# Limiting the statistical behavior of interference: the concept of equivalent limiting masks

Alexander Beremiz Hilario Tacuri and José Mauro P. Fortes

***Abstract*— The use of masks to limit the statistical behavior of interference is very common in communications. The ITU-R Radio Regulations has a number of examples of interference masks that are used to guarantee that the external interferences reaching a communication system would not prevent it from meeting its performance requirements. Each of these masks refers to a specific situation that depends on the characteristics of the interfering and the interfered-with systems. Currently, for each specific situation there is only a single mask to be applied. This paper presents a new concept in which, instead of a single mask, a set of *equivalent masks* is used to guarantee that, even in the presence of interference, a given communication link meets its performance requirements.**

***Keywords*— Interference Masks, satellite, communications.**

## I. Background

When a number of communications systems share a common frequency band, each of them operates subject to the interferences generated by the others. In such a situation it is important that studies and analyses be conducted to define the constrains (masks) that should be imposed to the interfering systems so that the protection of the victim system from harmful interference is guaranteed. These analyses are usually complex and strongly depend on the technical characteristics of the systems involved. The mask is usually obtained iteratively: starting with a given mask (and assuming that all interfering systems transmitting powers satisfy the mask with zero margins), calculations are performed to verify if the performance requirements of the victim system are satisfied. In case they are not satisfied, the initial mask is modified to increase the constraints it imposes on the interfering systems transmitting powers. If, on the other hand, the performance requirements are satisfied with significant margins, the initial mask is modified so that no undue constrains are imposed on the interference systems. This process is repeated iteratively leading to the definition of the mask that would guarantee the protection of the victim system without imposing undue constrains on the interfering systems.

As an alternative to this iterative process, more complex techniques to obtain limiting masks for the statistical behavior of interference can be found in Annex I to ITU-R Recommendation S. 1323 [1]. These techniques are based on methodologies that consider the join effect of the degradations due to rain and external interferences. Due to the complexity of such techniques and to the difficulty involved in applying them to situations of real interest, the example applications in [1] are academic and involve very simple mathematical models for the probability density function of the degradation due to external interferences. This fact has motivated the work in [2] that, by using a basis of orthonormal functions to represent the probability density function of the degradation due to external interferences and by solving a conveniently defined constrained optimization problem, makes it possible to apply, in a systematic way, methodologies such as those in [1] to situations of real interest. As a result, the technique in [2] produces a single limiting mask for the aggregate external interference affecting a given communication link. If the statistical behavior of the interference satisfies the limiting mask, the performance requirements of the link are met.

As shown in Section II, the mathematical modeling in [2] can also be used to define a new concept: the concept of equivalent limiting masks. Under this new approach, instead of a single limiting mask, a set of *equivalent limiting masks* is used. Interference would be acceptable if its statistical behavior satisfies any of the masks in the set.

## II. Mathematical Modeling

To guarantee the adequate performance of a communication link it is usual to establish a set of constrains that are to be satisfied by the link bit error rate (BER). These constraints are usually specified by limiting the percentage of time during which pre-specified levels of BER can be exceeded, that is,

$$P(b > BER_j) \leq p_j \quad ; \quad j = 1, ..., m \tag{1}$$

where $b$ is a random variable characterizing the link BER, $\{BER_j \ , \ j = 1, ..., m\}$ denote pre-specified levels of BER and $\{p_j \ , \ j = 1, ..., m\}$ are probability values that reflect the percentages of time these levels should not be exceeded. Note that the BER is being here modeled as a random variable mainly due to its dependency on degrading random factors such as rain attenuation and external interferences.

