Underwater Acoustic Noise Model for Shallow Water Communications

José S. G. Panaro, Fábio R. B. Lopes, Leonardo M. Barreira, Fidel E. Souza

***Resumo*—Este artigo apresenta um modelo empírico para o ruído do canal acústico submarino em águas rasas a partir da análise de dados provenientes de medições realizadas em campo. Uma função de densidade de probabilidade para a distribuição da amplitude do ruído é proposta e funções de verossimilhança são obtidas. Em decorrência, uma expressão para a probabilidade de erro de símbolo para sinalização binária no canal é apresentada. Além disso, são fornecidos os resultados de simulação realizada com as amostras reais de ruído coletadas em campo, de modo a verificar o efeito do ruído no desempenho de sistemas acústicos de comunicação submarina com sinalização binária.**

Palavras-Chave—Comunicação Submarina, Ruído Acústico Submarino, Distribuição de Ruído.

***Abstract*—This article presents an empirical model for the noise of the acoustic underwater channel in shallow water from the analysis of field data measurements. A probability density function for the noise amplitude distribution is proposed and the associated likelihood functions are derived. As a result, an expression to the probability of symbol error for binary signaling is presented for the channel. Additionally, the results of simulations conducted using the field collected noise samples are presented, in order to verify the noise effect on the performance of underwater acoustic communication binary signaling systems.**

Keywords—Underwater Communications, Underwater Acoustic Noise, Noise Distribution.

I. INTRODUCTION

The capability to communicate efficiently underwater has important applications including oceanographic studies, off-shore oil prospection and extraction, and defense operations. As electromagnetic waves cannot propagate over long distances underwater, acoustic communication assumes an important role for such applications.

Underwater acoustic communications has been a difficult problem due to unique channel characteristics such as fading, extended multipath, and refractive properties of the sound channel [1]-[3]. Moreover, underwater acoustic noise (UWAN) for shallow coastal water with presence of snapping-shrimp biological noise is neither white nor Gaussian distributed, presenting an accentuated impulsive behavior [3]-[6]. Therefore, a conventional communication receiver, designed for the AWGN channel, performs suboptimally in the presence of non-Gaussian noise.

In this paper, an empirical model for the noise of the acoustic underwater channel is developed from the analysis of field data measurements and a probability density function (pdf) is proposed. For binary signaling, the related likelihood functions are obtained and an expression for the symbol error probability is estimated for the channel. Additionally, simulations were conducted with experimentally collected noise, in order to estimate the performance of uncoded binary underwater communication systems.

II. NOISE MODEL

Several publications report that UWAN does not follow the normal distribution. In fact, this type of noise shows probability density function with extended tails shape, reflecting an accentuated impulsive behavior due to the high incidence of large amplitude noise events [5]-[10]. From these sources, it is known that UWAN follows the alpha-stable distribution class.

An symmetrical alpha-stable (SαS) distribution has a characteristic function given by

$$\phi_\alpha(\omega) = E[\exp(j\omega X)] = \exp\left(j\mu\omega - \gamma|\omega|^\alpha\right) \quad (1)$$

where μ is the location parameter and $\gamma > 0$ is the dispersion parameter. Consequently, the SαS probability density function (pdf) can be represented by the Fourier transform of the characteristic function, expressed as

$$f_\alpha(x) = \int_{-\infty}^{\infty} \exp\left(j\mu\omega - \gamma|\omega|^\alpha\right) e^{-j\omega x}\, d\omega \quad (2)$$

Cauchy and Gaussian distributions are particular cases of the alpha-stable distribution for $\alpha = 1$ and $\alpha = 2$ respectively. However, for intermediate values of α there is not analytical representation of the distribution SαS. Without knowledge of pdf in closed form, only the possibility of calculating (1) numerically remains. However, this introduces additional complexity in the study and implementation of the optimal receiver for underwater acoustic communication. In order to overcome this difficulty, it is desirable another characterizing method for the UWAN.

