Effects of diversity on the sum rate of a multiuser M-FSK system over fast Rayleigh fading channel

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***Abstract*— This paper shows how spatial diversity affects multiuser system rate when a multilevel frequency shift keying modulation signal is transmitted over a fast Rayleigh fading channel with additive white Gaussian noise. No specific multiple access method is used. The channel's short coherence time precludes the system from estimation or tracking channel parameters. Results show that, in this situation, transmitter diversity is ineffective while receiver diversity has a considerable influence on system rate, as well as increasing the optimum number of users that can share the channel at the same time. It is also shown that in this scenario spectral efficiency can be improved by increasing the number of frequency slots, in opposition to the single user Gaussian noise case.**

***Keywords*— MFSK, Multiuser system, diversity, fading channel.**

I. Introduction

There are situations in which fading channels are frequency selective and have short coherence time, making it impossible to use equalizers or other methods that rely on channel estimation. Such a situation could occur for instance in a network of low altitude aerial vehicles that have to transmit information to a base station in an urban environment. Radio signals would be corrupted by path loss, noise, Doppler shift and multipath fading [1].

A possible solution is to use Multiple Frequency Shift Keying (MFSK) modulation. Fast Frequency Hoping Code Division Multiple Access (FFH-CDMA)[2] uses MFSK and a predefined hoping sequence [3] to allow multipoint to point communication. However, the use of a hoping code is equivalent to a repetition code, greatly reducing spectral efficiency.

One question that arises is how much information could be transmitted by a multiuser MFSK system, regardless of the multiple access method. The capacity of two noiseless MFSK cases was studied in [4]. A noisy case for a fading channel was studied in [5]. One important result is that, in the presence of fading and noise, there is an optimum number of users that maximizes system rate, beyond which the rate declines.

The use of diversity has been shown to provide significant gains in capacity [6], [7], [8]. Diversity can be defined as the transmission of the same information through various channels, each with its own (and possibly independent) statistical realization, with the intent to improve reliability. This can be achieved by using time, spatial, polarization or other diversity schemes. This work intends to study the effects of spatial diversity on the system rate of multiuser MFSK communication over frequency selective Rayleigh fading channels. System rate is analyzed by calculating mutual information for a multiuser detection scheme.

To do so we develop a system model in section II based on [4], [9], and [5]. Channel statistics are derived for the diversity scenario. Multiuser mutual information is considered in section III. Numerical results and analysis of their consequences are in section IV. Section V summarizes our conclusions and highlights the key findings.

Regarding mathematical notation, lower case bold letters such as $\mathbf{v}$ are vectors. Upper case bolt letters such as $\mathbf{M}$ are matrices. Probability mass functions are written as $P(\cdot)$ and probability density functions (p.d.f.'s) are written as $p(\cdot)$; in both cases the argument is sufficient to define the function. Expected value, entropy and mutual information are respectively written as $\mathcal{E}\{\cdot\}$, $\mathcal{H}\{\cdot\}$, $\mathcal{I}\{\cdot,\cdot\}$; function arguments are the random variables involved. Capacity is written as $\mathcal{C}()$, but function arguments are system properties.

II. System Model

This model is graphically depicted in Fig. 1.

A user, indexed by $u = 0, 1, ..., U-1$, randomly chooses at time i (omitted for simplicity) a message m^u from the set $\{0, 1, ..., N-1\}$ with probabilities $P(m^u = n) = \mu_n$, $\sum_{n=0}^{N-1} \mu_n = 1$. Choosing a message is equivalent to choosing $k = log_2[N]$ bits. The ensemble of messages is grouped into the random vector $\mathbf{m} = [m^0 \;\; m^1 \;\; \cdots \;\; m^{U-1}]$. User messages can be converted to frequency utilization factors c_n^u that equals 1 if $m^u = n$ or 0 if $m^u \neq n$. The number of active users per frequency is c_n, defined as:

$$c_n = \sum_{u=0}^{U-1} c_n^u, \quad n = 0, 1, ..., N-1, \tag{1}$$

which can be grouped into the vector $\mathbf{c} = [c_0 \;\; c_1 \;\; \cdots \;\; c_{N-1}]$

