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Estimating Brasilia Rain Attenuation at THz Frequencies from Historical Data Based in Monte Carlo Simulation and Unscented Transform

Lucas Morais, Leonardo Menezes, and Pedro Moraes

***Abstract*— This paper proposes a case study for estimating rain attenuation from rainfall data collected in Brasilia, Brazil at THz frequencies using Mie Theory and Drop Size Distribution (DSD). To address this goal, we used measured rainfall rate data from the past 19 years collected at the National Institute of Meteorology (INMET). A statistical approach that uses Monte Carlo (MC) simulation and Unscented Transform (UT) was applied to obtain a reasonable estimation of rainfall attenuation in the terahertz spectrum. To evaluate the accuracy of the method, we performed a comparison between the rain attenuation calculated by Mie Theory and ITU-R recommendation. Also, a comparison between MC and UT were made to analyze the accuracy of the estimations. The results calculated in this work showed that the mean attenuation varies from 1.918 to 2.66 dB/km. For rain events higher than 10 mm/h, results showed that the mean attenuation varies from 8.53 to 15.96 dB/km.**

***Keywords*— Drop Size Distribution, Terahertz, Mie Theory, Monte Carlo, Rain Attenuation, Unscented Transform, Weibull.**

I. Introduction

Rain has significant impacts on terahertz communications due to attenuation that is caused by absorption and scattering of electromagnetic (EM) radiation. These effects are essentially produced by water droplets. There are two main approaches to calculate rain attenuation: i) empirically using well-known models as ITU-R; and ii) theoretically using Mie theory, which considers a drop of water a perfect sphere and a Drop Size Distribution (DSD). The DSD provides the distribution of raindrops according to their diameter [5], [11], [16]. The emergence of new technologies in communication systems has stimulated studies towards higher data transmission capacity. The Terahertz (THz) band consists of EM waves within frequencies between 0.3 to 3 THz. The short wavelengths provide a high ability to improve the rate of transmission and recently have been used in many indoor applications. A major limitation of outdoor propagation at THz is related to the high absorption and scattering from atmospheric gases and particles. Also, the rain might be a large source of scattering and absorption of EM at THz.

There are many published studies based on rain attenuation that covers a specific frequency [7], [8], [10], [12], [14]. Ishii [8] uses experimental values of 96, 140, 225, 313, and 355 GHz to estimate rain attenuation and concluded that the best DSD fit to the frequency of 313-355 GHz was using Mie theory combined with the Weibull distribution. Norouzian et al. [12] compared the rain attenuation at 77 GHz and 300 GHz by using ITU-R and Mie Theory with DSD distributions. They concluded that the model that had the best fit to the data collected in Birmingham, UK was with Weibull distribution. Therefore, the approach using Mie Theory and Weibull distribution for THz attenuation shows the best fit for DSD theoretical model.

This study provides an estimation of rainfall in THz communication based on Mie/Weibull theory, ITU-R recommendations, MC simulation, and UT. Due to the lack of studies related to the theoretical calculation of rain attenuation at THz spectrum in Brazil, the main goal of this work is to clarify and study the methods already described in the literature to find the influence of rainfall using different approaches and real data collected over 19 years in Brasilia, DF.

This paper is structured as follows: Section II introduces the theory behind rainfall attenuation and the procedure used to statistically estimate attenuation. Section III focuses on the behavior of rainfall in Brasilia. Section IV describes the attenuation dynamics due to rain in the wet season in Brasilia. Finally, Section V gives the main conclusions of this work.

II. Theory of Rainfall Attenuation

A. Mie Theory

Analyzing attenuation from scattering might be a challenging task. Mie Theory is based on a solution of Maxwell's Equations assuming that the drop of water is a perfect sphere and solving the equations using spherical Bessel functions. The cross-sections are obtained for a sphere by calculating the rate W_a at which electromagnetic energy crosses the surface of an imaginary sphere [3]. Then, it is possible to define the extinction energy rate as being:

$$W_{ext} = W_a + W_s \tag{1}$$

where W_a is the absorption energy rate; W_s is the scattering energy rate. The cross-sections coefficient can be defined in (2) as being a rate between power $[W]$ and intensity of radiance $[W/m^2]$.

$$C_{ext} = \frac{W_{ext}}{I} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)Re(a_n + b_n) \tag{2}$$

where the coefficients a_n and b_n are based on the spherical Bessel functions [3].

