Evaluation of the Effects of the Co-Channel Interference on the Bit Error Rate of Cellular Systems for BPSK Modulation

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***Abstract*—A performance analysis in terms of the bit error rate for digital systems with co-channel interference is done. In order to evaluate the effects of the interference on systems performance, expressions for the bit error rate are obtained for different scenarios. In these scenarios we consider one, two or K interferers that have the same power. If K is large, the interference is gaussian and the interference can be treated as equivalent noise. Scenarios with two interferers, where one of them is dominant and where the interference is asynchronous to the signal are also considered. Although the gaussian approximation is easy to be obtained, it is considered not a good model for cellular networks, because the number of main interferers is in essence one or two. This paper presents an effective tool to evaluate how much effective are the diverse schemes proposed to mitigate the co-channel interference in cellular networks.**

***Index Terms*—2-PAM, BPSK, Co-Channel Interference, SIR, BER.**

I. Introduction

The characterization of the co-channel interference is a fundamental question to the correct evaluation of the performance of cellular networks in terms of the bit error rate (BER).

The interference in cellular systems is produced by the co-cells that use the same resources of the central cell. As the propagation loss is proportional to the distance between the transmitter and receiver raised to an exponent (typically equal to 4), the interference is mainly produced by the first tier of co-channel cells [1].

Some papers have evaluated the BER for different digital modulations in the presence of co-channel interference [2], [3], [4], [5], [6]. As the exact analysis is difficult to do, the interference is sometimes modelled as gaussian [7]. In this paper, we present much simpler and intuitive expressions of the BER just in the presence of co-channel interference.

With the crescent widespread of cellular networks (e.g. WiMax and LTE), the correct performance evaluation of these systems in the presence of co-channel interference is an important item that deserves consideration. Besides, another important question is how effectively some known techniques in the literature mitigate the co-channel interference, as the reuse factor, antenna arrays, etc.

In this paper, we present expressions for the BER for BPSK modulation in the presence of K co-channel interferers with same power. It is supposed that the signal and interference are synchronous. We also present an expressions for a scenario with two interferers, where one of them prevails and for the case where signal and interference are asynchronous. An expression for the gaussian approximation is also presented, with the purpose of comparing the obtained results.

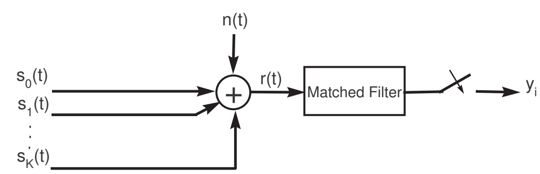

Figure 1. Receiver with K Interferers.

Initially, we are going to obtain BER expressions for a bandbase 2-PAM system. Later, we are going to show that these results are directly applied to BPSK systems.

This paper is organized as follows. Section II presents the system model. Section III derives BER expressions for different scenarios. Section IV presents the numerical results and section V shows the conclusions.

II. System Description

Consider the bandbase system shown in Fig. 1, where the received signal is given by:

$$r(t) = \sum_{k=0}^{K} s_k(t) + n(t) \tag{1}$$

where $s_k(t)$ are 2-PAM signals. Specifically, $s_0(t)$ represents the signal component, $s_k(t)$ for $k = 1, 2, \cdots, K$ are the K interferers and $n(t)$ is the additive white gaussian noise with bilateral power spectral density equal to $N_0/2$.

The PAM signal component is given by:

$$s_0(t) = \sum_{i=-\infty}^{\infty} A b_{i,0} p(t - iT_b) \tag{2}$$

where A is the amplitude, $b_{i,0}$ is the transmitted bit at the i-th time interval, that is modelled as a Bernoulli random variable that assumes ± 1 and $p(t)$ is the pulse format with duration T_b.

The k-th PAM source of interference is given by:

$$s_k(t) = \sum_{i=-\infty}^{\infty} \alpha A b_{i,k} p(t - iT_b) \tag{3}$$

where αA is the amplitude and $b_{i,k}$ is the transmitted bit at i-th time interval by k-th interferer.