Signals from each user will be received with energy E_b per bit. They last τ seconds and occupy the appropriate frequency slot, called a chip. Path loss effects are compensated by each user so that average received energy is the same. This is equivalent to modeling the signal that each user transmits as having $E_c = k \cdot E_b$ energy. The nature of MFSK modulation has it that all of this energy is transmitted in a single chip. As a result, each user generates a signal $s^u(t)$ defined as:

$$s^u(t) = \sum_{n=0}^{N-1} c_n^u \cdot x_n(t), \tag{2}$$

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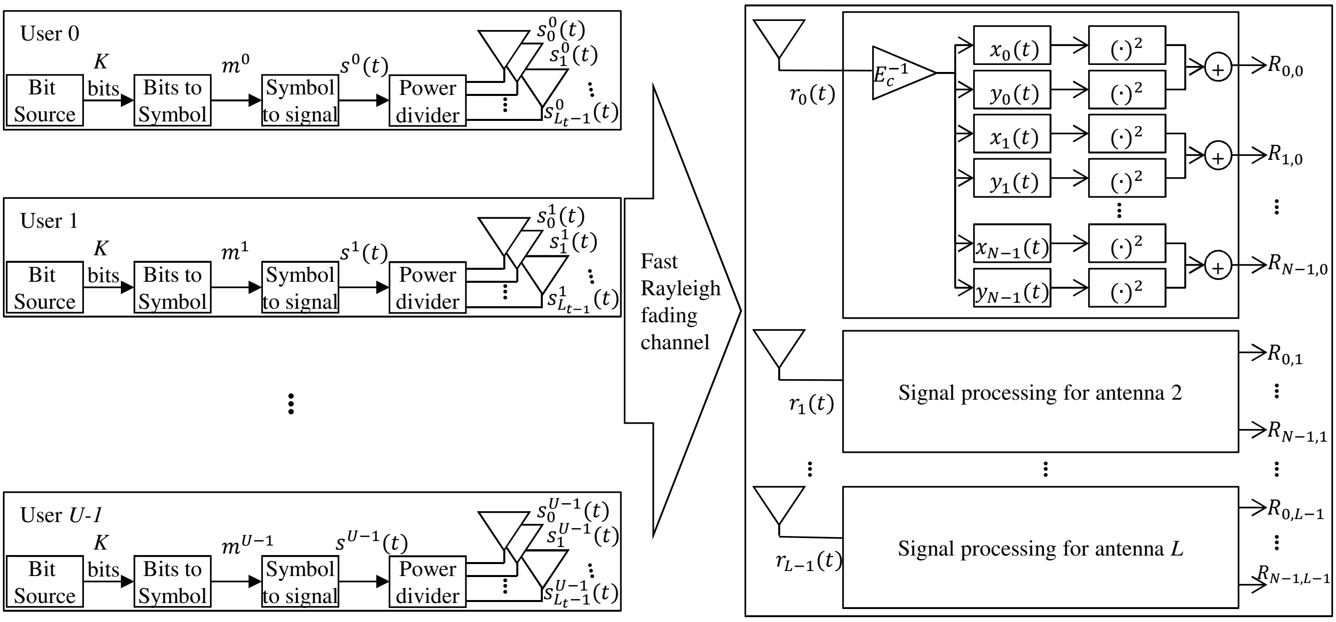

Fig. 1. Multiuser system

where $x_n(t)$ belongs to the set of basis functions:

$$
\begin{aligned}
x_n(t) &= \sqrt{\frac{2E_c}{\tau}} \cdot cos\left[2\pi t\left(f_0 + \frac{n}{\tau}\right)\right], \\
y_n(t) &= \sqrt{\frac{2E_c}{\tau}} \cdot sin\left[2\pi t\left(f_0 + \frac{n}{\tau}\right)\right],
\end{aligned} \tag{3}
$$

for a given base frequency $f_0 >> \tau^{-1}$. Considering a frequency separation of τ^{-1}, necessary for non coherent orthogonal detection, total channel bandwidth is $W = \frac{N}{\tau}$.