Let then, $e$ be a random variable characterizing the degraded $E_b/N_0$ ratio (energy per bit to thermal noise density ratio), expressed in dB. Note that $e$ can be written as

$$e = E_{CS} - z \tag{2}$$

where $E_{CS}$ is the clear sky $E_b/N_0$ ratio and $z$ is the degradation due to factors such as rain attenuation and external interferences, both in dB. Considering that the dependency of $b$ on $e$ is usually given by a monotone decreasing function $f$, it is possible to show that the constrains in (1) are equivalent to

$$P(f(E_{CS} - z) > BER_j) \leq p_j \quad ; \quad j = 1, ..., m \tag{3}$$

Alexander Beremiz Hilario Tacuri and José Mauro P. Fortes, Centro de estudos em Telecomunicações da PUC-Rio, Rio de Janeiro, Brasil, E-mails: ahilario@cetuc.puc-rio.br, jmfortes@cetuc.puc-rio.br. Este trabalho foi parcialmente financiado pelo CNPq.

or, considering the decreasing nature of $f$,

$$1 \geq F_z(Z_j) \geq 1 - p_j \quad ; \quad j = 1, ..., m \tag{4}$$

where $F_z(Z)$ denotes the probability distribution function of $z$ and the values $\{Z_j, j = 1, \ldots, m\}$ are given by

$$Z_j = E_{CS} - f^{-1}(BER_j) \tag{5}$$

with $f^{-1}$ denoting the inverse function of $f$. Note that, in (4), the fact that $F_z(Z) \leq 1$ was also considered.

Here, as in [2], $z$ reflects the joint effect of the degradation due to rain and to external interferences, that is, $z = x + y$ with $x$ and $y$ being random variables characterizing, respectively, $E_b/N_0$ degradations due to rain and to external interferences, both expressed in dB. Assuming that these two random variables are statistically independent, the probability distribution function of $z$ is given by

$$F_z(Z) = F_x(Z) * p_y(Z) \tag{6}$$

where the symbol $*$ denotes the convolution operation. The statistical characterization of rain attenuation, expressed by $F_x(Z)$ in (6), is usually obtained from well known rain attenuation models. In [2], for example, the model in ITU-R Recommendation P.618-7 [3] was used. For the probability density function of the degradation due to external interferences, the technique in [2] uses the parametric representation given by

$$p_y(Y) = \alpha_0 \, \delta(Y - Y_{min}) + \alpha_{n+1} \, \delta(Y - Y_{max}) + \sum_{i=1}^{n} \alpha_i \, \phi_i(Y) \tag{7}$$

where $\delta(.)$ is the Dirac Impulse Function, $\{\phi_i(Y) \; , \; i = 1, ..., n\}$ is a basis of orthonormal functions and $\{\alpha_0, ..., \alpha_{n+1}\}$ is the set of parameters used to represent $p_y(Y)$. Considering (6) and (7) it is possible to show [2] that the constraints in (4) can be written as

$$1 \geq f_j^{max} + \mathbf{k}_j^T \boldsymbol{\alpha} \geq 1 - p_j \quad ; \quad j = 1, 2, \ldots, m \tag{8}$$

where

$$\boldsymbol{\alpha} = (\alpha_0 \;\; \alpha_1 \;\; \cdots \;\; \alpha_n)^T \tag{9}$$

and

$$f_j^{max} = F_x(Z_j - Y_{max}) \tag{10}$$

with $Z_j$, given by (5). Also in (8),

$$\mathbf{k}_j = \mathbf{m}_j - f_j^{max} \mathbf{c} \tag{11}$$

where

$$\mathbf{m}_j = \left( f_j^{min} \;\; M_1(Z_j) \;\; \cdots \;\; M_n(Z_j) \right)^T ; \; j = 1, 2, \ldots, m \tag{12}$$

with

$$M_i(Z_j) = \phi_i(Z) * F_x(Z) \Big|_{Z = Z_j} \; ; \; \begin{array}{rcl} j & = & 1, 2, \ldots, m \\ i & = & 1, 2, \ldots, n \end{array} \tag{13}$$

and

$$f_j^{min} = F_x(Z_j - Y_{min}) \tag{14}$$

In (11),

$$\mathbf{c} = (1 \;\; c_1 \;\; \cdots \;\; c_n)^T \tag{15}$$

with $\{c_i \; ; \; i = 1, \cdots, n\}$ given by

$$c_i = \int_{Y_{min}}^{Y_{max}} \phi_i(Y) \; dY \quad ; \quad i = 1, 2, \ldots, n \tag{16}$$

Observe that, in obtaining (8), $\alpha_{n+1}$ was expressed as a function of $\alpha_0, \alpha_1, \ldots, \alpha_n$ by considering that

$$\int_{Y_{min}}^{Y_{max}} p_y(Y) \; dY \; = 1 \tag{17}$$

This way,

$$\alpha_{n+1} = 1 - \alpha_0 - \sum_{i=1}^{n} \alpha_i c_i \tag{18}$$

with the coefficients $c_i$ given by (16).