An alternative modeling method is by means of empirical analysis of noise samples obtained directly from the underwater environment. Thus, as part of this work, different segments of noise, collected during experiments conducted in shallow water at Arraial do Cabo, RJ, Brazil, have been analyzed. The signals were received through a broadband hydrophone (300 Hz ~ 11 kHz), located about 200 meters from the beach and 6 meters deep, with sea floor at a depth of 12 meters. The samples were digitalized at the rate of 44.1 kHz and collected from diverse continuous measurement intervals with duration of some seconds and separated each other by several minutes. Consequently, it was possible to carry out a frequency analysis on the amplitude of the background noise

José S. G. Panaro, Fábio R. B. Lopes e Fidel E. Souza¸ Departamento de Engenharia de Telecomunicações, Universidade Federal Fluminense, Niterói-RJ, Brasil, Leonardo M. Barreira, Instituto de Estudos do Mar Almirante Paulo Moreira, Arraial do Cabo-RJ, Brasil. E-mails: panaro@telecom.uff.br, fabiorbl@hotmail.com, fidelramber@hotmail.com, barreira@ieapm.mar.mil.br.

with about 20 million samples in total. Figure 1 shows the waveform of a small section of the collected UWAN, where the impulsive nature of the noise can be clearly observed.

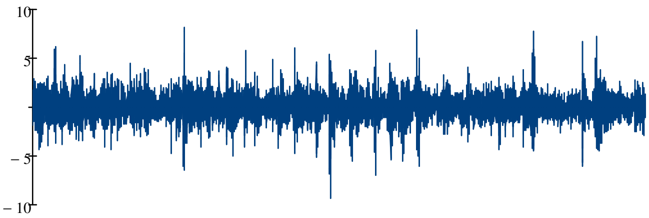

Fig. 1. Typical waveform of the underwater acoustic noise signal.

Figure 2 shows the normalized histogram derived from the frequency analysis on the amplitude of noise, compared to the Gaussian distribution. As expected, it appears that the probability density for large noise amplitudes is significantly higher and has a lower decay rate in relation to the normal distribution.

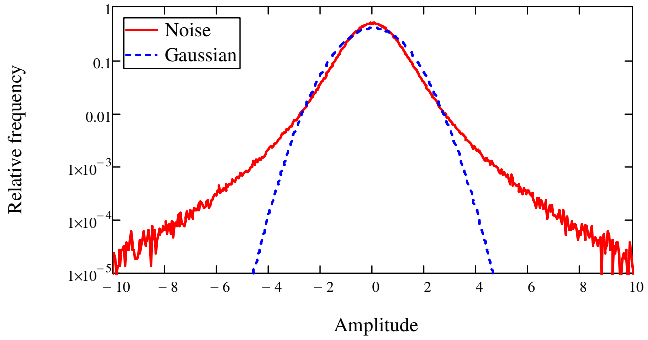

Fig. 2. Comparison between the amplitude distribution of the underwater acoustic noise and the Gaussian pdf.

The Student's t distribution is a well known statistical distribution associated with the standard Gaussian distribution, but presenting wider tails. Although the most common application of this distribution is the estimation of the mean of a population when the number of samples is small and the population standard deviation is unknown, the Student's t distribution has been used for the statistical modeling of wide tail processes [11] and Bayesian analysis of data [12].

The Student's t pdf is expressed by

$$f_T(t,d) = \frac{\Gamma\left((d+1)/2\right)}{\sqrt{\pi d}\ \Gamma\left(d/2\right)}\left(1+\frac{t^2}{d}\right)^{-(d+1)/2} \tag{3}$$

where $\Gamma(\cdot)$ is the gamma function and d is the parameter which controls the dispersion of the distribution. The lower the value of d, wider the tails of the pdf becomes and vice versa. For sufficiently large values of d, the Student pdf converges to the Gaussian distribution.

The pdf represented in (3) has zero mean and variance equal to $d/(d-2)$ for $d > 2$. However, to allow modeling a random variable X with variance σ^2, it is possible to make the following change of variables

$$t = \sqrt{\frac{d}{\sigma^2(d-2)}}\ x \tag{4}$$

and consequently, a new scaled pdf function can be written as

$$f_X(x,d) = \frac{\Gamma\left((d+1)/2\right)}{\sigma\sqrt{\pi(d-2)}\ \Gamma\left(d/2\right)}\left(1+\frac{x^2}{(d-2)\sigma^2}\right)^{-(d+1)/2} \tag{5}$$

