The Joint Phase-Envelope Distribution of the κ-μ Fading Channel

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***Resumo*— Este artigo apresenta uma expressão exata e em forma fechada para a distribuição conjunta da envoltória e fase para o ambiente de desvanecimento κ-μ, um modelo de desvanecimento generalizado que inclui como casos especiais os modelos de Rice e Nakagami-*m*. As estatísticas conjuntas são obtidas em compatibilidade com os casos particulares de Rice e Nakagami-*m*. Simulações de Monte Carlo são usadas para validar as formulações.**

***Palavras-Chave*— Desvanecimento, distribuição conjunta, distribuição κ-μ.**

***Abstract*— This paper derives an *exact* closed-form expression for the phase-envelope joint distribution of the κ-μ fading environment, a general fading model that includes the Rice and the Nakagami-*m* models as special cases. The derived joint statistics are obtained so that compatibility with both Rice as well as Nakagami-*m* cases are kept. Monte Carlo simulations are used to validade the formulations.**

***Keywords*— Fading, joint distribution, κ-μ distribution.**

I. Introduction

The κ-μ distribution is a general fading distribution that includes as special cases other important distributions such as Rice (Nakagami-*n*) and Nakagami-*m* [1]. The One-Sided Gaussian and the Rayleigh distributions also constitute special cases of it. The κ-μ distribution can be used to represent the small-scale variation of the fading signal under line-of-sight (LOS) conditions. It also describes the fading observed when clusters of multipath waves propagate in a non-homogeneous environment. Its flexibility renders it suitable to better fit field measurements data in a variety of scenarios, both for low- [1] and high-order statistics [2].

In contrast to the Rice, Rayleigh, and Hoyt fading models, for which the derivation of the envelope probability density function (PDF) produced as an intermediate step the corresponding joint phase-envelope PDF, for the κ-μ distribution, as well as for Nakagami-*m*, no information on the signal phase was provided when these distributions were described. In [3], a simple and closed-form Nakagami-*m* joint distribution for the phase and envelope was proposed. The corresponding phase PDF was then obtained and a compatibility with the PDFs comprised by Nakagami-*m* or approximated by it was achieved.

The envelope and phase distributions have found a great variety of applications in communications systems, such as radar clutter modeling, signal detection, statistics, and fields of multipath fading signal modeling [4]–[10]. In special, the phase PDF finds applications, for instance, in the estimation of probabilities of error for *M*-phase signaling over fading channels using diversity [11] and in the detection of *M*-ary phase shift keying (MPSK) signal constellations [12]. In some situations, the joint envelope-phase distribution may be used, for instance, to facilitate the determination of high-order statistics in multibranch diversity scenarios [13].

The aim of this paper is to derive the κ-μ envelope-phase distribution joint PDF in an exact and closed-form manner. It is certainly desirable that compatibility with the PDFs comprised by the κ-μ distribution be accomplished. In this case, the envelope-phase joint PDFs of Rice as well as that of Nakagami-*m* are to be found as special cases of the κ-μ PDF. Finally, in order to validate the formulations, simulations of the κ-μ fading channels are developed and compared against the theoretical data.

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II. The κ-μ Fading Model Revisited

The κ-μ distribution is a general fading distribution that represents the small-scale variations of the fading signal under a LOS condition [1]. As its name implies, it is written in terms of two physical parameters, namely κ and μ. The parameter μ is related to the multipath clustering, whereas the parameter κ is the ratio between the total power of the dominant components and the total power of the scattered waves.

For a fading signal with envelope R with $\hat{r} = \sqrt{E(R^2)}$ being the *rms* value of R, the κ-μ envelope probability density function, $f_R(r)$, is written as [1]

$$f_R(r) = \frac{2\mu\left(1+\kappa\right)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp\left(\mu\kappa\right)\hat{r}}\left(\frac{r}{\hat{r}}\right)^{\mu}\exp\left[-\mu\left(1+\kappa\right)\left(\frac{r}{\hat{r}}\right)^2\right] \times I_{\mu-1}\left[2\mu\sqrt{\kappa\left(1+\kappa\right)}\frac{r}{\hat{r}}\right] \quad (1)$$

where $\kappa > 0$, $\mu > 0$ is given by $\mu = \frac{E^2(R^2)}{Var(R^2)}\frac{1+2\kappa}{(1+\kappa)^2}$, $I_\nu(\cdot)$ is the modified Bessel function of the first kind and order ν [14, Equation 9.6.20], and $E(\cdot)$ and $Var(\cdot)$ denote the expectation and variance operators, respectively.