Note that, in a more compact notation, (18) can be written as

$$\alpha_{n+1} = 1 - \mathbf{c}^T \boldsymbol{\alpha} \tag{19}$$

with $\boldsymbol{\alpha}$ and $\mathbf{c}$ respectively given by (9) and (15).

As explained in [2], additional constraints had to be imposed to the parameters $\alpha_0, \alpha_1, \ldots, \alpha_n$ to guarantee that $p_y(Y)$ in (7) have the characteristics of a probability density function. They are given by

$$0 \leq \alpha_0 \leq 1 \tag{20}$$

$$0 \leq \mathbf{c}^T \boldsymbol{\alpha} \leq 1 \tag{21}$$

and

$$\boldsymbol{\Phi}^T(Y) \boldsymbol{\alpha} \geq 0 \quad ; \quad \forall \quad Y \in (Y_{min}, Y_{max}) \tag{22}$$

At this point, it is worth noting that any set $\boldsymbol{\alpha}$ of coefficients satisfying (8), (20), (21) and (22) corresponds to a probability density function $p_y(Y)$ (and consequently to a probability distribution function $F_y(Y)$) for which the performance requirements in (1) are met. The methodology in [2] searches for the set $\boldsymbol{\alpha}^\star$ that corresponds (see (7)) to a probability density function $p_y^\star(Y)$ for which the probability of having $y$ in a given subset $\mathcal{S}$ of the interval $(Y_{min}, Y_{max})$ is maximized. In other words, the technique in [2] consists in finding a solution for the constrained optimization problem of maximizing

$$f(\boldsymbol{\alpha}) = P(y \in \mathcal{S}) = \int_{\mathcal{S}} p_y(Y) dY \tag{23}$$

subject to (8), (20), (21) and (22). The optimal solution to this problem leads to a cumulative distribution function $C_y^\star(Y) = 1 - F_y^\star(Y)$ that is used as a limiting mask for the statistical behavior of the degradation $y$ due to external interferences. A limiting mask for the statistical behavior of the interference to thermal noise ratio $i/n$ can then be obtained considering that $i/n$ and $y$, when both are expressed in dB, are related by

$$\frac{i}{n} = 10 \log \left( 10^{\frac{y}{10}} - 1 \right) \tag{24}$$

and, as a consequence, the cumulative distribution function of the $i/n$ ratio can be obtained from the cumulative distribution function of $y$ using the relation

$$C_{\frac{i}{n}}^\star(\Gamma) = C_y^\star \left( 10 \log \left( 10^{\frac{\Gamma}{10}} - 1 \right) \right) \tag{25}$$

The cumulative distribution function $C_{\frac{i}{n}}^\star(\Gamma)$ in (25) defines a limiting mask for the statistical behavior of the interference to thermal noise ratio $i/n$. Interferences corresponding to $i/n$ ratios having a statistical behavior satisfying this mask are acceptable since they do not prevent the performance requirements in (1) from being met.

Instead of using a single limiting mask, the idea here is to have a family of *equivalent limiting masks*. The constraints in (8), (20), (21) and (22) are then used to define this new concept. Masks $C_{\frac{i}{n}}(\Gamma)$ corresponding to values of $\boldsymbol{\alpha}$ satisfying (8), (20), (21) and (22) are said to be *equivalent*. Interferences having $i/n$ ratios with statistical behavior satisfying any of these masks would be acceptable since they do not prevent the performance requirements in (1) from being met. The benefits of this approach can be appreciated in the numerical example presented in Section III.