For completeness, transmitting diversity is achieved by dividing total energy between L_t transmitting antennas, indexed by $l_t = 0, 1, ..., L_t - 1$,. This is done by choosing real valued coefficients $w_{l_t}^u$ such that $\sum_{l_t=0}^{L_t-1} w_{l_t}^u = 1$. The signal transmitted by each antenna can be written as:

$$
s_{l_t}^u(t) = \sqrt{w_{l_t}} \sum_{n=0}^{N-1} c_n^u \cdot x_n(t), \tag{4}
$$

All users transmit at the same time through a channel with Rayleigh fading and additive white Gaussian Noise $\omega(t)$ with power spectral density $\frac{N_0}{2}$. The signals are simultaneously received by L receiving antennas, indexed by $l = 0, 1, ..., L-1$. Fading is such that each transmitting/receiving antenna pair (l_t, l) suffers statistically independent attenuation $\alpha_{n,l}^{u,l_t}$ with Rayleigh distribution, $\mathcal{E}\{\alpha_{n,l}^{u,l_t}\} = 1$, and uniformly distributed phase rotation $\theta_{n,l}^{u,l_t}$. The received signal in each antenna is then the combination of noise and faded transmitted signals:

$$
\begin{aligned}
r_l(t) = \omega(t) + \sum_{n=0}^{N-1} \sum_{l_t=0}^{L_t-1} \sum_{u=0}^{U-1} \Big\{ & c_n^u \alpha_{n,l}^{u,l_t} \sqrt{w_{l_t}^u} \\
& \cdot \left[cos\left(\theta_{n,l}^{u,l_t}\right) x_n(t) + \sin\left(\theta_{n,l}^{u,l_t}\right) \cdot y_n(t) \right] \Big\}.
\end{aligned} \tag{5}
$$

If the channel's coherence time is short, it would not be possible to estimate or track the channel's parameters $\alpha_{n,l}^{u,l_t}$ and $\theta_{n,l}^{u,l_t}$. The receiver could use non coherent detection using NL pairs of matched filters, one pair for each antenna and chip, resulting in channel outputs:

$$
\begin{aligned}
X_{n,l} &= \frac{1}{E_c} \int_0^\tau r_l(t) \cdot x_n(t) dt \\
&= \sum_{u=0}^{U-1} \sum_{l_t=0}^{L_t-1} c_n^u \alpha_{n,l}^{u,l_t} \sqrt{w_{l_t}^u} \cos(\theta_{n,l}^{u,l_t}) + \kappa_{n,l}, \\
Y_{n,l} &= \frac{1}{E_c} \int_0^\tau r_l(t) \cdot y_n(t) dt \\
&= \sum_{u=0}^{U-1} \sum_{l_t=0}^{L_t-1} c_n^u \alpha_{n,l}^{u,l_t} \sqrt{w_{l_t}^u} \sin(\theta_{n,l}^{u,l_t}) + \gamma_{n,l}.
\end{aligned} \tag{6}
$$

Each term inside the summations is a Gaussian random variable with zero mean and variance $\frac{c_n^u w_{l_t}^u}{2}$ each. For all $c_n^u \neq 0$, which happens c_n times, there are exactly L_t random variables with non zero variance $\frac{c_n^u w_1^u}{2}, \frac{c_n^u w_2^u}{2}, \cdots \frac{c_n^u w_{L_t}^u}{2}$. Since $\sum_{l_t=1}^{L_t} w_{l_t}^u = 1$, both summations result in Gaussian random variables with zero mean and variance $\frac{c_n}{2}$. This means that the value of L_t and the way each user distributes its power among transmission antennas are irrelevant for signal statistics. Random variables $\gamma_{n,l}$ and $\kappa_{n,l}$ are also Gaussian with zero mean and variance $\sigma^2 = \frac{N_0}{2E_c}$. Thus, given c_n, $X_{n,l}$ and $Y_{n,l}$ are Gaussian random variables with zero mean and variance $(c_n + \sigma^2)/2$.

