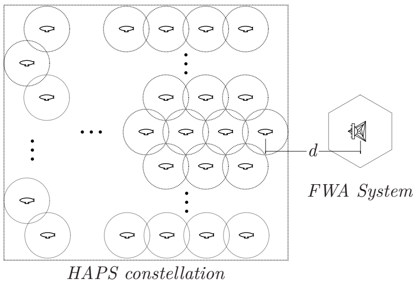

Fig. 4. Interfering Scenario from multiple HAPS systems into FWA system.

ITU-R F.1569 are shown in Table I. Each HAPS utilizes 367 elliptical beams defined by the optimized antenna array in [13], and operate on a four color code frequency reuse (91 co-channel interfering beams per HAPS). For the FWA system, the technical parameters were taken from Recommendation ITU-R F.1609 [7] and are also listed in Table I.

TABLE I
PARAMETERS FOR INTERFERING SCENARIOS

TYPICAL LINK BUDGET FOR HAPS (AT 20 KM)				
Parameter transmitting antennas	**Up link**	**Down link**	**Up link**	**Down link**
Frequency (GHz)	31.3	28	31.3	28
Transmission power density (dB(W/Mhz))	-29.3	-27.5	-29.3	-28.2
Maximum Gain (dBi)	36	29.5	35	16.5
Feeder Loss (dB)	0.5	0.5	0.5	0.5
Atmospheric gases attenuation (dB)	0.4	0.4	0	0
Rain attenuation (dB)	0	0	0	0
Elevation angle (degrees)	20		90	
Path length (km)	57.8		20	
Bandwith (MHz)	20		20	

PARAMETERS OF FWA SYSTEM	
Parameter	**Value**
FWA base station	
Antenna pattern (Azimuth and elevation)	Rec. ITU-R F.1336
Maximum gain (dBi)	15
Antenna elevation (degrees)	0
Noise spectral level (dB(W/MHz))	-137.93
Feeder loss (dB)	0
FWA subscriber station	
Transmission output (dBW)	-10
Noise spectral level (dB(W/Mhz))	-137.93
Antenna pattern	F.1245
Feeder loss (dB)	0
Antenna maximum gain (dBi)	36 for $e \in [0^\circ, 5^\circ)$ 42 for $e \in [5^\circ, 60^\circ]$
Antenna Diameter (cm)	30 for $e \in [0^\circ, 5^\circ)$ 60 for $e \in [5^\circ, 60^\circ]$

For the aggregate interference produced by all 144 HAPS-AS into the victim FWA-BS receiver (scenario $j = 1$), (24) was used to determine $P((i/N)_{dB}) > \alpha$. The result is shown in Figure 5 for two different values of d_1. For comparison purposes the result corresponding to the deterministic calculation of $(i/N)_{dB}$ is also shown in the figure. Note, for example, that for $d_1 = 496$ km if a maximum level of -39 dB is required for $(i/N)_{dB}$, the deterministic approach (solid line) indicates that the interfering HAPS systems do not attend this requirement. However, if the probabilistic approach (dashed line) is considered it shows that the value of -39 dB is exceeded with a very low probability (10^{-6}), indicating that it is reasonable to accept the HAPS systems operation.

Figure 6 shows curves of the $(i/N)_{dB}$ level exceeded with probability p (for different values of p) as a function of

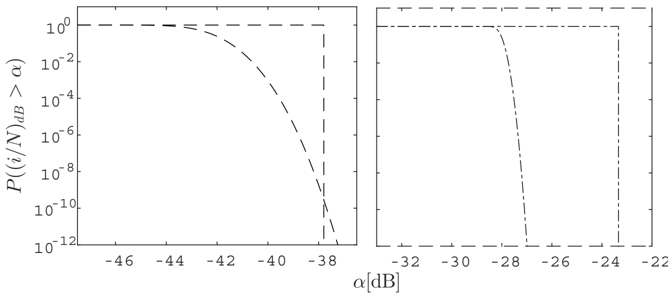

Fig. 5. Scenario 1: for distances, $d_1 = 496$ km left and $d_1 = 100$ right, probability density function for the interference to noise ratio interference from HAPS-AS into FWA-BS

the distance d_1. Note in this figure that if, for example, a $(i/N)_{dB} = -25$ dB is considered, the deterministic approach ($p = 0$) indicates that a minimum distance of 330 km is required for co-channel operation of HAPS and FWA systems. On the other hand, if the probabilistic approach is considered, this minimum operating distance drops down to 61 km for $p = 10^{-5}$ and to 70 km for $p = 10^{-8}$.