## III. Numerical results and Conclusion

The first example in [2] is used here as a reference to illustrate the advantage of using a family of *equivalent masks* to limit the statistical behavior of the interference affecting a satellite communication system operating in 19 GHz. In the example, scaled normalized *Shifted Lagrange Polynomials* [2], [4] were used as the basis $\{\phi_i(Y)\ ,\ i = 1, ..., n\}$ of orthonormal functions, with $Y_{min} = 0$, $Y_{max} = 5$ and $n = 7$. The performance requirements indicated in (3) were taken to be those in Table I. The probability distribution function $F_x(X)$ of the degradation due to rain was obtained using the rain attenuation model contained in ITU-R Recommendation P.618-7 [3]. More specifically, the procedure described in Item 2.2.1.1 of this recommendation was used to determinate the probability distribution function of $x$. In this procedure the parameter values presented in Table II were used to produce the probability distribution function $F_x(X)$.

TABELA I

Performance requirements for the 19 GHz satellite link

| $BER_j$ | $(E_b/N_0)_j$ (dB) | $p_j$ |
|---|---|---|
| $1 \times 10^{-6}$ | 6,5 | 0,0004 |
| $1 \times 10^{-8}$ | 7,6 | 0,006 |
| $1 \times 10^{-9}$ | 8,7 | 0,04 |

TABELA II

Attenuation Model Parameters (19 GHz link)

| | |
|---|---|
| point rainfall rate for 0.01% of an average year - $R_{0.01}$ - [mm/h] | 23 |
| earth station height above mean sea level - $h_s$ - [km] | 0 |
| rain height - $h_r$ - [km] | 3 |
| earth station antenna elevation angle - $\theta$ - [degree] | 25 |
| earth station latitude - $\varphi$ - [degree] | 40 |
| frequency - $f$ - [GHz] | 19 |
| effective radius of the Earth - $R_e$ - [km] | 8500 |

Under these assumptions, a single limiting mask $C^{\star}_{\frac{i}{n}}(\Gamma)$ was obtained using the procedure in [2], which maximizes (23) subject to the constraints in (8), (20), (21) and (22). This mask corresponds to $\boldsymbol{\alpha}^{\star}$ = (0.0731, 0.4146, -0.6158, 0.5896, -0.4387, 0.2558, -0.1078, 0.0262) and is shown in Figure 1, which also contains curves describing the statistical behavior of the $i/n$ ratios corresponding to two different interferences: Interference 1 and Interference 2. Note in Figure 1 that, according to this single limiting mask, only Interference 1 is acceptable.

To illustrate the concept of *equivalent limiting masks* defined in this paper, two additional masks were considered. They were obtained by choosing two different values of $\boldsymbol{\alpha}$, each of them satisfying the constraints in (8), (20), (21) and (22), guaranteeing that they are equivalent to the single mask in Figure 1. More specifically these two equivalent masks correspond to $\boldsymbol{\alpha}$ = (0.7147, 0.1275,-0.1304, 0.0302, 0.0503, -0.0583, 0.0280, -0.0056) and $\boldsymbol{\alpha}$ = (0.9177, 0.0367, -0.0051, -0.0129, -0.0158, 0.0095, 0.0153, -0.0101), respectively. The three equivalent masks are shown in Figure 2, where the statistical behavior of interferences 1 and 2 are also illustrated. Note that if a family of equivalent masks is used instead of a single one, both interferences are acceptable since, according to the proposed definition of *equivalent masks*, none of them prevent the victim link performance requirements in Table I from being met (Interference 1 satisfies Mask 1 and Interference 2 satisfies Mask 2). Concluding, this example clearly shows the benefit of using a set o equivalent masks: some acceptable interferences which may not be allowed under a single limiting mask approach are allowed under a family of equivalent limiting masks.