III. The κ-μ Phase-Envelope Joint Distribution

Let R and $\mathbf{\Theta}$ be random variates representing, respectively, the envelope and the phase of the κ-μ signal. The corresponding joint PDF, $f_{R,\Theta}\left(r,\theta\right)$, is given by (2), as shown at the top

$$f_{R,\Theta}(r,\theta) = \frac{r^{\mu+1}}{2}\frac{\mu^2}{\Omega^{\frac{\mu}{2}+1}}\frac{(1+\kappa)^{\frac{\mu}{2}+1}}{\kappa^{\frac{\mu}{2}-1}}\frac{|\sin(2\theta)|^{\frac{\mu}{2}}}{|\sin(2\phi)|^{\frac{\mu}{2}-1}}\exp\left[-\mu\left(\frac{(1+\kappa)r^2}{\Omega} - 2r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta-\phi)+\kappa\right)\right]$$
$$\times\,\mathrm{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta)\cos(\phi)\right)\mathrm{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\sin(\theta)\sin(\phi)\right)$$
$$\times\, I_{\frac{\mu}{2}-1}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}|\cos(\theta)\cos(\phi)|\right) I_{\frac{\mu}{2}-1}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}|\sin(\theta)\sin(\phi)|\right) \quad (2)$$

of the this page. Integrating (2) with respect to θ, the envelope PDF $f_R(r)$ is obtained as in (1). In the same way, integrating $f_{R,\Theta}(r,\theta)$ with respect to r, the phase PDF $f_\Theta(\theta)$ is found, unfortunately, in this case, not in closed form.

IV. Derivation of the κ-μ Phase-Envelope Joint Distribution

Let $K = R\exp(j\Theta)$ be the fading signal following the κ-μ distribution, in which R represents the envelope and Θ the phase. The complex κ-μ signal can be written as

$$K = X + jY \quad (3)$$

Of course, $R^2 = X^2+Y^2$, $\Theta = \arg(X+jY)$, $X = R\cos\Theta$, and $Y = R\sin\Theta$. In accordance with the definition of the κ-μ signal [1]

$$R^2 = \sum_{i=1}^{\mu}(X_i+p_i)^2 + \sum_{i=1}^{\mu}(Y_i+q_i)^2 \quad (4)$$

where X_i and Y_i are mutually independent Gaussian processes; $E(X_i) = E(Y_i) = 0$; $E(X_i^2) = E(Y_i^2) = \sigma^2$; p_i and q_i are respectively the mean values of the in-phase and quadrature components of the multipath waves of cluster i; and μ initially assumed to be integer. As defined in [1], $p^2 = \sum_{i=1}^{\mu} p_i^2$ and $q^2 = \sum_{i=1}^{\mu} q_i^2$, and

$$\kappa = \frac{p^2+q^2}{2\mu\sigma^2} \quad (5)$$

Accordingly,

$$\sigma^2 = \frac{\Omega}{2\mu(1+\kappa)} \quad (6)$$

where $\Omega = E(R^2)$. Defining $\phi = \arg(p+jq)$ as a phase parameter, then

$$p = \sqrt{\frac{\kappa\Omega}{1+\kappa}}\cos(\phi) \quad (7)$$

$$q = \sqrt{\frac{\kappa\Omega}{1+\kappa}}\sin(\phi) \quad (8)$$

Let $Z^2 = \sum_{i=1}^{\mu} Z_i^2$, where $Z = X$ or $Z = Y$ and $Z_i = X_i+p_i$ or $Z_i = Y_i+q_i$, as required. From the given definition, $E(Z_i) = \lambda_i$, $E(Z_i^2) = \sigma^2$, $\lambda_i = p_i$ or $\lambda_i = q_i$. Now, the aim is to obtain the distribution of Z. As an intermediate step, we find the PDF $f_W(w)$ of $W = Z^2$, which is known to follow a non-central chi-squared distribution with μ degrees of freedom, i.e.