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B. Drop Size Distribution

The DSD has been an important topic of research in the last few decades [1], [2], [9], [17]. Norouzian et al. [12] empirically calculated the DSD using measured attenuation data at 77 GHz and 300 GHz and they concluded that the best fit to the real data was using ITU-R and Mie Scattering Theory with the Weibull distribution.

The Weibull distribution was first used to explain rainfall attenuation in 1982 by Sekine and Lind [14]. Where the parameters N_0, η, and σ were fitting by real data and R is the rainfall rate given in mm/h. Norouzian et al. [12] used the data collected to the 77 GHz and 300GHz to fit the parameters in (3) and (4).

$$N(D) = N_0 \frac{\eta}{\sigma} \left(\frac{D}{\sigma}\right)^{\eta-1} exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right] \tag{3}$$

$$\begin{aligned} N_0 &= 1100R^{0.53} \quad m^{-3} \\ \eta &= 0.9R^{0.05} \\ \sigma &= 0.24R^{0.2} \quad mm \end{aligned} \tag{4}$$

C. Rainfall Attenuation

Using Mie theory and the concepts presented by Olsen [13], it is well-known that the rain attenuation can be calculated by integrating (5) over all raindrop diameters.

$$A\left(\frac{db}{km}\right) = 4.343 \int_0^{a_{max}} C_{ext}(a, \lambda, m) N(a, R) da \tag{5}$$

Where N(a,R) is the DSD which is the number of drops per unit of volume and diameter of drops and the variables a, λ, and m are respectively the diameter of a rain drop, wavelength, and refractive index . Applying (2), (3), (4), and (5), it is possible to calculate a theoretical value for rainfall attenuation in a range of THz frequency.

The classical approach to find attenuation is using the empirical model from ITU-R [15] that describes rainfall attenuation throughout 1 to 1000 GHz. Equation 6 shows the ITU-R model to calculate the attenuation due to rain. The parameters k and α depend on the frequency and polarization.

$$A_{ITU} = kR^{\alpha} \tag{6}$$

D. Statistical Methods

In order to achieve an estimation of rain attenuation with data collected at INMET, we used two simulation approaches: i) Monte Carlo based on the distribution of rainfall rates throughout the wet season [6]. ii) Unscented Transform, which can give an estimation of rain attenuation based on few simulations with calculated weights [4].

III. Calculated Attenuation in Brasilia, DF, Brazil

A. Rainfall Data in Brasilia

It is crucial to know the rainfall behavior throughout the year and seasons to correctly analyze and predict rainfall attenuation. Therefore, a third important phase of this work is based on the analyses of data collected at the National Institute of Meteorology (INMET). The rainfall rate was collected in Brazil between 2001 and 2019 at the weather station localized in Brasilia, Brazil (latitude: -15.789343, Longitude: -47.925756, Height: 1160.9 m). Fig. 1 shows the total volume of rain and the mean rate of rainfall throughout 19 years of data. It is interesting to note that April is not the rainiest month by analyzing the total volume of rain, but it has the greatest mean rate in mm/hr. The rainfall intensity is classified according to the rate of precipitation. The following categories are used to classify rainfall intensity:

- Light Rain: $R \leq 2.5$ mm/h
- Moderate Rain: $2.5 \leq R < 10$ mm/h
- Heavy Rain: $10 \leq R < 50$ mm/h
- Violent Rain: $R \geq 50$ mm/h