Total received energy per antenna per chip is:

$$
R_{n,l} = X_{n,l}^2 + Y_{n,l}^2, \tag{7}
$$

which is an exponential random variable if c_n is known:

$$
p(R_{n,l}|\mathbf{c}) = p(R_{n,l}|c_n) = \frac{1}{c_n + \sigma^2} \cdot exp\left(-\frac{R_{n,l}}{c_n + \sigma^2}\right). \tag{8}
$$

The set of received signals can be grouped in an $N \times L$ matrix $\mathbf{R}$. Given $\mathbf{c}$, the values of $R_{n,l}$ are statistically independent. Defining $\Gamma(\mathbf{c})$ as the set of all possible values that $\mathbf{c}$ can assume, an expression for $p(\mathbf{R})$ can be obtained from (8) using the following:

$$p(\mathbf{R}|\mathbf{c}) = \prod_{n=0}^{N-1} \prod_{l=0}^{L-1} p(R_{n,l}|c_n), \tag{9}$$

resulting in:

$$\begin{aligned} p(\mathbf{R}) &= \sum_{\mathbf{c}\in\Gamma(\mathbf{c})} P(\mathbf{c}) \cdot p(\mathbf{R}|\mathbf{c}) \\ &= \sum_{\mathbf{c}\in\Gamma(\mathbf{c})} P(\mathbf{c}) \prod_{n=0}^{N-1} \prod_{l=0}^{L-1} p(R_{n,l}|c_n) \\ &= \sum_{\mathbf{c}\in\Gamma(\mathbf{c})} P(\mathbf{c}) \prod_{n=0}^{N-1} \left[\frac{1}{(c_n+\sigma^2)^L} exp\left(-\frac{\sum_{l=0}^{L-1} R_{n,l}}{c_n+\sigma^2} \right) \right]. \end{aligned} \tag{10}$$

From (10) it is clear that, since there is no attempt to estimate any of the channel fading parameters, transmit diversity has no influence in $p(\mathbf{R}|\mathbf{c})$ or $p(\mathbf{R})$. Therefore, L_t has no influence in this channel's mutual information. This result is expected since the detector is a non coherent energy detector.

A second conclusion is that an equal gain combiner [10] is sufficient to provide the required statistics to perform maximum likelihood detection. The combiner should combine the received energy per chip from each antenna to generate values R_n defined as:

$$R_n = \sum_{l=0}^{L-1} R_{n,l} \tag{11}$$

This value is used in calculating both $p(\mathbf{R}|\mathbf{c})$ and $p(\mathbf{R})$, used in maximum likelihood and maximum *a posterior* decision methods. Thus, diversity does not increase detector complexity other than the circuitry needed to generate R_n. This simplifies receiver design and could be used when other similar types of diversity are available.

III. Multiuser System Rate

A single user detector (SUD) would attempt to estimate the message from one user considering the others as unknown interference, and do so for all U users, that is, it would estimate m^u for $u = 0, 1, ..., U-1$, one at a time. On the other hand, a multiuser detector (MUD) would attempt to estimate all messages at the same time, that is, it would estimate $\mathbf{m}$ once. Maximum value of the mutual information $\mathcal{I}\{\mathbf{R}, \mathbf{m}\}$ is an upper bound on the sum rate of the system.

The capacity of two noiseless MFSK channel was investigated in [4]. In the case with intensity information, channel input is $\mathbf{m}$ and channel output is $\mathbf{c}$. Since knowledge of $\mathbf{m}$ completely defines $\mathbf{c}$, channel capacity is achieved when entropy of $\mathbf{c}$ is maximized. This was shown to happen when $\mu_n = \frac{1}{N}$ for all $n = 0, 1, ..., N-1$. The resulting capacity is denoted by $\mathcal{C}_N$ and was found to be a function of U and N:

$$\begin{aligned} \mathcal{C}_N(N,U) &= \max_{\{\mu_n\}} [\mathcal{I}\{\mathbf{c}, \mathbf{m}\}] = \max_{\{\mu_n\}} [\mathcal{H}\{\mathbf{c}\}] \\ &= \frac{U!}{N^U} \sum_{\mathbf{c}\in\Gamma(\mathbf{c})} \frac{log_2\left(\frac{N^T \cdot c_0! c_2! \cdots c_{N-1}!}{T!}\right)}{c_0! c_2! \cdots c_{N-1}!}. \end{aligned} \tag{12}$$

