# Impairment Aware Routing Algorithm for All-Optical Networks Based on Power Series and Particle Swarm Optimization

Daniel A. R. Chaves, Douglas O. Aguiar, Carmelo J. A. Bastos-Filho, Joaquim F. Martins-Filho

*Abstract*— In all-optical networks signals are transmitted through physical layer with no regeneration. Therefore, physical impairments along lightpath can severely reduce network performance. For this reason, many efforts have been made to develop impairment aware routing and wavelength assignment algorithms (IRWA) in order to mitigate the impairments effects, improving the network performance. In this paper we propose and analyze the performance of an adaptive impairment aware routing algorithm based on a set of chosen input network parameters. The cost function of this routing algorithm is based on a power series expansion. The routing algorithm, called Power Series Routing (PSR), is trained by an optimization technique called Particle Swarm Optimization. We show that this IRWA algorithm can learn during the training stage and adapt itself to the network conditions.

## I. INTRODUCTION

All-optical networks have been considered as the most reliable and economic solution to achieve high transmission capacities with proper quality of service (QoS). Nevertheless, there are two main challenges to manage these networks providing QoS: define an appropriate routing and wavelength assignment algorithm (RWA) and obtain acceptable optical signal-to-noise ratio (OSNR) for every established lightpath.

The RWA problem can be divided in the routing process and the wavelength assignment process. A classical approach to solve routing problem is to represent the network topology by a graph, then use some metrics to evaluate the cost of each branch of the graph, and finally, use an algorithm that finds the minimum cost path between two given nodes [1], [2]. The wavelength assignment algorithm has to decide which available channel should be used to establish the call [1], [3]. Some routing algorithms use heuristics based on a predefined metric, such as the shortest path (SP), minor delay, load balance [4] and lower noise figure in lightpath [5].

Some RWA algorithms designed for opaque networks just consider the wavelength availability. The main aim of these approaches is to achieve an improved load distribution or to minimize the use of the physical layer resources [6], [7].

In transparent all-optical networks there is no signal regeneration at intermediate nodes along the lightpaths. Therefore, the signals accumulate noise due to transmission impairments. For this reason, the routing algorithm must

Carmelo J. A. Bastos-Filho, is with the Department of Computing and Systems, University of Pernambuco, CEP: 50720-001, email:cjabf@dsc.upe.br

be aware of these physical penalties to fetch routes that minimize OSNR degradation due to optical noise. Recently, many efforts have been made to develop RWA algorithms that consider physical impairments (IRWA) [5], [8], [9], [10]. The main goal of these approaches is to minimize the blocking probability by finding routes considering physical layer status. Although routing schemes based on optical impairments outperform the most common approaches, the use of these algorithms implies in higher computational complexity. Some algorithms have been proposed to achieve a good performance with a lower computational time [9].

In this paper we propose a method to build the link cost function based on a set of relevant network parameters. This is an important tool for routing algorithm design, since the parameters selection is a relatively easy task for a network specialist. Nonetheless, combining these parameters to obtain optimal network performance is a complex task. For this reason we propose an adaptive cost function for impairment aware routing, which we call PSR (Power Series Routing). We use PSR to provide the link cost for a lowest cost routing algorithm (*e.g.* Dijkstra's algorithm). The PSR training is performed by the particle swarm optimization technique.

### **II. POWER SERIES AND ALGORITHM DESCRIPTION**

In this section we present a method to determine a link cost function for network routing. The proposed approach consists of three steps: first, a number of input variables for the cost function is chosen by a network specialist. Then, the cost function is written in terms of a series of functions. And finally, an optimization algorithm is used to find the series coefficients that minimizes the network blocking probability.

In this paper, we focus our analysis in the series that make use of a set of orthogonal polynomials

$$f(x) = \sum_{n=0}^{\infty} a_n x^n.$$
 (1)

Assuming the continuity of the function and its derivatives, the expansion in Eq. (1) can also be done for a multivariable functions

$$f(x_0, x_1, \dots, x_k) = \sum_{n_0=0}^{\infty} \sum_{n_1=0}^{\infty} \dots \sum_{n_k=0}^{\infty} b_{n_0, n_1, \dots, n_k} \prod_{j=0}^k x_j^{n_j}.$$
(2)

It is well known that one can find  $b_{n_0,n_1,...,n_k}$  by means of derivatives (multivariable Taylor's series) [11]. However,

Daniel A. R. Chaves, Douglas O. Aguiar and Joaquim F. Martins-Filho are with the Department of Electronics and Systems, Federal University of Pernambuco, Recife-PE, CEP: 50740-530, Brazil, email:jfmf@ufpe.br

this approach works only for a function with derivatives. Nevertheless, the lack of an analytical form to find  $b_{n_0,n_1,...,n_k}$  is not an obstacle if one is able to find these coefficients by a non analytical procedure. Considering this, we used the proposed approach to build an adaptive cost function for impairment aware routing, which we call power series routing (PSR).