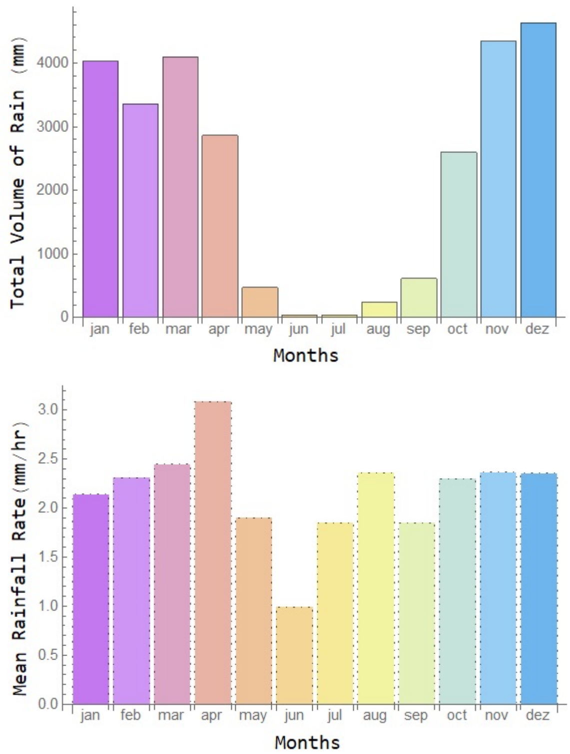


Fig. 1. Total Volume and Mean Rate of Rainfall Between 2001-2019

Figure 1 shows that the dry season in Brasilia is from May to September. Even though there is some rain throughout the dry season, it is clear by analyzing the data that they are quite rare events. To analyze the attenuation throughout the wet season, we collected the rainfall data from October to April. Fig. 2 shows the histogram and the cumulative distribution function (CDF) of 19 years of rainfall versus the rain rate measured in mm/h for the months from October to April. Most events of rain, more precisely 86.62 %, occur at the range of 0 to 5 mm/h, and an event of heavy rain, which is rain above 10 mm/hr, has the probability of 5.41 % occurring during the wet season.

The rainfall needs to be studied in its extreme cases to predict damage attenuation. Figure 3 shows the PDF of all

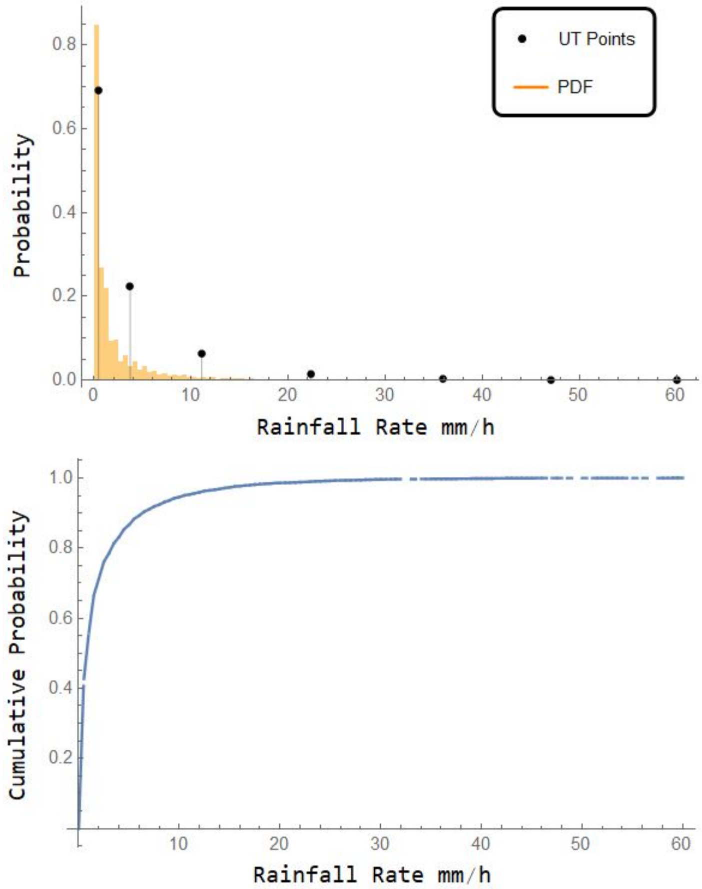


Fig. 2. PDF with several rainfall events throughout the wet season (October to April) along 2001 to 2019 and CDF of Rainfall rate in mm/hr. The histogram contains the Unscented Transform points (UT).

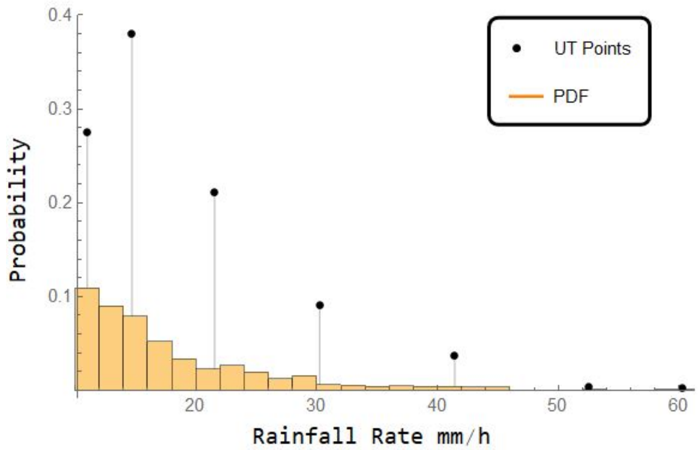


Fig. 3. PDF with several rainfall events throughout the wet season (October to April) along 2001 to 2019 for events of rain above 10 mm/h. The PDF contains the Unscented Transform points (UT).