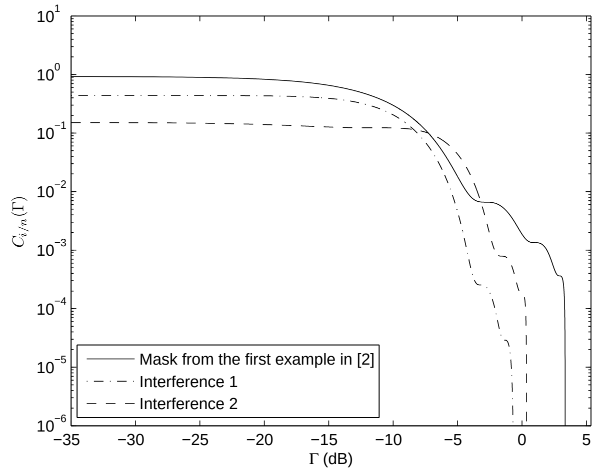


Fig. 1. Single limiting mask.

Another possible sub-product of the presented approach would be a new procedure to check if a given interference is acceptable or not. To briefly describe this new procedure, let $\hat{C}_{\frac{i}{n}}(\Gamma)$ denote the cumulative distribution function of the interference to be checked. The cumulative distribution function of the degradation due to interference $\hat{C}_y(Y)$ is then determined using the relation in (25). The next step would be to represent de probability density function

$$\hat{p}_y(Y) = \frac{d}{dY}\left(1 - \hat{C}_y(Y)\right) \quad (26)$$

in the parametric form given in (7). This can be done, for

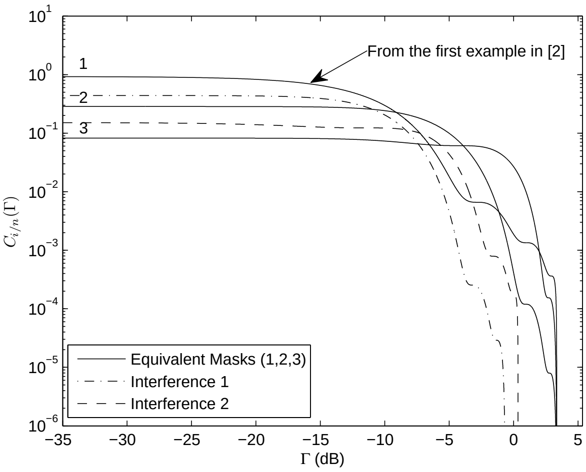


Fig. 2. The concept of Equivalent Masks.

example, by determining the value $\hat{\boldsymbol{\alpha}}$ that minimizes the function

$$g(\boldsymbol{\alpha}) = \int_{[Y_{min}, Y_{max}]} \Big[ \alpha_0 \delta(Y - Y_{min}) + \alpha_{n+1} \delta(Y - Y_{max}) + \sum_{i=1}^{n} \alpha_i \; \phi_i(Y) - \hat{p}_y(Y) \Big]^2 \, dY. \quad (27)$$

Remember that, in (27),

$$\alpha_{n+1} = 1 - \alpha_{n+1} = \mathbf{c}^T \boldsymbol{\alpha} = 1 - \alpha_0 - \sum_{i=1}^{n} \alpha_i \; c_i \quad (28)$$

with $\{c_i, i = 1, \ldots, n\}$ given by (16). Finally, the interference in question would be considered acceptable if $\hat{\boldsymbol{\alpha}}$ satisfies (8), (20), (21) and (22) and not acceptable otherwise.

## REFERÊNCIAS


[1] *Maximum permissible levels of interference in a satellite network (GSO/FSS; non-GSO/FSS; non-GSO/MSS feeder links) in the fixed-satellite service caused by other codirectional FSS networks below 30 GHz*, ITU-R Recommendation S.1323-2, International Telecommunication Union, Geneva 2002.

[2] Fortes, J. M. and Baptista, A. J., *Satellite link performance under rain and external interferences: Constrains to be imposed to the Statistical behavior of external interferences*, International Journal of Satellite Communications and Networking, (Published on line on October 2010 - DOI 10.1002/sat.985), Vol. 29, pp 333-348, 2011.

[3] *Propagation data and prediction methods required for the design of Earth-space telecommunication systems*, ITU-R Recommendation P.618-7, International Telecommunication Union, Geneva 2001.

[4] Abramowitz M., Stegun I. A., *Handbook of Mathematical Functions*, Dover Publication Inc., New York, 9th Edition, 1972.