$$f_W(w) = \frac{1}{2\sigma^2}\left(\frac{w}{|\lambda|^2}\right)^{\frac{\mu-2}{4}}\exp\left(-\frac{w+\lambda^2}{2\sigma^2}\right) I_{\frac{\mu}{2}-1}\left(\frac{\sqrt{w}|\lambda|}{\sigma^2}\right) \quad (9)$$

where $\lambda^2 = \sum_{i=1}^{\mu}\lambda_i^2$. The variate Z can be written as $Z = sgn(Z)\times|Z|$, where $sgn(\cdot)$ denotes the sign function. Note that $|Z| = \sqrt{W}$. Therefore

$$f_{|Z|}(|z|) = \frac{|z|^{\frac{\mu}{2}}}{\sigma^2|\lambda|^{\frac{\mu}{2}-1}}\exp\left(-\frac{z^2+\lambda^2}{2\sigma^2}\right) I_{\frac{\mu}{2}-1}\left(\frac{|\lambda z|}{\sigma^2}\right) \quad (10)$$

Motivated by the fact that for $\mu = 1$ the κ-μ envelope PDF, $f_R(r)$, reduces to the Rice one, then, in order to keep compatibility with such a model also in the phase case, for the same condition the PDF of its in-phase (or quadrature) component, $f_Z(z)$, must reduce to a non-zero mean Gaussian distribution. One possible solution for this problem is to obtain $f_Z(z)$ as

$$f_Z(z) = f_{|Z|}(|z|)\frac{f_Z(z)|_{\mu=1}}{f_{|Z|}(|z|)|_{\mu=1}} \quad (11)$$

where $-\infty < z < \infty$. The required compatibility is obtained if $f_Z(z)|_{\mu=1} = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(z-\lambda)^2}{2\sigma^2}\right]$. By carrying out the necessary calculations and simplifications, then

$$f_Z(z) = \frac{|z|^{\frac{\mu}{2}}}{2\sigma^2|\lambda|^{\frac{\mu}{2}-1}}\frac{\exp\left(-\frac{(z-\lambda)^2}{2\sigma^2}\right)}{\cosh\left(\frac{\lambda z}{\sigma^2}\right)} I_{\frac{\mu}{2}-1}\left(\frac{|\lambda z|}{\sigma^2}\right) \quad (12)$$

which is the in-phase ($Z = X$, $\lambda = p$) or quadrature ($Z = Y$, $\lambda = q$) κ-μ PDF. Knowing that X and Y are independent, then the joint PDF $f_{X,Y}(x,y)$ of X and Y is $f_{X,Y}(x,y) = f_X(x)\times f_Y(y)$. It follows that

$$f_{X,Y}(x,y) = \frac{|xy|^{\frac{\mu}{2}}}{4\sigma^4|pq|^{\frac{\mu}{2}-1}}\exp\left[-\frac{(x-p)^2+(y-q)^2}{2\sigma^2}\right]$$
$$\times\,\mathrm{sech}\left(\frac{px}{\sigma^2}\right)\mathrm{sech}\left(\frac{qy}{\sigma^2}\right) I_{\frac{\mu}{2}-1}\left(\frac{|px|}{\sigma^2}\right) I_{\frac{\mu}{2}-1}\left(\frac{|qy|}{\sigma^2}\right) \quad (13)$$

Using the standard statistical procedure of transformation of variates, then $f_{R,\Theta}(r,\theta) = |J|f_{X,Y}(x,y)$, where $|J| = r$ is the Jacobian of the transformation. Substituting (6), (7), (8), and expressing x and y in terms of r and θ in (13), the resulting joint PDF $f_{R,\Theta}(r,\theta)$ is given by (2). Integrating (2) with respect to r it is possible to obtain the phase PDF $f_\Theta(\theta)$, unfortunately not in closed form. Although derived for integer values of μ, there are no mathematical constraints for these expressions to be used for any $\mu > 0$.