A. K Interferers

Let's consider first a system with only one interferer and later we generalize to K interferers. Suppose that there is no noise and that signal and interference are synchronous, specifically in the time interval $iT_b \leq t \leq (i+1)T_b$. The matched filter output is sampled at $t = (i+1)T_b$ and is given by:

$$y_{(i+1)T_b} = Ab_{i,0} + \alpha A b_{i,1} \tag{4}$$

where we used that $\int_{iT_b}^{(i+1)T_b} p^2(t)dt = 1$. The mean power of the received signal given by (1) with just one interferer is given by:

$$P = A^2 + \alpha^2 A^2 \tag{5}$$

From (2), the signal mean power is equal to $S = A^2$. From (3), the interference mean power is $I = \alpha^2 A^2$. As a consequence, the signal to interference ratio (SIR) is given by:

$$\frac{S}{I} = \frac{1}{\alpha^2} \tag{6}$$

Developing the same reasoning for two interferers with equal power, the sample at the matched filter output is given by:

$$y_{(i+1)T_b} = Ab_{i,0} + \frac{1}{\sqrt{2}} \sum_{k=1}^{2} \alpha A b_{i,k} \tag{7}$$

where the factor $1/\sqrt{2}$ maintains the total interference power equal to $I = \alpha^2 A^2$ and as a consequence the SIR is given by (6).

Generalizing for K interferers with equal power, the sample at the matched filter output is given by:

$$y_{(i+1)T_b} = Ab_{i,0} + \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \alpha A b_{i,k} \tag{8}$$

For a large number of K interferers with equal power, the interference becomes gaussian and as a consequence we can write that noise plus interference power is equal to an equivalent noise power, that is:

$$\left(\frac{S}{N}\right)_{eq} = \frac{S}{N+I} = \frac{1}{\left[\left(\frac{S}{I}\right)^{-1} + \left(\frac{S}{N}\right)^{-1}\right]} \tag{9}$$

B. One Prevailing Interferer

It is not probable at all that in a cellular network the power of the interferers be all equal, nor that the interference be gaussian. In fact, as there are just 6 co-channel cells nearer a given central cell, there is potentially just 6 main interferers. As the propagation loss is proportional to the distance between transmitter and receiver raised to a propagation exponent of γ (typically 4), any difference between the interferers distances to a given receiver is amplified by the propagation exponent. Thus, it is much more probable that there is one prevailing interferer.

Let's suppose now the case of two interferers, where one of them prevails. In this case, the sample at the matched filter output is given by:

$$y_{(i+1)T_b} = Ab_{i,0} + \alpha A b_{i,1} + \beta A b_{i,2} \tag{10}$$

where α and β are multiplicative constants that define the power of both interferers.

In this case, the total mean power is given by:

$$P = A^2 + \alpha^2 A^2 + \beta^2 A^2 \tag{11}$$

From the mean power given by (11), we can write that the SIR is given by:

$$\frac{S}{I} = \frac{1}{\alpha^2 + \beta^2} \tag{12}$$

When $\beta \ll \alpha$ the first interferer is dominant in relation to the second and (6) becomes a good approximation for the SIR.

C. One Asynchronous Interferer

Consider now that the interference is asynchronous to the signal, that is:

$$s_1(t) = \sum_{i=-\infty}^{\infty} \alpha A b_{i,1} p(t - iT_b - \tau) \tag{13}$$

where τ is a delay between interference and signal, that is a random variable uniform in the interval $0 \leq \tau \leq T_b$.

In this case, the sample at the matched filter output is given by:

$$y_{(i+1)T_b} = Ab_{i,0} + \alpha A b_{i-1,1} \frac{\tau}{T_b} + \alpha A b_{i,1} \frac{(T_b - \tau)}{T_b} \tag{14}$$

The asynchronism between signal and interference does not modify the SIR, that is given by (6).