The fading channel studied here can be seen as a concatenation of a noiseless channel with a noisy, fading channel, forming a Markov Chain. Since $\mathbf{m}$ completely defines $\mathbf{c}$, $\mathcal{H}\{\mathbf{R}|\mathbf{m}\} = \mathcal{H}\{\mathbf{R}|\mathbf{c}\}$, and $\mathcal{I}\{\mathbf{R}, \mathbf{m}\} = \mathcal{I}\{\mathbf{R}, \mathbf{c}\} = \mathcal{H}\{\mathbf{R}\} - \mathcal{H}\{\mathbf{R}|\mathbf{c}\}$. Thus, by the data processing theorem [11]:

$$\mathcal{I}\{\mathbf{R}, \mathbf{m}\} = \mathcal{I}\{\mathbf{R}, \mathbf{c}\} \leq \mathcal{I}\{\mathbf{c}, \mathbf{m}\} \leq \mathcal{C}_N(N,U). \tag{13}$$

Calculation of $\mathcal{I}\{\mathbf{R}, \mathbf{m}\}$ requires calculation of $\mathcal{H}\{\mathbf{R}\}$ and $\mathcal{H}\{\mathbf{R}|\mathbf{c}\}$, whose corresponding p.d.f.'s were obtained in the previous section. Since $R_{n,l}$ are statistically independent given $\mathbf{m}$, entropy $\mathcal{H}\{\mathbf{R}|\mathbf{c}\}$ can be obtained in bits as:

$$\mathcal{H}\{\mathbf{R}|\mathbf{c}\} = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} \mathcal{H}\{R_{n,l}|c_n\} = L \sum_{n=0}^{N-1} \mathcal{H}\{R_{n,0}|c_n\}, \tag{14}$$

since $\mathcal{H}\{R_{n,l}|c_n\} = \mathcal{H}\{R_{n,0}|c_n\}, \forall l = 0, 1, ..., L-1$:

$$\mathcal{H}\{R_{n,l}|c_n\} = \frac{1}{ln(2)} \sum_{c_n=0}^{U} P(c=c_n) \cdot [1 + ln(c_n + \sigma^2)]. \tag{15}$$

Given (10), however, there is no known closed form for $\mathcal{H}\{\mathbf{R}\}$. It can be calculated by Monte Carlo integration.

The special case when $L = 1$ was studied in [5] and [12]. It was shown that when users choose messages m_j independently with $\mu_n = \frac{1}{N}$, $\mathcal{I}\{\mathbf{R}, \mathbf{c}\}$ reaches an inflexion point. Moreover, the uniform distribution on $\mathbf{m}$ maximizes a tight upper bound on $\mathcal{I}\{\mathbf{R}, \mathbf{c}\}$ when $U < N$. Any gains by using a different distribution, if existent, would be small and cause an increase in transmitter and receiver complexity. This value of $\mathcal{I}\{\mathbf{R}, \mathbf{c}\}$ is also an upper bound on the system rate when input symbols are uniformly distributed and denoted as $\mathcal{R}(N, U, L)$. In this case, $P(\mathbf{c})$ in (10) becomes [4]:

$$P(\mathbf{c}) = \frac{U!}{N^U c_0! c_1! \cdots c_{N-1}!} \tag{16}$$

It is possible to conclude from (10) and (14) that receiver diversity should always increase mutual information because $\mathcal{H}\{\mathbf{R}|\mathbf{c}\}$ increases linearly with L but $\mathcal{H}\{\mathbf{R}\}$ increases at a lower rate. The first part of this statement is evident from (14). The second part is true because, without knowledge of $\mathbf{c}$, random variables $R_{n,l}$ are not statistically independent and:

$$\begin{aligned} \mathcal{H}\{\mathbf{R}\} < & \sum_{l=0}^{L-1} \mathcal{H}\{R_{0,l}, R_{1,l}, \cdots R_{N-1,l}\} \\ & = L \cdot \mathcal{H}\{R_{0,l}, R_{1,l}, \cdots R_{N-1,l}\} \end{aligned} \tag{17}$$

Since $\mathcal{H}\{R_{0,l}, R_{1,l}, \cdots R_{N-1,l}\} = \mathcal{H}\{\mathbf{R}\}$ when $L = 1$, this entropy increases at a slower rate than $\mathcal{H}\{\mathbf{R}|\mathbf{c}\}$.