The first step is to choose the input variables for the cost function. In optical networks the information about link length, link availability and number of hops have high correlation with noise accumulated along the lightpath.

Furthermore, excessive noise can damage the signal transmission quality as the link length increases, and higher gains must be provided by the optical amplifiers to compensate for the losses. Therefore, more ASE noise is added by optical amplifiers in the lightpath. Link usage has impact in amplifier saturation and ASE noise generation, since the amplifier gain and noise figure depends on the total input signal power [12], [13]. Besides, as the number of hops increases, more crosstalk noise is added in intermediate nodes. Therefore, these elementary network parameters could be used to build a simple routing scheme, instead of using the noise information, obtaining similar network performance as for schemes that use optical noise information to compose the cost function [9]. For these reasons, we choose as input variables for the cost function two simple network parameters: normalized link availability and normalized route length.

The second step of the algorithm is to describe the cost function in terms of a series as in Eq. (2), according to the number of network parameters chosen. Therefore, the link cost between nodes i and j can be expressed in a two variables form of Eq. (2) by:

$$f(x_{i,j}, y_{i,j}) = \sum_{n_0=0}^{\infty} \sum_{n_1=0}^{\infty} b_{n_0,n_1} x_{i,j}^{n_0} y_{i,j}^{n_1},$$
 (3)

where  $x_{i,j}$ , and  $y_{i,j}$  are, respectively, the link availability and normalized link length between the nodes *i* and *j*.  $x_{i,j}$  is defined as:

$$x_{i,j} = \frac{\lambda_{i,j}^a}{\lambda_{i,j}^T},\tag{4}$$

where  $\lambda_{i,j}^a$  and  $\lambda_{i,j}^T$  are, respectively, the number of unused and total number of wavelengths in the link between nodes *i* and *j*. The normalized link length  $y_{i,j}$  is defined as:

$$y_{i,j} = \frac{d_{i,j}}{d_{max}},\tag{5}$$

where  $d_{i,j}$  is link length between nodes *i* and *j* and  $d_{max}$  is the maximum link length in the network. Since it is not computationally possible to have an infinite number of terms in Eq.(3), one must truncate the series in order to obtain an approximation with *N* terms:

$$f(x_{i,j}, y_{i,j}) = \sum_{n_0=0}^{N} \sum_{n_1=0}^{N} b_{n_0,n_1} x_{i,j}^{n_0} y_{i,j}^{n_1}.$$
 (6)

One can note from Eq. (6) that this function has a constant term, which can represent the hop cost.

The third step consists of using PSO to find the series coefficients that optimizes a network performance parameter. We used PSO because it achieves a better performance in high dimensionality problems than other optimization techniques (e.g. Genetic Algorithms) [14]. The optimization algorithm can be either used to maximize the network throughput or minimize network blocking probability. In this paper we find the  $b_{n_0,n_1,\ldots,n_k}$  coefficients that minimize blocking probability as will be described in the next section.

It must be highlighted that one can include an arbitrary number of input parameters in order to build the cost function, including direct information about the physical impairments.

#### **III. PARTICLE SWARM OPTIMIZATION**

In order find the  $b_{n_0,n_1}$  coefficients, as discussed in previous section, we used an intelligent optimization technique called Particle Swarm Optimization (PSO) [14]. PSO was proposed by Kennedy and Eberhart in 1995 and it is inspired in bird flocking [15]. In PSO, each particle i is a possible solution of the problem and it has some properties such as its current velocity  $\vec{v}_i$ , its current position  $\vec{x}_i$  and its best position in the past  $\overrightarrow{p}_i$ . For the Swarm communication topology we used the local topology in a ring model, also known as Lbest, in which each particle has information about only two neighborhoods of the swarm [16]. It is recommend in [16] to use local best model, instead of global best model used in first PSO definition, since the global best approach has a higher probability to be trapped in local minima. Denoting by  $v_{i,d}$  the  $d^{th}$  component of  $\overrightarrow{v}_i$  vector and using the same notation for the other vectors we can state the pseudo code algorithm that we used to implement the PSO optimizer as shown in table I. g() returns the fitness of one particle and  $min_i(\overrightarrow{p}_{neighbors})$  returns the position  $\overrightarrow{p}_{neighbor}$  of the fitter particle among the two neighbors of the particle *i*.