D. BPSK Modulation

Consider the same receiver given in Fig. 1 for the BPSK modulation.

The received signal is now given by:

$$s_0(t) = \sum_{i=-\infty}^{\infty} \sqrt{2} A b_{i,0} p(t - iT_b) \cos(2\pi f_0 t + \phi) \tag{15}$$

where $\sqrt{2}$ maintains the power equal to the bandbase case and ϕ is the received phase. The k-th interferer is given by:

$$s_k(t) = \sum_{i=-\infty}^{\infty} \sqrt{2}\alpha A b_{i,k} p(t - iT_b)\cos(2\pi f_0 t + \phi) \quad (16)$$

Considering absence of noise and synchronism between signal and interference, we can write the sample at the matched filter output for the time interval $iT_b \leq t \leq (i+1)T_b$ as:

$$y_{(i+1)T_b} = Ab_{i,0} + \alpha A b_{i,1} \quad (17)$$

The mean power of the received signal is given by (1), where the PSK signal and interference are given by (15) and (16). For the case with just one interferer, the mean power for the PSK modulation is also equal to (5). As a consequence, the SIR is also given by (6). Based on this, we can extend all analysis developed for 2-PAM to the BPSK case. In the following, all the analysis is valid for both BPSK and 2-PAM modulation, but we will refer just as BPSK that is the focus of the paper.

III. BER Analysis

The BER for a BPSK system without interference is given by [8]:

$$P_b = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \quad (18)$$

where $E_b = A^2 T_b$ and $\sigma^2 = N_0/2T_b$, for A and T_b given in (2). Using the same reasoning when developing (18), we can obtain the BER for different scenarios of interference.

A. K Interferers

Let's consider first one interferer and then extend to K interferers. The BER in this case is given by:

$$P_b = \frac{1}{2}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right) + \frac{1}{2}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right) \quad (19)$$

The BER for two interferers with same power is given by:

$$\begin{aligned} P_b &= \frac{1}{4}Q\left(\left(1+\frac{2}{\sqrt{2}}\alpha\right)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left(\left(1-\frac{2}{\sqrt{2}}\alpha\right)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{2}Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \quad (20) \end{aligned}$$

For the general case of K interferers, we can write that:

$$P_b = \sum_{k=0}^{K} \frac{\binom{K}{k}}{2^K} Q\left(\left[1 - \frac{K-2k}{\sqrt{K}}\alpha\right]\sqrt{2\frac{E_b}{N_0}}\right) \quad (21)$$

When K is large, the interference is gaussian. Using (9) in (18), we can obtain that:

$$\begin{aligned} P_b &= Q\left(\sqrt{\left(\frac{S}{N}\right)_{eq}}\right) \\ &= Q\left(\sqrt{\left[\left(\frac{S}{I}\right)^{-1} + \left(\frac{S}{N}\right)^{-1}\right]^{-1}}\right) \quad (22) \end{aligned}$$

B. One Prevailing Interferer

The BER for the case of two interferers, where one of them is dominant is given by:

$$\begin{aligned} P_b &= \frac{1}{4}Q\left([1+(\alpha+\beta)]\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left([1+(\alpha-\beta)]\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left([1-(\alpha+\beta)]\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left([1-(\alpha-\beta)]\sqrt{2\frac{E_b}{N_0}}\right) \quad (23) \end{aligned}$$

where (19) can be a good approximation when $\beta \ll \alpha$.