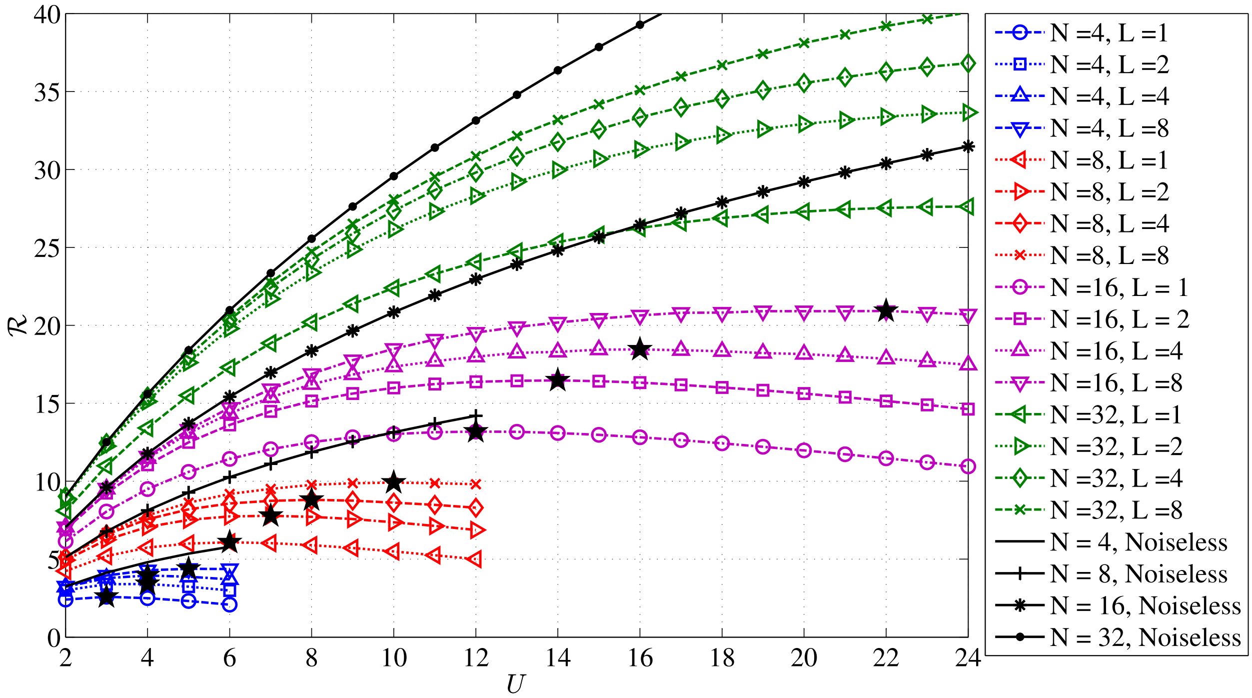

Fig. 2. $\mathcal{R}(N, U, L)$ for some combinations of N, U and L, and $\frac{E_b}{N_0} = 10dB$, except were noted as noiseless. Black stars indicate the value of U that maximizes $\mathcal{R}$ for a given N, L.

IV. Results and Analysis

Numerical results were obtained for some combinations of $N = 4, 8, 16, 32$, $L = 1, 2, 4, 8$ and $U = 2$ to $1.5 \cdot N$, limited by computational complexity to $U \leq 24$. Fig. 2 shows how $\mathcal{R}(N, U, L)$ varies as a function of U, for some values of N,L, and $E_b/N_0 = 10dB$. Results are similar in shape for other values of E_b/N_0. For all cases, doubling L increases $\mathcal{R}(N, U, L)$. This gain can be significant depending on the choice of N and U. It can also be seen that the uniform distribution seems to be a good choice, if not the best, for μ_i. The reasoning is that $\mathcal{R}(N, U, L)$ is close to the noiseless capacity $\mathcal{C}_N(N, U)$(an upper bound as indicated in (13)). The values get closer by increasing L or decreasing U.

In all cases there is a number of users U_{MAX} such that $\mathcal{R}(N, U_{MAX}, L) \geq \mathcal{R}(N, U, L), \forall U \neq U_{MAX}$, with equality possibly holding in the neighborhood of U_{MAX} for some particular values of E_b/N_0. For a given N, the value of U_{MAX} varies with E_b/N_0 and L as shown in Fig. 3. It seems that U_{MAX} decreases with E_b/N_0 for low L but increases for high L. For some cases, $U_{MAX} > N$. As a result, a new spreading scheme is required to allow multiple access for this number of users, since the method provided in [3], used for example in [13] and [14], provides at most N frequency hoping sequences.