V. DISCUSSIONS AND NUMERICAL RESULTS

With $\mu = 1$ in (2), then

$$f_{R,\Theta}(r,\theta) = \frac{r(\kappa+1)}{\pi\Omega} e^{-\frac{r^2(\kappa+1)}{\Omega} + 2r\sqrt{\frac{\kappa(\kappa+1)}{\Omega}}\cos(\theta-\phi)-\kappa} \quad (14)$$

which is the exact joint Rice phase-envelope PDF. In the same way, by setting $\kappa = 0$ in (2), it should be possible to obtain the Nakagami-*m* phase-envelope PDF. However, inserting $\kappa = 0$ in the κ-μ distribution leads to indeterminacy. Appendix I-A shows that in the limit, as $\kappa \to 0$, then

$$f_{R,\Theta}(r,\theta) = \frac{\mu^\mu |\sin 2\theta|^{\mu-1} r^{2\mu-1}}{2^{\mu-1}\Omega^\mu \Gamma^2(\mu/2)} \exp\left(-\frac{\mu r^2}{\Omega}\right) \quad (15)$$

which is the exact required joint PDF. Of course, both the envelope and the phase PDFs can be obtained in an exact manner, by performing the appropriate integrations of the corresponding formulas. As can be seen from (2), there are also indeterminacies for $\phi = \pm\frac{n\pi}{2}$, $n \in \mathbb{N}$. Appendix I-B shows *exact* formulations for these cases, i.e., the exact κ-μ joint density functions for $q = 0$ ($\phi = \pm n\pi$) and $p = 0$ ($\phi = \pm\frac{(2n+1)\pi}{2}$) are obtained. Obviously, these cases also reduce to the corresponding Rice as well as Nakagami-*m* ones.

Now, some plots of the κ-μ phase distribution are shown. In addition, the validity of the expressions is checked by comparing the theoretical curves against Monte Carlo simulation. As will be observed, an *excellent* agreement has been achieved between the simulation results and the formulations derived here. The Monte Carlo simulation was performed by considering an integer number of Gaussian processes, which are then generated in accordance with the well-established Jakes/Clark model [15]. Fig 1 depicts the phase PDF for arbitrary values of κ, μ and ϕ. For $\phi \neq \pm\frac{n\pi}{2}$, there is no symmetry around their minima, which occur at integers multiples of $\pm\frac{\pi}{2}$. On the other hand, for values of $\phi = \pm\frac{n\pi}{2}$, the curves are symmetrical about the minimum point, as show Fig. 2. A great number of other examples have been investigated by the authors and an excellent agreement has always been attained.

Fig. 3 plots the phase PDF for $\kappa = 0.81$, $\phi = \pm\frac{(2n+1)\pi}{2}$, and μ varying. For values of μ smaller than 1, the curves are convex tending to infinity at $\pm\frac{(2n+1)\pi}{2}$. For $\mu = 1$, the plots are those for the Rice case. For values of μ greater than 1, the curves are symmetrical at $\pm\frac{(2n+1)\pi}{2}$. Fig 4 plots the phase PDF for $\mu = 1.3$, $\phi = \pm n\pi$, and κ varying. As expected, for $\kappa \to 0$, the κ-μ phase PDF reduces to that of the Nakagami-*m* model. Fig 5 shows the shapes of the κ-μ phase PDF in polar coordinates with $\mu = 1.3$, $\phi = \pm n\pi$, and κ varying.

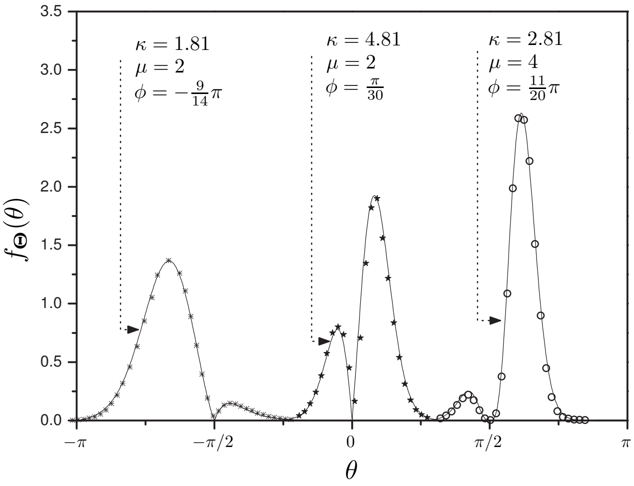

Fig. 1. Phase PDF of the κ-μ fading model. Comparison between theoretical (solid lines) and Monte Carlo simulated (stars and balls) curves.