C. One Asynchronous Interferer

For the asynchronous case, the BER using (14) is given by:

$$\begin{aligned} P_b &= \frac{1}{4}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left(\left(1+\alpha-\alpha\frac{2\tau}{T_b}\right)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left(\left(1-\alpha+\alpha\frac{2\tau}{T_b}\right)\sqrt{2\frac{E_b}{N_0}}\right) \quad (24) \end{aligned}$$

which is a function of τ.
The mean value of the BER is given by:

$$\begin{aligned} \overline{P_b} &= \frac{1}{4}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right) \\ &+ \frac{1}{4}\int_0^{T_b} Q\left(\left(1+\alpha-\alpha\frac{2\tau}{T_b}\right)\sqrt{2\frac{E_b}{N_0}}\right)\frac{1}{T_b}d\tau \\ &+ \frac{1}{4}\int_0^{T_b} Q\left(\left(1-\alpha+\alpha\frac{2\tau}{T_b}\right)\sqrt{2\frac{E_b}{N_0}}\right)\frac{1}{T_b}d\tau \quad (25) \end{aligned}$$

These integrals have closed form and consequently the mean BER is given by:

$$\begin{aligned}\overline{P_b} &= \frac{1}{4}Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right)\\ &+ \frac{1}{4}Q\left((1-\alpha)\sqrt{2\frac{E_b}{N_0}}\right)+\frac{1}{4}\\ &+ \frac{1}{8\alpha\sqrt{\pi\frac{E_b}{N_0}}}\exp\left[-(-1+\alpha)^2\frac{E_b}{N_0}\right]\\ &- \frac{1}{8\alpha\sqrt{\pi\frac{E_b}{N_0}}}\exp\left[-(1+\alpha)^2\frac{E_b}{N_0}\right]\\ &+ \frac{(-1+\alpha)}{8\alpha}\left[1-2Q\left((-1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right)\right]\\ &- \frac{(1+\alpha)}{8\alpha}\left[1-2Q\left((1+\alpha)\sqrt{2\frac{E_b}{N_0}}\right)\right] \qquad (26)\end{aligned}$$

IV. Numerical Results

In order to evaluate the BER for cellular networks in the presence of co-channel interference, we are going to plot the expressions developed in section III.

Fig. 2 presents the BER as a function of E_b/N_0 in dB for just one interferer, where we used (19) for $S/I = -3$, 0, 3, 6, 9 and ∞ dB. For $S/I = -3$ dB, there is a BER floor that is equal to $1/2$ due to the fact that the interference power is larger than signal power. For $S/I = 0$ dB, the BER floor is equal to $1/4$ due to the fact that the interference power is equal to the signal power. In both cases the system performance can not be improved even increasing the E_b/N_0. On the other hand, for $S/I = 3$, 6 and 9 dB we observe that BER decreases with E_b/N_0 with a cost of some dB in relation to the free interference case. When $S/I \to \infty$ the case of no interference given in (18) is achieved. Here we also plot the case without interference with the purpose of comparing the obtained results.

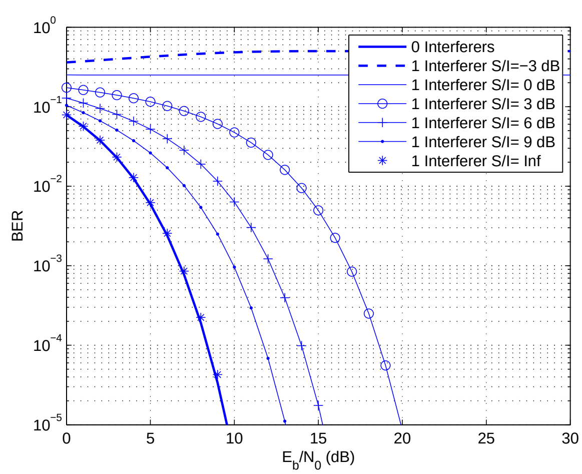

Figure 2. BER as a function of E_b/N_0 in dB, for one interferer and $S/I = -3$, 0, 3, 6, 9 and ∞ dB.

Fig. 3 presents the BER as a function of E_b/N_0 in dB for two interferers, where we used (20) for $S/I = 0$, 3, 6, 9 and ∞ dB. For $S/I = 0$ dB, the BER floor is equal to $1/4$ and for $S/I = 3$ dB, the BER floor is equal to $1/8$ due to the fact that interference power is larger than and equal to the signal power respectively. For $S/I = 6$, and 9 dB we observe that BER decreases with E_b/N_0 and when $S/I \to \infty$ there is the same behavior as in the free interference case.