The effects of E_b/N_0 on $\mathcal{R}(N, U_{MAX}, L)$ are shown in Fig. 4. It shows that , after $\mathcal{R}(N, U_{MAX}, L)$ approaches a ceiling, increase in E_b/N_0 has very little influence. Increasing L has two effects on this limit: it makes it closer to $C_N(N, U_{MAX})$ and the limit is reached for a lower value of E_b/N_0. This result could be used to design systems by choosing L such that $\mathcal{R}(N, U_{MAX}, L)$ is close to the ceiling when operating with the available E_b/N_0. Values of $\mathcal{R}(N, U_{MAX}, L)$ for high L seem to be unachievable for low L, even with high E_b/N_0. In all cases $\mathcal{R}(N, U_{MAX}, L)$ is much lower than $C_N(N, U_{MAX})$.

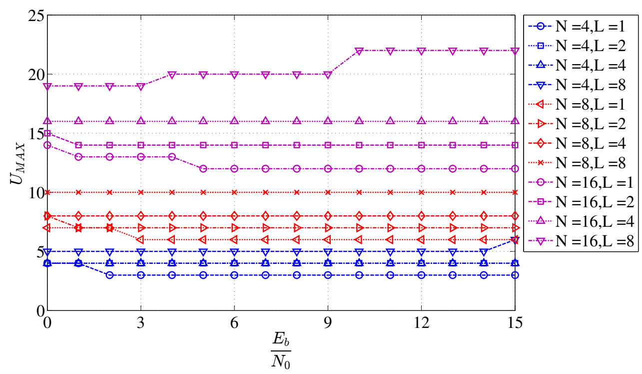

Fig. 3. U_{MAX} as a function of $\frac{E_b}{N_0}$.

A. Spectral Efficiency

Given W, U and E_b/N_0, spectral efficiency depends on N. Considering that a channel use lasts $\tau = N/W$ seconds, spectral efficiency can be defined as, in bits/seconds/Hertz:

$$\eta = \frac{1}{W} \cdot \frac{\mathcal{R}(N, U, L)}{\tau} = \frac{\mathcal{R}(N, U, L)}{N}. \tag{18}$$

Results for some combinations of N, L and E_b/N_0 are shown in Fig. 5. Three features are of interest:

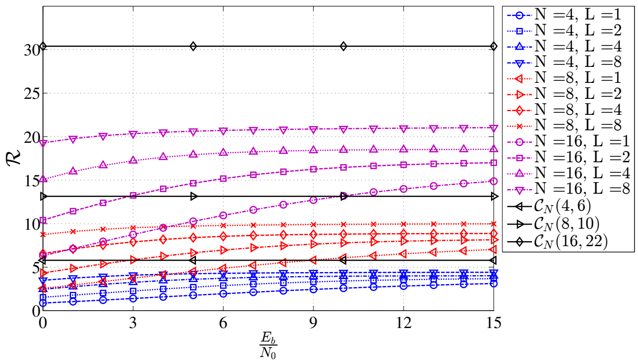

Fig. 4. $\mathcal{R}(N, U_{MAX}, L)$ as a function of $\frac{E_b}{N_0}$. Noiseless case $\mathcal{C}_N(N, U)$ shown for comparison.

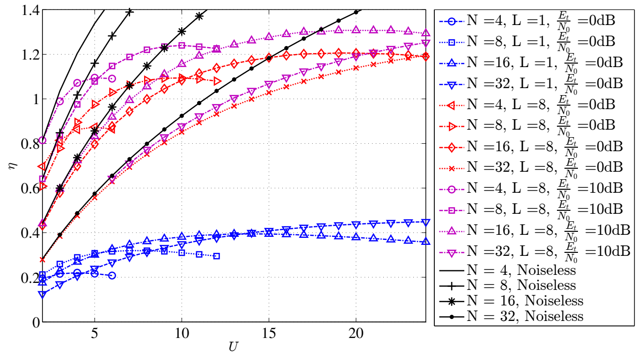

Fig. 5. Spectral efficiency for selected values of N, L and $\frac{E_b}{N_0}$.