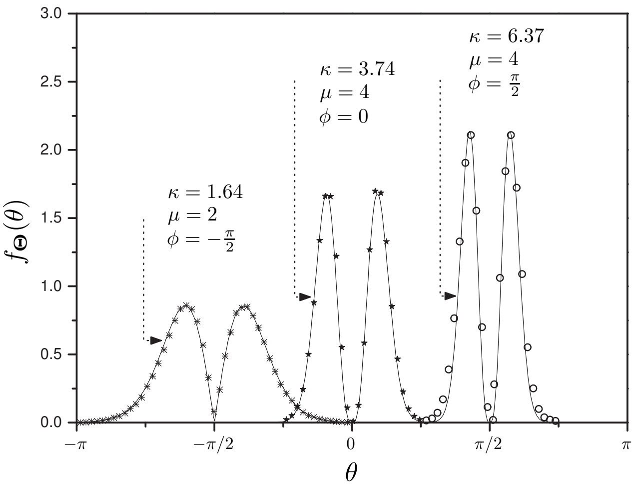

Fig. 2. Phase PDF of the κ-μ fading model. Comparison between theoretical (solid lines) and Monte Carlo simulated (stars and balls) curves.

VI. CONCLUSIONS

In this paper, the κ-μ phase-envelope joint PDF has been obtained in an exact manner. In addition, its limiting cases, which includes indeterminacies, have also been obtained. The formulations derived here comprise those of Rice and Nakagami-*m* statistics, found elsewhere in the literature. Sample numerical results obtained by means of Monte Carlo simulation were presented that validate the formulations developed here.

APPENDIX I
THE DISTRIBUTION FOR LIMITING VALUES OF THE PARAMETERS

This appendix obtains the expressions of the distribution for the cases in which the simple substitution of the parameters in the formulas leads to indeterminacy.

A. The κ-μ Distribution for $\kappa \to 0$

For small arguments of the Bessel function, the relation $I_{\nu-1}(z) \approx (z/2)^{\nu-1}/\Gamma(\nu)$ holds [14, Equation 9.6.7]. Using this in Equation (2), and after some algebraic manipulation, the corresponding joint PDF is given by (16). As $\kappa \to 0$, Equation (16) reduces to (15), which is the Nakagami-*m* phase-envelope

$$f_{R,\Theta}(r,\theta) = \frac{\mu^{\mu}|\sin(2\theta)|^{\mu-1} r^{2\mu-1}}{2^{\mu-1}\Omega^{\mu}\Gamma(\mu/2)}(1+\kappa)^{\mu}\exp\left[-\mu\left(\frac{r^2(1+\kappa)}{\Omega} - 2r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta-\phi)+\kappa\right)\right] \times \operatorname{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta)\cos(\phi)\right)\operatorname{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\sin(\theta)\sin(\phi)\right) \quad (16)$$

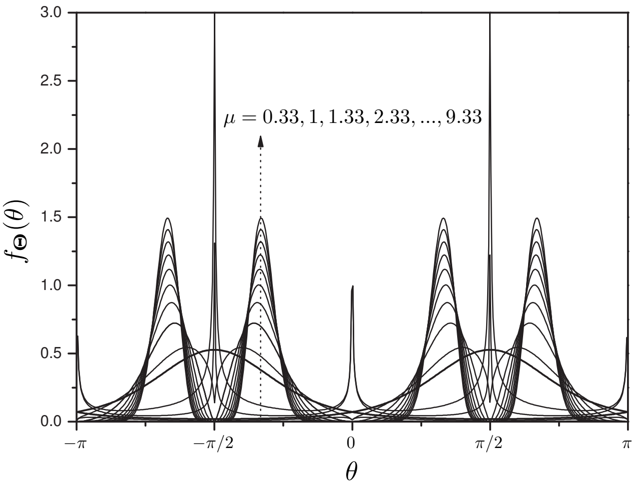

Fig. 3. Phase PDF of the κ-μ fading model ($\kappa = 0.81$, $\phi = \pm\frac{(2n+1)\pi}{2}$, and μ varying).