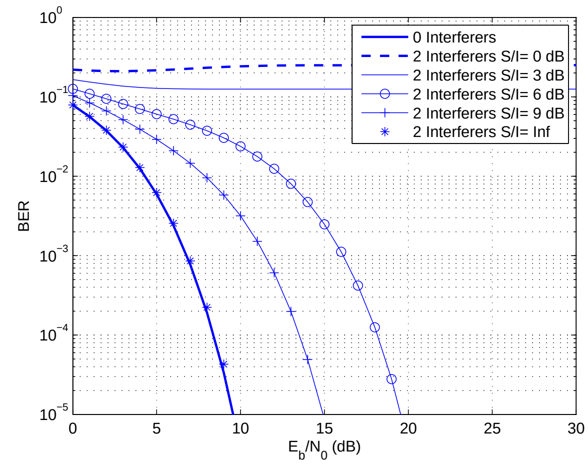

Figure 3. BER as a function of E_b/N_0 in dB, for two interferers and $S/I = 0$, 3, 6, 9 and ∞ dB.

In (21) when the SIR is equal to the number of interferers K, the BER presents a floor given by $P_b = 1/2^{K+1}$. When the SIR is greater than K, there is no floor at all.

Fig. 4 presents the gaussian approximation BER as a function of E_b/N_0 in dB, where we used (22) for $S/I = -3$, 0, 3, 6, 9 and ∞ dB. Observe when $K \to \infty$ interferers the BER presents a floor for any S/I, and when $S/I = \infty$ there is no BER floor.

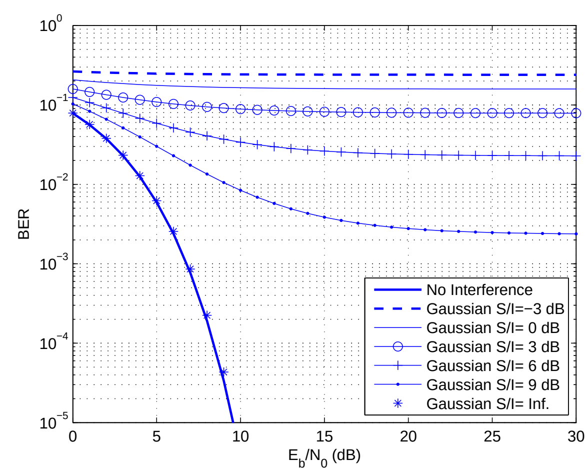

Figure 4. BER as a function of E_b/N_0 in dB, for ∞ interferers and $S/I = -3$, 0, 3, 6, 9 and ∞ dB.

Fig. 5 presents the BER as a function of E_b/N_0 in dB, where we used (21) and simulation results, for 0, 1, and 6 interferers and $S/I = 9$ dB. Notice that the BER worsen by increasing the number of interferers, although S/I is kept constant. For a BER of 10^{-5}, one interferer causes a degradation of approximately 3 dB in E_b/N_0 and six interferers a degradation of 15 dB.

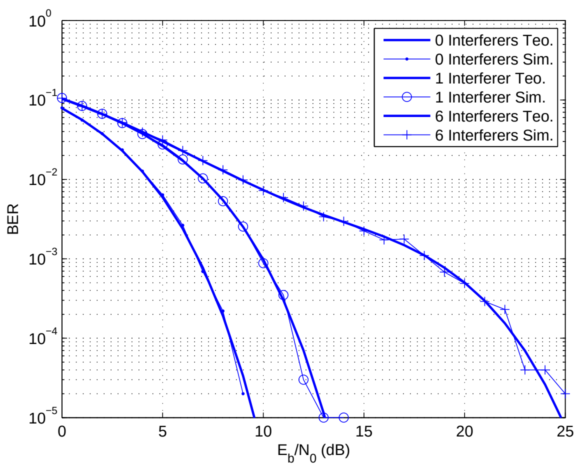

Figure 5. BER as a function of E_b/N_0 in dB, for 0, 1 and 6 interferers and $S/I = 9$ dB.