 TABLE I

 PSO ALGORITHM FOR MINIMIZATION USED IN OUR METHOD.

initialize randon population				
Do				
For $i = 1$ to Population Size				
if $g(\overrightarrow{x}_i) < g(\overrightarrow{p}_i)$ then $\overrightarrow{p}_i = \overrightarrow{x}_i$				
$\overrightarrow{p}_{g} = min_{i}(\overrightarrow{p}_{neighbors})$				
For $d=1$ to Dimension				
$v_{i,d} = \chi(v_{i,d} + c_1\epsilon_1(p_{i,d} - x_{i,d}) + c_2\epsilon_2(p_{g,d} - x_{i,d}))$				
$x_{i,d} = (v_{i,d} + x_{i,d})$				
Next d				
Next i				
Until termination criterion is met				

The particle velocities are updated using the constriction factor approach [17]. In this approach the particle velocity is updated using the following equation:

$$v_{i,d} = \chi [v_{i,d} + c_1 \epsilon_1 (p_{i,d} - x_{i,d}) + c_2 \epsilon_2 (p_{g,d} - x_{i,d})], \quad (7)$$

where the  $\chi$  is evaluated by:

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|}, \quad \varphi = c_1 + c_2.$$
 (8)

In [17] the authors found that if  $\varphi > 4$ , the algorithm runs properly. For this reason, we have chosen the same approach for our PSO implementation.

## **IV. SIMULATIONS SETUP**

Our simulation software uses the following steps for solve RWA problem. Upon a call request it selects an available wavelength from a list, using the first fit algorithm. The route is defined by a routing algorithm that uses one of the following weight functions: Shortest Path algorithm (SP), with physical length as the cost function, Least Resistance Weight (LRW) described in [7], an algorithm that uses the total OSNR of the lightpath as the cost function (OSNR-R) proposed in [5], and our proposal. The OSNR of each lightpath is evaluated using the same model used in [9]. This model considers the following impairments: ASE noise, amplifier gain saturation effect, saturation of ASE noise in EDFAs and homodyne crosstalk in optical switches. If the OSNR of the lightpath is above the pre-determined level  $(OSNR_{QoS})$ , then it is established. Otherwise the call is blocked. Our algorithm also blocks a call if there is no wavelength available. The blocked calls are lost. The blocking probability is obtained from the ratio of the number of blocked calls and the number of call requests. For each network simulation a set of  $10^7$  calls are generated by choosing randomly (uniform distribution) the source-destination pair. The call request is characterized as a Poisson process. We assume circuit switched bidirectional connections in two different fibers and no wavelength conversion capabilities. The default optical parameters used in our simulations are: amplifier output saturation power  $P_{Sat} = 19 \, \text{dBm}$ , transmitter output power  $P_{in} = 0 \, dBm$ , input optical signal-to-noise ratio  $OSNR_{in} = 40 \, dB$ , optical signal-to-noise ratio for QoS criterion  $OSNR_{QoS} = 23 \, dB$ , optical filter bandwidth  $B_o = 100 \,\mathrm{GHz}$ , channel spacing  $\Delta f = 100 \,\mathrm{GHz}$ , the lower wavelength of the grid  $\lambda_i = 1550.12 \,\mathrm{nm}$ , zero dispersion wavelength  $\lambda_0 = 1510 \,\mathrm{nm}$ , fiber loss coefficient  $\alpha =$  $0.2 \,\mathrm{dB/km}$ , multiplexer loss  $L_{Mux} = 3 \,\mathrm{dB}$ , demultiplexer loss  $L_{Demux} = 3 \,\mathrm{dB}$ , switch loss  $L_{Switch} = 3 \,\mathrm{dB}$ , amplifier noise factor that corresponds to  $NF = 5 \, dB \, F_0 = 3.162$ , noise factor model parameter  $A_1 = 100$  noise factor model parameter  $A_2 = 4 \text{ W}$  [13], switch isolation factor  $\epsilon = -41 \text{ dB}$ .

Amplifier gains are set to compensate link losses. We used the network topology shown in Figure 1, which is the optical network of Finland, a well known network that is often used as a benchmark.