The characterization problem consists in finding the value of the parameter d^* that provides the best fit of the pdf represented in (5) to the sample distribution of the noise, according to some criterion. For the purpose of digital communication, it is important that the pdf to be obtained accurately tracks the density decrease at high amplitudes, so that the nature of the impulsive noise is preserved and, therefore, the associated likelihood function represents the additive noise adequately. For this reason, the error function to be minimized was chosen as:

$$e(\mathbf{x},d) = \left\| \log f_X(\mathbf{x},d) - \log h(\mathbf{x}) \right\| \tag{6}$$

where $\mathbf{x}$ is a vector of M discrete amplitude levels of noise, for that the normalized histogram $h(x_k) = n_k / N$, $k = 1, 2, \ldots, M$ was built by analyzing relative frequency of the noise samples, n_k is the number of samples in the interval $[x_{k-1}, x_k)$, N is the total number of samples and $\|\cdot\|$ means Euclidian Norm.

The minimization process for the dispersion parameter, using the function defined in (6) on the set of noise samples, resulted to an optimum value $d^* \cong 4.5$. Thus, substituting this value in (5), the empirical estimation for the pdf of the noise is obtained as

$$\hat{f}(x) = f_X(x,d^*) = \frac{\kappa}{\sigma}\left(\frac{2x^2}{\sigma^2}+5\right)^{-2.75} \tag{7}$$

where $\kappa \cong 42.3423$.

Figure 3 shows the comparison of the relative frequency of the noise samples and the estimated distribution. In fact, as can be seen, there is an excellent agreement between the obtained pdf and the experimental data. It should be emphasized that the above result is an approximation for specific short-term test conditions and by additional measurements will be possible to determine the variability of the model.

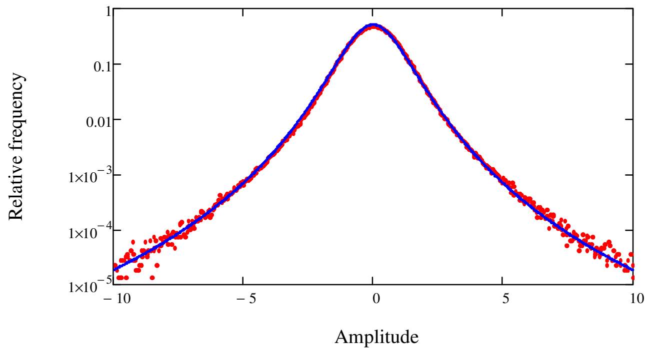

Fig. 3. Comparison between the noise sample amplitude distribution and the estimated probability density function ($d^* = 4.5$).

III. Error Performance Analysis

From the previously estimated pdf, it is possible to evaluate the likelihood functions for decision support in the reception of binary signals through the UWAN channel. For binary antipodal baseband signaling the amplitudes of the transmitted symbols s_m are given by $+A$ or $-A$, and then these functions can be defined as

$$L_0(x) = \hat{f}(x \mid s_k = +A) = \hat{f}(x - A) \qquad (8)$$

and

$$L_1(x) = \hat{f}(x \mid s_k = -A) = \hat{f}(x + A) \qquad (9)$$

Thus, for a binary equiprobable source, the probability of symbol error in the detection of an antipodal signal corrupted by additive noise can be directly calculated by integration of any of the likelihood functions, i.e.

$$P_e = \int_{-\infty}^{0} L_0(x)\,dx = \int_{0}^{\infty} L_1(x)\,dx \qquad (10)$$

The energy of each bit is given by $E_b = A^2 T_b$, where T_b is the bit interval. In addition, the average power spectral density of the noise can be expressed by $N_o = \sigma^2/B$, where $B = 1/2T_b$ is the bandwidth occupied by the baseband signal. Without generality loss, assuming that the amplitude of the pulses is unitary, i.e. $A = 1$, then the noise variance σ^2 can be related to the signal-to-noise ratio (SNR) per bit E_b/N_o, according to the following relationship:

$$\sigma^2 = \frac{1}{2E_b/N_o} \qquad (11)$$

Finally, changing the variables in (7) and applying the result in (10), it follows that the symbol error probability of the binary UWAN channel can be estimated as

$$P_e = \kappa\sqrt{2E_b/N_o}\int_0^{\infty}\left[(4E_b/N_o)(x+1)^2 + 5\right]^{-2,75} dx \qquad (12)$$

Figure 4 shows graphs of the symbol error probability P_e as a function of E_b/N_o for binary bipolar signaling. The continuous trace is the estimation for the error probability for the UWAN channel obtained directly from (12), while dashed trace shows the theoretical error performance of the additive white Gaussian noise (AWGN) channel. Simulation results, shown as small circles, are also presented for the UWAN.