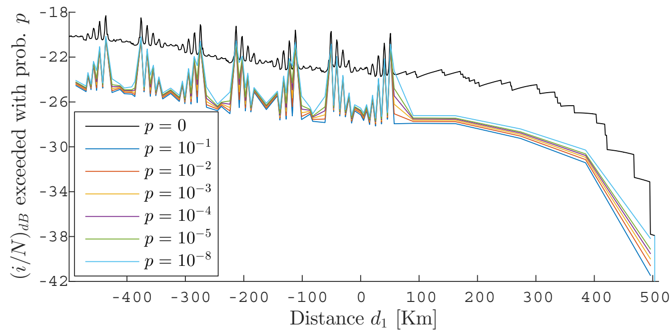

Fig. 6. Scenario 1: Interference from HAPS-AS into FWA-BS.

In the case of interference from HAPS-AS into FWA-SS, the results are shown in Figure 7. Note that the deterministic approach indicates that in the visibility range (FWA-SS can see the HAPS-AS), where $0 \leq d_2 \leq 500$, $(i/N)_{dB}$ is always grater than -10 dB. The same occurs when the probabilistic approach is used, indicating that the use of the probabilistic model produces no gain, in terms of minimal operating distance. These relatively high levels of interference results from the worst case assumption considered in this specific scenario (that at any distance d_2, the FWA-SS has its receiving antenna pointing toward the HAPS-AS).

For scenarios that consider interference from HAPS-GS transmitters into FWA receivers (BS or SS), due to the spherical geometry assumed for the earth surface, only HAPS-GS operating with the nearest HAPS are visible to the victim receiver. For this reason only one interfering HAPS system is considered and the aggregate interference form its co-channel visible HAPS-GS is evaluated. For a victim FWA-BS, Figure 8 shows curves of the $(i/N)_{dB}$ level exceeded with different values of probability p as a function of the distance d_3. Note in this figure that if, for example, a $(i/N)_{dB} = -25$ dB is considered, the deterministic approach ($p = 0$) indicates that a minimum distance of 66 km is required for co-

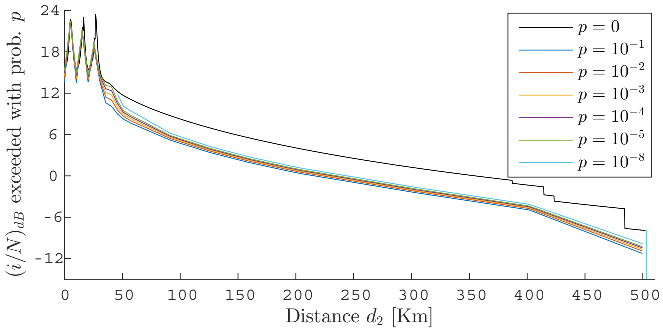

Fig. 7. Scenario 2: Interference from HAPS-AS into FWA-SS.

channel operation of both systems. On the other hand, if the probabilistic approach is considered, this minimum operating distance drops down to 56 km for $p = 10^{-5}$. In the case

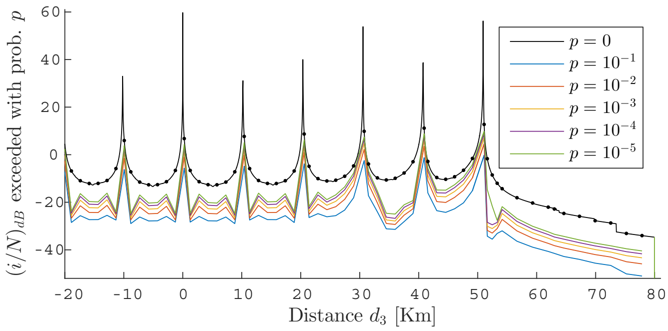

Fig. 8. Scenario 3: Interference from HAPS-GS into FWA-BS.

of a victim FWA-SS, Figure 9 shows curves of the $(i/N)_{dB}$ level exceeded with probability p (for different values of p) as a function of the distance d_4. Note in this figure that for a level of $(i/N)_{dB} = -25$ dB considered, the deterministic approach ($p = 0$) indicates that a minimum distance of 68 km is required for co-channel operation of the HAPS and the FWA systems. On the other hand, if the probabilistic approach is considered, this minimum operating distance drops down to 57 km for $p = 10^{-5}$.