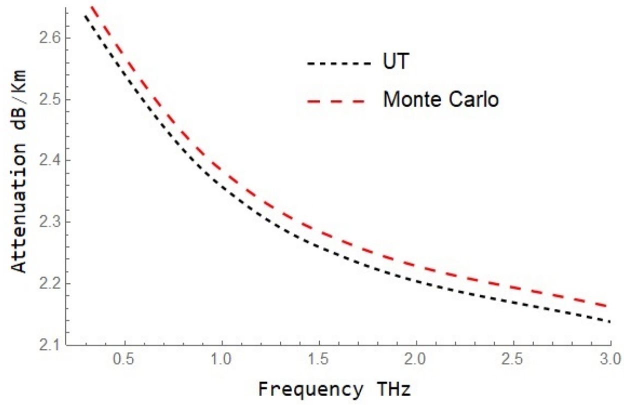
rainfall with a rate above 10 mm/h that occurs from October to April throughout 19 years of data. The UT points were also plotted in Fig.2 and Fig.3.

IV. Rainfall Attenuation

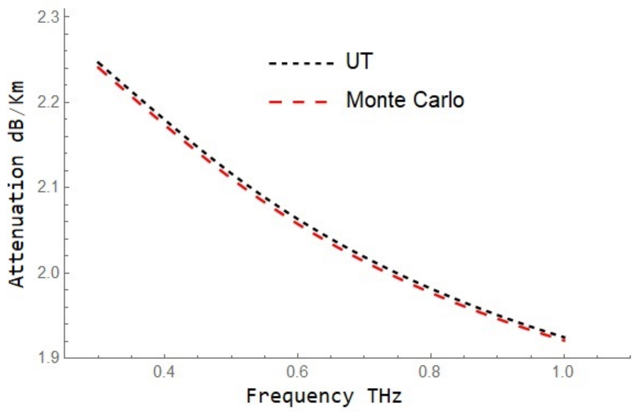
A. Mean Attenuation in the Wet Season

To analyze the dynamic of rainfall in Brasilia, it is important to observe the absence of rain between the months of May to September, which is the dry season in the Midwest of Brazil. Considering the events of rain between these months rare, it is not necessary to analyze the dynamics of attenuation due to rain throughout these months.

The method used to estimate the attenuation was done by Monte Carlo simulation and Unscented Transform. By using Monte Carlo, the Probability Distribution Function (PDF) of the wet season (October to April) was collected and then used as input data for the attenuation function calculated by (5) using Mie/Weibull theory and as an input in (6) that uses the ITU-R recommendation. The seven UT points based on the rain rate distribution (Fig.2/ Fig.3) were used as input and a comparison between both approaches is disposed in Fig.4, which shows the attenuation as a function of frequency for MC and UT methods. Fig.4a shows the attenuation calculated from Mie/Weibull theory and Fig.4b from ITU-R data.



a)



b)

Fig. 4. (a) Mean attenuation versus frequency in the wet season by using Mie/Weibull theory. (b) Attenuation versus frequency in the wet season by using ITU-R theory.

The mean attenuation disposed in Fig.4 is not enough to have a complete picture about the behavior of attenuation due to rain in Brasilia, DF. The amount of dispersion in the values is also an important parameter to have a better idea of attenuation. Tab.I shows a comparison for values of mean and standard deviation (SD) using MC and UT methods for attenuation values that were calculated using (5), which is Mie/Weibull theory. Table II shows the same information, but now calculated using (6), which is ITU-R.

Fig. 5 is the smooth probability distribution that was calculated using MC and Mie/Weibull Theory. It shows how the probability distribution of attenuation is varying for some THz frequencies. As expected, the attenuation tends to decrease as the frequency increase for terahertz applications.