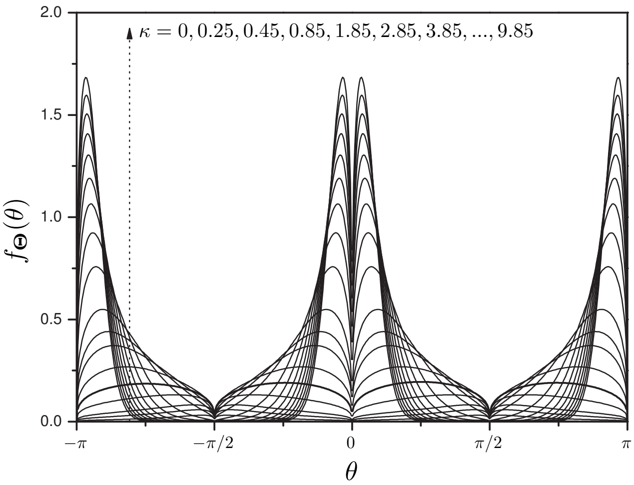

Fig. 4. Phase PDF of the κ-μ fading model ($\mu = 1.3$, $\phi = \pm n\pi$, and κ varying).

PDF [3, Equation 1]. In this case, the parameter μ coincides with the well-known Nakagami-*m* parameter m.

B. The κ-μ Distribution for $q = 0$ or $p = 0$

For $\phi = \pm n\pi$, $n \in \mathbb{N}$, the quadrature component becomes null ($q = 0$) then the κ-μ distribution has only the in-phase component p. Using $f_X(x)$ as in (13) and $f_Y(y)$ as the equivalent Nakagami-*m* PDF [3, Equation 11],

$$f_Y(y) = \frac{\mu^{\mu/2}|y|^{\mu-1}}{\Omega_n^{\mu/2}\Gamma(\mu/2)}\exp\left(-\frac{\mu y^2}{\Omega_n}\right) \quad (17)$$

which corresponds to the κ-μ $f_Y(y)$ with $q = 0$, where $\Omega_n = \Omega/(1+\kappa)$ is the relation between the Nakagami-*m* and the κ-μ mean powers. Carrying through the same standard statistical procedure of variates transformation then $f_{R,\Theta}(r,\theta) = |J| f_{X,Y}(x,y)$, where $|J| = r$, and substituting (7) in $f_X(x)$, the corresponding κ-μ joint phase-envelope PDF $f_{R,\Theta}(r,\theta)|_{\phi=\pm n\pi}$ is given by (18). Analogously, for $\phi = \pm\frac{(2n+1)\pi}{2}$, the in-phase component becomes null ($p = 0$) then the κ-μ distribution has only the quadrature component q. Therefore, the corresponding κ-μ joint phase-envelope PDF $f_{R,\Theta}(r,\theta)|_{\phi=\pm\frac{(2n+1)\pi}{2}}$ is obtained as (19).

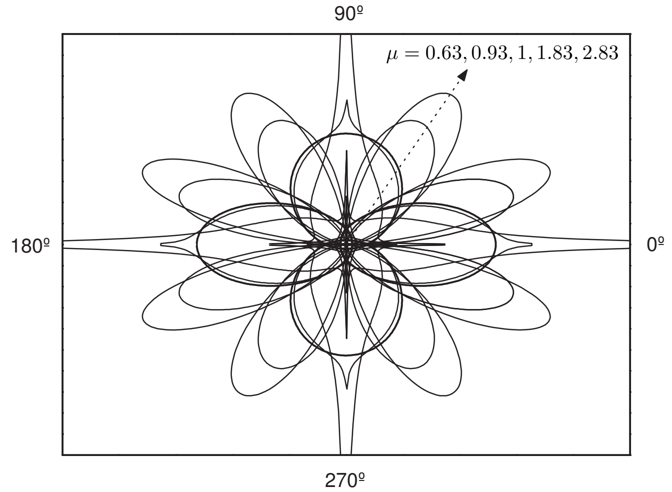

Fig. 5. Phase PDF of the κ-μ fading model - polar coordinates ($\kappa = 0.81$, $\phi = \pm\frac{n\pi}{2}$, and μ varying).