1) For the noiseless case, multiuser spectral efficiency decreases for all U as N increases, as in the case of single user noisy MFSK[15]. However, in the presence of noise and fading, it is possible to increase spectral efficiency by doing the opposite, that is, increasing N. This gain is only possible up to a certain value of N, after which spectral efficiency will start to fall.
2) For a relatively low number of users ($U < N/2$), diversity brings η close to the noiseless case.
3) Diversity greatly increases η and has a larger impact on it than increasing signal power. For example, η for $N = 16$, $L = 8$, $U = 22$ and $E_b/N_0 = 10dB$ would be very close to the case where $U = 20$ and $E_b/N_0 = 6dB$, as it can be deduced from Figs. 3 and 4.

V. Conclusions

The are three main results in this work. The first is that, in the scenario of absence of channel state information, transmitter diversity does not improve system rate since it has no influence in the received signal statistics.

The second is that receiver diversity increases $\mathcal{R}(N, U, L)$ and changes the number of users that maximize its value. Receiver diversity does not significantly increase complexity since an equal gain combiner provides sufficient statistics for detection. Also, receiver diversity can lead to situations in which the ideal number of users is larger than the number of frequencies, turning it impossible to use previously used multiple access methods. A new multiple access method would be required if a system intends to reach $\mathcal{R}(N, U_{MAX}, L)$. Research is required to find alternatives to the method proposed in [3].

The third main result is that, as opposed to single user MFSK, it is possible to increase spectral efficiency by increasing the number of frequency slots. The ideal number of frequency slots depends on the number of users.

References

[1] B. Sklar, "Rayleigh fading channels in mobile digital communication systems. I. Characterization," *IEEE Communications Magazine*, vol. 35, no. 9, pp. 136-146, September 1997.
[2] D. J. Goodman, P. S. Henry, and V. K. Prabhu, "Frequency-hopped multilevel fsk for mobile radio," *Bell Systems Technical Journal*, vol. 59, no. 7, pp. 1257–1275, September 1980.
[3] G. Einarsson, "Address assignment for a time-frequency-coded, spread-spectrum system," *Bell Systems Technical Journal*, vol. 59, no. 7, pp. 1241–1255, September 1980.
[4] S.-C. Chang and J. Wolf, "On the t-user m-frequency noiseless multiple-access channel with and without intensity information," *IEEE Transactions on Information Theory*, vol. 27, no. 1, pp. 41–48, January 1981.
[5] M. Sharma and J. Portugheis, "On the sum capacity of a t-user n-frequency multiple access channel with noise," *European Transactions on Telecommunications*, vol. 21, no. 1, pp. 23–29, 2010.
[6] J. Winters, "On the capacity of radio communication systems with diversity in a rayleigh fading environment," *Selected Areas in Communications, IEEE Journal on*, vol. 5, no. 5, pp. 871–878, Jun 1987.
[7] S. Alamouti, "A simple transmit diversity technique for wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.
[8] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of mimo channels," *Selected Areas in Communications, IEEE Journal on*, vol. 21, no. 5, pp. 684–702, June 2003.
[9] O.-C. Yue, "Maximum likelihood combining for noncoherent and differentially coherent frequency-hopping multiple-access systems," *IEEE Transactions on Information Theory*, vol. 28, no. 4, pp. 631–639, July 1982.
[10] Y. Nathan, J.-P. M. Linnartz, and G. Fettweis, "Multi-carrier cdma in indoor wireless radio networks," *IEICE Transactions on Communications*, vol. 77, no. 7, pp. 900–904, 1994.
[11] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.
[12] M. Sharma and J. Portugheis, "Exit chart optimization of repeat-accumulate codes for an n-frequency t-user multiple access channel with noise," *Journal of Communication and Information Systems*, pp. 26–33, 2013.
[13] T. Mabuchi, R. Khono, and H. Imai, "Multiuser detection scheme based on canceling cochannel interference for mfsk/fh-ssma systems," *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 4, pp. 593–604, May 1994.
[14] U.-C. Fiebig, "Iterative interference cancellation for ffh/mfsk ma systems," *IEE Proceedings-Communications*, vol. 143, no. 6, pp. 380–388, December 1996.
[15] J. Proakis, *Digital Communications*, McGraw-Hill, 2001.