TABLE I
MEAN ATTENUATION AND SD USING MIE/WEIBULL THEORY, MONTE CARLO, AND UT FOR TERAHERTZ FREQUENCIES

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
3.	2.161	2.137	3.198	3.255
2.142	2.217	2.192	3.277	3.325
1.578	2.274	2.249	3.355	3.396
1.035	2.375	2.349	3.492	3.522
0.625	2.513	2.486	3.691	3.712
0.517	2.561	2.533	3.768	3.787
0.3	2.666	2.635	3.981	3.998

TABLE II
MEAN ATTENUATION AND SD USING ITU-R, MONTE CARLO, AND UT FOR TERAHERTZ FREQUENCIES

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
1	1.918	1.925	2.084	2.058
0.9	1.944	1.951	2.096	2.068
0.8	1.974	1.982	2.114	2.085
0.7	2.011	2.019	2.141	2.111
0.6	2.055	2.063	2.179	2.148
0.5	2.108	2.116	2.231	2.200
0.4	2.171	2.179	2.302	2.269
0.3	2.238	2.246	2.388	2.355

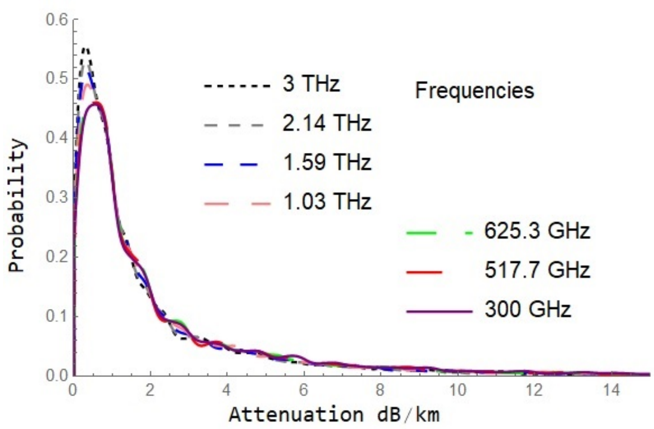
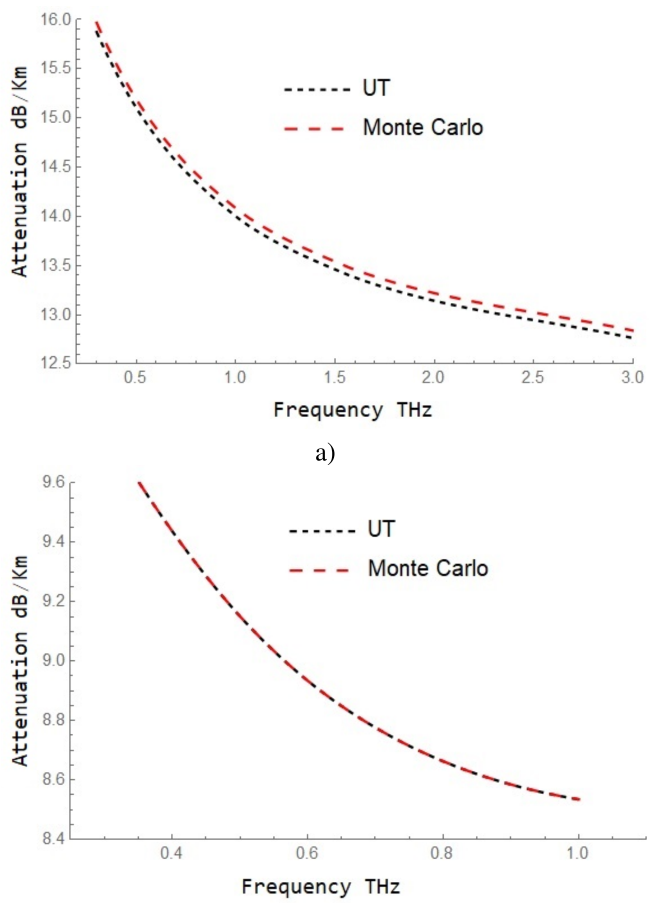


Fig. 5. Smooth PDF of rain attenuation for some THz frequencies based on Monte Carlo simulation.

B. Mean Attenuation for Extreme Events

As already observed, attenuation due to scattering and absorption in THz frequencies is a major limitation to the application of these technologies. To fully understand the behavior of a system working on these frequencies, it is necessary to comprehend the attenuation response due to extreme cases. Extreme cases can be understood as events of rainfall with a rate greater than 10 mm/h. The probability of a rain event of this magnitude is around 5.41% in the wet season.