We used the following PSO parameters in our simulations: 50 particles, 500 interactions, velocity update parameters  $c_1 = c_2 = 2.05$ ,  $\epsilon_1$  and  $\epsilon_2$  random numbers with uniform distribution in the interval [0,1], Constriction factor  $\chi =$ 0.72984, PSO search space interval [-1,+1], maximum and minimum velocity equals to +1 and -1 respectively.

The network parameters, physical layer parameters and devices characteristics were set for two different situations. In the first scenario  $(S_1)$  the blocked calls are mainly due to OSNR degradation along the lightpath, i.e., the blocked calls are due to crosstalk and amplifiers impairments and blocking



Fig. 1. Network topology used in our simulations.

due to lack of available wavelength is negligible. In the second scenario ( $S_2$ ) the blocked calls are due to both, the lack of wavelength and OSNR degradation. In the second situation, the number of blocked calls by lack of wavelengths is quite similar to number of blocked calls due to OSNR degradation. The main difference between  $S_1$  and  $S_2$  scenarios is the number of total wavelengths available in each link. In  $S_1$ scenario we set the number of available wavelengths to 36 in order to provide very low call blockings due to lack of wavelengths. In the  $S_2$  situation we decreased the number of available wavelengths to 21.

### V. RESULTS

The first step before the assignment of the Eq. (6) as a cost function for routing is to find the optimum values for the  $b_{n_0,n_1}$ parameters. We have performed a search in  $b_{n_0,n_1}$  space using PSO as described in section III. The search was done using network load of 80 Erlangs. We propose to optimize for higher network loads since it is the worst case. The goal of this search is to minimize the network blocking probability (*BP*). In order to evaluate the fitness for a given particle, each network was simulated for a set of  $10^5$  calls. The returned blocking probability *BP* is assigned as the fitness value for this particle. We call these network simulations as offline training process since it should be done prior to network operation.

Figure 2 shows the convergence of PSO algorithm. The lowest blocking probability found in each PSO iteration is shown. We performed the same optimization 3 times for N = 4. This value was chosen as a commitment between computational time spent to solve the problem and resulting network blocking probability found by PSO. As N increases the computational time necessary for PSO convergence also increases. On the other hand, if N is too small, it compromises the cost function representation. We can see that for early iterations, the function found by PSR has higher blocking probability than OSNR-R algorithm. As the number of generations increases, PSO converges and PSR reaches lower blocking probabilities than the OSNR-R scheme.

As it was discussed in section II we chose two variables as input parameters for PSR cost function: link availability  $x_{i,j}$ 



Fig. 2. PSO convergence analysis for scenario  $S_1$ .



Fig. 3. PSO convergence analysis for scenario  $S_2$ .

and normalized link length  $y_{i,j}$ . Using the best parameters  $b_{n_0,n_1}$  found by PSO, we can plot the link cost as a function of  $x_{i,j}$  and  $y_{i,j}$  in terms of level curves, for both  $S_1$  and  $S_2$  as shown in Figure 4 and Figure 5, respectively. One can note that in both cases the link cost is high for long distances and low link availabilities (white regions in graph) and the cost is low for short distances and high link availabilities (black regions in graph), as expected.

The cost function is found for the entire search space i.e. for all possible choices of link length and link availability. However, during the routing process, some values of link length and link availability are more often checked than others by the routing algorithm. In order to find which are these more often checked values, we built an histogram with the number of times the routing algorithm consults the cost of a given link (in term of length and availability). These histograms are shown in Figure 6 and Figure 7 for  $S_1$  and  $S_2$ scenarios, respectively. Each line in the staked graph represents the normalized availability distribution for a given link. The numbers in brackets indicate the source and destination nodes for the link according to the node numbers presented in Figure 1. We can see from Figure 6 and Figure 7 that the most checked availability values are between 0.6 and 0.9 for  $S_1$ 



Fig. 4. Cost function  $f(x_{i,j}, y_{i,j})$  of Eq. 6 found by PSO as a function of normalized link length and link availability for  $S_1$  scenario.