Fig. 6 presents the BER as a function of E_b/N_0 in dB, where we used (22) for 2 interferers and $S/I = 9$ dB. Two cases are considered: one of the users is a dominant strong with $\beta/\alpha = 0.1$ or a dominant weak with $\beta/\alpha = 0.6$ for a $S/I = 9$ dB. For comparison purposes, the curves for one and two interferers are also shown. Observe that, as expected, the strong dominant case is near the one interferer curve and the weak dominant is near the two interferer curve.

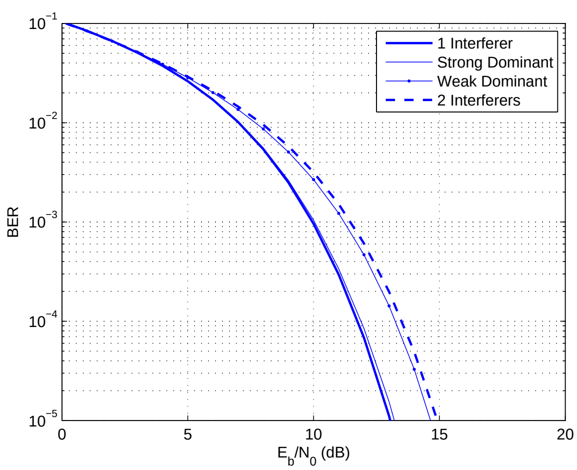

Figure 6. BER as a function of E_b/N_0 in dB, for dominant strong $\beta/\alpha = 0.1$ and dominant weak $\beta/\alpha = 0.6$ and $S/I = 9$ dB.

Fig. 7 presents the BER as a function of E_b/N_0 in dB, where we used (26) for 1 interferer and $S/I = 9$ dB. This curve presents a comparison between the synchronous and asynchronous case for a $S/I = 9$ dB. Observe that when the interferer is synchronous to the signal, the BER is worse by approximately 0.7 dB. As the difference between both cases is small and as the synchronous case is in fact the worst case, we should prefer the synchronous analysis because these expressions are easier to obtain and to manipulate.

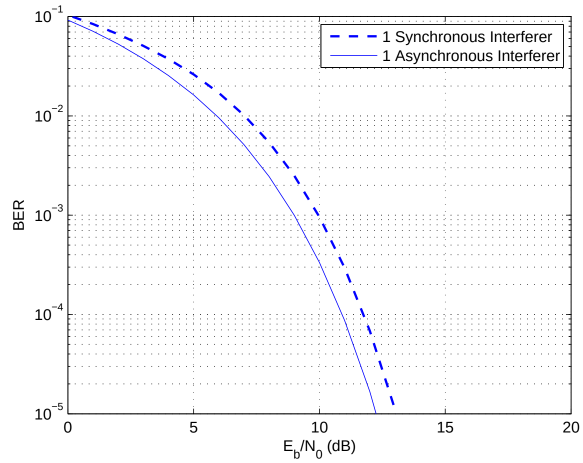

Figure 7. BER as a function of E_b/N_0 in dB, for one Synchronous and Asynchronous Interferer with $S/I = 9$ dB.

V. Conclusions

In this paper, we have presented expressions for the BER for a BPSK system that were evaluated in different scenarios of co-channel interference. One scenario that we have examined present K synchronous interferers for a given S/I. We have concluded that there is a floor in the BER when $S/I \leq K$ or when $K \to \infty$ for any S/I. Other important scenario that we have examined is when there is one prevailing interferer. Finally, the scenario where the asynchronous interference presents better performance than the synchronous interference. We have concluded that the case with just one synchronous interferer is a good case to study. These expressions are also valid for Q-PSK since this has the same performance of BPSK. These results are important to correctly evaluate the performance of cellular networks that use BPSK and Q-PSK modulations.

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