Fig. 6 is the mean attenuation of extreme events over wet season using Mie/Weibull and ITU-R. As expected, the divergence between ITU-R and Mie/Weibull grows for extreme events. Table III shows the mean and SD calculated for some THz frequencies using Mie/Weibull. Also, it shows a comparison of mean and SD between MC and UT methods. Table IV shows the same parameters but now calculated using ITU-R recommendations. Figure 7 shows the probability distribution of attenuation for some frequencies at THz spectrum using Mie/Weibull theory. As expected, the attenuation curve goes to the right showing that the mean attenuation is increasing as the frequency decreases.



a)

b)

Fig. 6. (a) Mean attenuation versus frequency for extreme cases in the wet season by using Mie/Weibull theory. (b) Attenuation versus frequency for extreme cases in the wet season by using ITU-R theory.

TABLE III

MEAN ATTENUATION AND SD USING MIE/WEIBULL THEORY, MONTE CARLO, AND UT FOR TERAHERTZ FREQUENCIES BASED ON THE RAINFALL RATE ABOVE 10 MM/HR

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
3.	12.838	12.7624	4.63476	4.87904
2.14286	13.1551	13.0778	4.75035	5.19163
1.57812	13.473	13.3939	4.85625	5.33217
1.03555	14.0254	13.9432	5.0383	5.74728
0.625391	14.8253	14.7386	5.31335	6.0626
0.517777	15.1318	15.0432	5.42732	6.25045
0.3	15.9675	15.8733	5.77921	6.43654

TABLE IV

MEAN ATTENUATION AND SD USING ITU-R, MONTE CARLO, AND UT FOR TERAHERTZ FREQUENCIES BASED ON RAINFALL RATE RATE DATA ABOVE 10 MM/HR

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
1	8.53461	8.5344	2.28378	2.05812
0.9	8.58465	8.58445	2.2799	2.06835
0.8	8.66282	8.66262	2.28529	2.08551
0.7	8.77638	8.77618	2.30256	2.11144
0.6	8.93424	8.93404	2.33481	2.14838
0.5	9.15136	9.15115	2.38772	2.20016
0.4	9.43829	9.43808	2.46654	2.26959
0.3	9.78778	9.78755	2.57338	2.35537

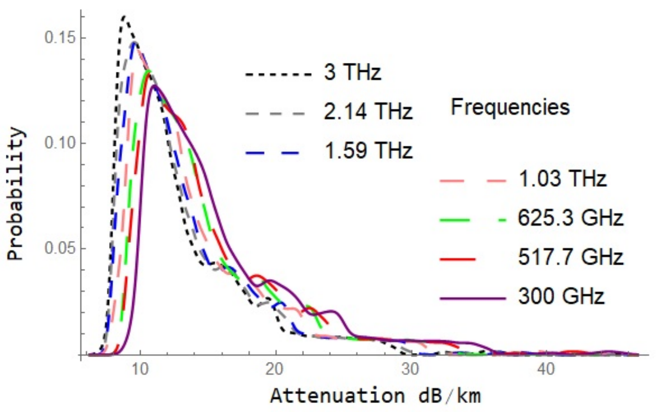


Fig. 7. Smooth PDF of attenuation for rainfall rate greater than 10 mm/h at Thz frequencies based on Monte Carlo simulation.

V. CONCLUSION

Electromagnetic attenuation due to rainfall at THz frequency is studied in this paper. To predict the mean attenuation at THz spectrum in Brasilia, MC and UT statistical method was applied with historical data collected in INMET. To calculate the attenuation, it was used the theoretical approach using Mie scattering theory together with DSD based in Weibull and also with the empirical model proposed by ITU-R. The mean attenuation throughout the wet season (October to April) was calculated and the results show that MC and UT have converged to very similar values. To evaluate the mean attenuation for extreme cases, which is included rainfall rate greater than 10 mm/h, we collected the data of extreme rains events and used the same approach with MC and UT to calculate mean attenuation.

The results show that considering the minimum/maximum value of rainfall attenuation using ITU-R, Mie/Weibull, MC, and UT, the mean attenuation along the wet season in a year will be from 1.918 to 2.666 dB/km. For extreme rain events with rate higher than 10 mm/h, results showed that the mean attenuation varies from 8.534 to 15.967 dB/km. A further step to this work would be to validate the estimations proposed by using experimental data collected using a transmitter/receiver working in THz frequencies along the year. This work is fundamental to understand THz signal propagating phenomena in a rainy environment in Midwest of Brazil to further outdoor applications.

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