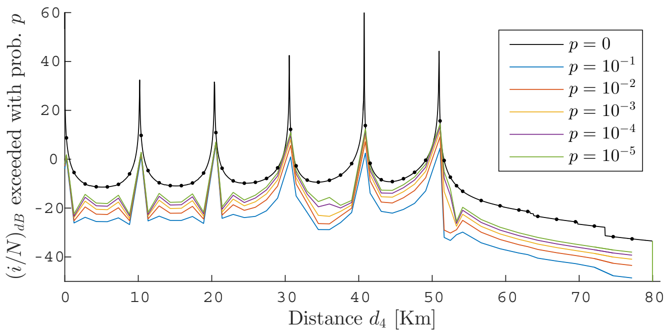

Fig. 9. Scenario 4: Interference from HAPS-GS into FWA-SS.

V. Conclusion

This paper presented a probabilistic model for the assessment of interference between HAPS and FWA systems. The proposed model avoids the worst case assumption that, in all single-entry interference contributions, the transmitting antenna gains in the antenna sidelobe region, are given by a reference radiation pattern envelope. In the model these antenna gains are characterized by statistically independent random variables. Results obtained using this more realistic model have indicated a reduction in the minimal operating distances between HAPS and FWA systems, when compared to the results provided by the usual conservative deterministic model. A reduction of approximately 80% was observed in the case of interference from HAPS-AS into FWA-BS. The relatively high levels of interference observed when HAPS-AS interferes into FWA-SS resulted from the worst case assumption that at any distance, the FWA-SS has its receiving antenna pointing toward the HAPS-AS. In the case of interference from HAPS-GS into FWA receivers (BS and SS), the reduction in the minimal operating distance, although not so large, reached values on the order of 15%.

References

[1] *X-station*, StratXX, `http://www.stratxx.com/products/x-station/`, 2014.
[2] *QucomHaps, for a better connected world*, QucomHaps Co., `http://www.qucomhaps.net/`, 2017.
[3] *Project Loon*, Google, `https://x.company/loon/`, 2015.
[4] *Aquila Project*, Facebook, `https://code.facebook.com/posts/268598690180189`, 2016.
[5] M. Oodo, R Miura *et al*, *Sharing and compatibility study between fixed service using high altitude platform stations (HAPS) and other services in the 31/28 GHz bands*, Wireless Personal Communications, pp. 3–14, 2002.
[6] L. F. Abdulrazak, T. A. Rahman *et al*, *Study HAPS interference power to noise level ratio of fixed services and related separation distance*, Wireless Personal Communications, pp. 3–14, 2002.
[7] *Interference evaluation from fixed service systems using high altitude platform stations to conventional fixed service systems in the bands 27.5-28.35 GHz and 31-31.3 GHz*, Recommendation ITU-R F.1609, Genebra, 2006.
[8] J. P. A. Albuquerque and J. M. P. Fortes, *Statistical aspects in the evaluation of interference among satellite networks*, International Communications Conference - ICC'88, pp. 210–214, 1988.
[9] N. Makhijani, *Statistical analysis of link budgets satellite communications*, International Communications Conference - ICC'88, pp. 220–224, 1988.
[10] J. M. P. Fortes and J. P. A. Albuquerque, *Análise Estatística de Interferências em Sistemas de Telecomunicações via Satélite*, CETUC D-ST-01/87, PUC-Rio, Jan 1987.
[11] *Mathematical model of average and related radiation patterns for line-of-sight point-to-point fixed wireless system antennas for use in certain coodination studies and interference assessment in the frequency range from 1 GHz to about 70 GHz*, Recommendation ITU-R F.1245-2, Genebra, 2012.
[12] *Satellite antenna radiation patterns for non-geostationary orbit satellite antennas operating in the fixed-satellite service below 30 GHz*, Recommendation ITU-R F.1528, Genebra, 2001.
[13] J. Thornton, D. Grace, M. H. Capstick and T. C. Tozer *Optimizing an array of antennas for cellular coverage from a high altitude platform*, IEEE Transactions on Wireless Communications, pp. 484–492, 2003.
[14] R. H. Lang *Statistical Model for Sidelobe Interference in Satellite Networks*, Institute for Information Science and Technology, George Washington University, 1982.
[15] *Proposed Amendment to Report 391-2: Some statistical Properties of Antenna Sidelobes*, Document CCIR 4/32, Japan Administration Contribution, CCIR Meeting, Genebra, Jun 2010.
[16] P. G. Moschopoulos, *The distribution of the sum of independent gamma random variables*, Annals of the Institute of Statistical Mathematics, pp. 541–544, 1985.
[17] *Technical and operational characteristics for the fixed service using high altitude platform stations in the bands 27.5-28.35 GHz and 31-31.3 GHz*, Recommendation ITU-R F.1569, Genebra, 2002.