Fig. 5. Cost function  $f(x_{i,j}, y_{i,j})$  of Eq. 6 found by PSO as a function of normalized link length and link availability for  $S_2$  scenario.

scenario and between 0.3 and 0.8 for  $S_2$  scenario. Therefore, the analysis of the generated PSR function must be realized on these ranges. Comparing the regions of more often checked values for availability in both scenarios, one can note that the dependence of the cost function with availability is more pronounced in  $S_2$  than in  $S_1$  as one can see from Figures 4 and 5. As most of the links have normalized length between 0.2 and 0.5, one can note from Figures 4 and 5 that the link availability has a greater influence is  $S_2$  than in  $S_1$ . The same behaviour can be noted for the dependence with length i.e. as the link length increases the cost function increases faster for  $S_2$  than for  $S_1$  as one can see from Figures 4 and 5. It means that PSR is able to find different cost functions for different network scenarios and for this reason it can optimize the network performance. Thus, the PSR algorithm has the ability to learn with the changes in the network characteristics, in this case, with the change in the number of available wavelengths.

Since we have found a link cost function (Figures 4 and 5) we can assign it as the network link cost and evaluate de network performance of the proposed routing scheme. We compared the PSR against three other cost function



Fig. 6. Histogram for the number of consults made by the PSR algorithm as a function of availability for all the links of network for scenario  $S_1$ .

reported in literature: SP, LRW, OSNR-R. These algorithm were chosen for comparison due to following reasons: SP is simple and most largely used cost function for routing comparison purposes; LRW is an algorithm capable of finding less congested routes and, for this reason, leads to an improved network load distribution; and OSNR-R is a routing scheme that uses the physical impairments information for the routing procedure. Figure 8 shows the blocking probability as a function of total network load for these four different algorithms for  $S_1$ . One can note that our proposed PSR far outperforms the results obtained using either SP or LRW algorithms. Furthermore, when compared with the IRWA approach (OSNR-R), PSR has better network performance in terms of blocking probability. It means that PSR is capable to reach the high performance to the IRWA approach not evaluating directly the impairments in real time. The impairment information was considered in the offline (training) stage only. Performing the same analysis for  $S_2$ , PSR also far outperforms either SP or LRW algorithms and achieved a quite similar performance to the OSNR-R algorithm.

PSR and OSNR-R routing algorithms have quite similar performance in terms of blocking probability. However, we must also compare the time spent by these approaches to solve



Fig. 7. Histogram for the number of consults made by the PSR algorithm as a function of availability for all the links of network for scenario  $S_2$ .



Fig. 8. Network blocking probability as a function of network load for the LRW, SP, OSNR-R and PSR algorithms in  $S_1$  scenario.

the RWA problem for each call. We used an Intel® Core<sup>TM</sup>2 @2.13 GHz with 3 GB of RAM computer to perform this comparison. The results for the average time spent to solve the RWA per call, performing 50000 calls, are shown in table II. The PSR algorithm solves the RWA problem 9.4



Fig. 9. Network blocking probability as a function of network load for the LRW, SP, OSNR-R and PSR algorithms in  $S_2$  scenario.

times faster than OSNR-R. This occurs due to the offline training based on the physical impairments evaluation. In the OSNR-R algorithm, as well as in other physical impairment based algorithm, these calculations occur during the online (call by call) solution of the RWA problem. Table II also shows that PSR is up to 1.25 times slower than LRW. This small difference should be due to the simple mathematical formula of the LRW function, which involves just a single division operation. We did not consider the SP algorithm for computation time analysis since it has a fixed routing table.

### TABLE II

Average Time Spent to Solve RWA per Call in  $S_1$ .

Algorithm	Time	Normalized time
LRW	0.078ms	0.0975
PSR	0.098ms	0.11
OSNR-R	0.886ms	1.00

#### VI. CONCLUSIONS

In this paper we propose a systematic form to build the link cost function based on a set of relevant network parameters. We apply the proposed scheme to build an adaptive cost function (PSR) for impairment aware routing in all-optical networks. The proposed PSR is based on simple network parameters such as link availability, link length and hop count. Since PSR indirectly takes into account the network physical impairments we demonstrated that it outperforms or, in worst case, provides similar performance to other algorithm that use OSNR degradation as a weight function. However, the computation time for our weight function was 9.4 times faster than for the OSNR based one, for the network simulation conditions used.

It must be highlighted that the proposed weight function does not rely on online physical impairments evaluation to infer about signal noise in the network. Therefore, it is not mandatory to perform complex evaluations to obtain values for optical noise based weight functions. However, PSR requires an offline simulation to store the awareness of physical impairments in the series parameters. This characteristic of a priori knowledge brings to our weight function a drastic reduction in the computation time for real time routing decision as compared to noise based approaches.

#### VII. ACKNOWLEDGMENT

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