Simulation was performed by directly inspecting N noise samples x_k and comparing those to the signal amplitude A_s, properly scaled to the desired SNR. For each event occurrence such as $|x_k| > A_s$, one error was added up. After all samples were tested, the error rate was computed by the ratio $n_E/2N$, where n_E is the error count.

As can be viewed in Figure 4, the UWAN channel appears to be slightly less prone to errors compared to the AWGN channel, up to a SNR level of approximately 3 dB. After this point, the error probability for the UWAN channel becomes significantly larger in relation to the AWGN channel, and the performance gap enlarges for increasing SNR levels.

Although behavior in the UWAN channel at low SNR seems to be counterintuitive, it can be explained due to the shape of the probability density function. The UWAN pdf has wider tails and, consequently, slimmer and taller central body when compared with the normal curve. Thus, for low SNR conditions, i.e., high noise environments, the error probability in (10) corresponds to the area of one entire tail and a considerable portion of the central body half of the pdf. The tail area of the UWAN pdf is larger than the normal but, in compensation, there is section in the central body where the Gaussian pdf surpasses the UWAN pdf (see Figure 2). Since the probability density is higher in the central body, the difference between the areas in this region more than compensates for the area deviation in the tails.

For high SNR environments, as expected, the frequent incidence of impulsive events in the UWAN channel produces severe degradation in the system performance when compared to the AWGN channel. Figure 4 shows that the relative performance degradation exceeds 10 dB at error rates below 10^{-5}. Given the poor performance of the UWAN channel under these conditions, it is recognized that employing some error control coding technique would be essential to mitigate the unwanted effect of the noise and, in this way, contributing to get reliable underwater acoustic communications.

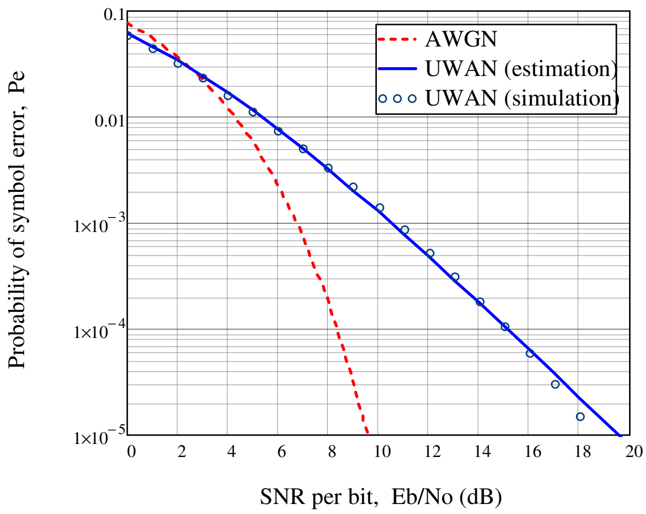

Fig. 4. Symbol error performance for AWGN and UWAN channels.

IV. CONCLUSION

Underwater acoustic noise (UWAN) in shallow coastal waters with presence of snapping-shrimp shows an accentuated impulsive behavior and, consequently, does not follow the Gaussian distribution. The literature reports that UWAN follows a symmetrical alpha-stable (SαS) distribution, but cannot described in closed form. The analysis of field data measurements has shown that the noise amplitude distribution presents good fitting with the Student's *t* distribution. Thus, in this article it has been proposed an empirical model for the distribution of the UWAN based on this distribution and the probability density function was derived by adjusting the dispersion parameter adequately.

The bit error probability could be estimated for the uncoded UWAN channel and it was observed that UWAN channel is slightly less prone to errors, compared with the AWGN channel, up to a SNR level of approximately 3 dB. After this point, the error probability for the UWAN channel surpasses the AWGN channel and the difference enlarges progressively for increasing SNR. In fact, for high SNR environments the effect of impulsive events produces severe performance degradation when compared to the AWGN channel, exceeding 10 dB for error rates below 10^{-5}.

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