REFERENCES

[1] M. D. Yacoub, "The κ-μ and the η-μ distribution," *IEEE Ant. and Prop. Magazine*, vol. 49, pp. 68–81, Feb. 2007.

[2] S. L. Cotton, and W. G. Scanlon, "High-order statistics for κ-μ distribution," *IEE Electron. Lett.*, vol. 43, no. 22, Oct. 2007.

[3] M. D. Yacoub, G. Fraidenraich, and J. C. S. Santos Filho, "Nakagami-*m* phase-envelope joint distribution," *Electron. Lett.*, vol. 41, no. 5, Mar. 2005.

[4] J. I. Marcum, "A statiscal theory of target detection by pulsed radar," *IRE Trans. Inf. Theory*, vol. 29, pp. 59–267, Nov. 1960.

[5] S. O. Rice, "Statistical properties of a sine wave plus random noise," *Bell System Technical Journal*, vol. 27, pp. 109–157, Jan. 1948.

[6] J. H. Roberts, *Angle Modulation*. Stevenage, U.K.: Peregrinus, 1977.

[7] R. F. Pawula, "On the theory of error rates for narrowband signals digital FM," *IEEE Trans. Commun.*, vol. 29, pp. 1634–1643, Nov. 1981.

[8] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and*

$$f_{R,\boldsymbol{\Theta}}(r,\theta) = \frac{\mu^{\frac{\mu}{2}+1} r^{\frac{3\mu}{2}}}{\kappa^{\frac{\mu}{4}-\frac{1}{2}}} \left(\frac{1+\kappa}{\Omega}\right)^{\frac{3\mu}{4}+\frac{1}{2}} \frac{|\sin\theta|^{\mu-1}}{\Gamma(\mu/2)} \frac{|\cos\theta|^{\frac{\mu}{2}}}{|\cos\phi|^{\frac{\mu}{2}-1}} \exp\left[-\mu\left(\frac{(1+\kappa)r^2}{\Omega} - 2r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta)\cos(\phi) + \kappa\cos(\phi)^2\right)\right]$$
$$\times \operatorname{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\cos(\theta)\cos(\phi)\right) I_{\frac{\mu}{2}-1}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}|\cos(\theta)\cos(\phi)|\right) \quad (18)$$

$$f_{R,\boldsymbol{\Theta}}(r,\theta) = \frac{\mu^{\frac{\mu}{2}+1} r^{\frac{3\mu}{2}}}{\kappa^{\frac{\mu}{4}-\frac{1}{2}}} \left(\frac{1+\kappa}{\Omega}\right)^{\frac{3\mu}{4}+\frac{1}{2}} \frac{|\cos\theta|^{\mu-1}}{\Gamma(\mu/2)} \frac{|\sin\theta|^{\frac{\mu}{2}}}{|\sin\phi|^{\frac{\mu}{2}-1}} \exp\left[-\mu\left(\frac{(1+\kappa)r^2}{\Omega} - 2r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\sin(\theta)\sin(\phi) + \kappa\sin(\phi)^2\right)\right]$$
$$\times \operatorname{sech}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}\sin(\theta)\sin(\phi)\right) I_{\frac{\mu}{2}-1}\left(2\mu r\sqrt{\frac{\kappa(1+\kappa)}{\Omega}}|\sin(\theta)\sin(\phi)|\right) \quad (19)$$

Techniques. IEEE Press, 1996.

[9] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, *Digital Communication Techniques: Signal Design and Detection*. Prentice Hall, 1995.

[10] P. Beckmann, *Probability in Communication Engineering*. New York: Harcourt, Brace & World, 1967.

[11] J. G. Proakis, "Probabilities of error for adaptive reception of M-phase signals," *IEEE Trans. on Commun. Technol.*, vol. 16, no. 1, pp. 71–81, Feb. 1968.

[12] ——, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[13] G. Fraidenraich, J. C. S. Santos Filho, and M. D. Yacoub, "Second-order statistics of maximal-ratio and equal-gain combining in Hoyt fading," *IEEE Commun. Lett.*, vol. 9, no. 1, Jan. 2005.

[14] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1972.

[15] W. C. Jakes, *Microwave Mobile Communications*. New York, USA: Wiley, 1974.