Feedback-Assisted Adaptive Network Coded Cooperation for Wireless Networks

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***Abstract*—In this work, we propose a feedback-assisted version of the Adaptive Network Coded Cooperation (ANCC) scheme, recently proposed by Bao and Li for a network consisting of M users having independent information to send to a common base station. The aim is to increase the system rate without compromising its error performance, based on a small amount of feedback sent by the base station (only one bit for each received block is sufficient). Two different approaches are proposed. The expected rate is analyzed, and simulation results agree with the analytical ones. The system's error performance is also evaluated through simulations, and we show that the performance is not degraded by the increase in the rate.**

***Index Terms*—Cooperative communication, LDPC, network coding.**

I. Introduction

As a way to exploit the random-fading nature of the wireless channel, cooperative communication has attracted the attention of many researchers in the last decade [1]–[3]. In cooperative networks, besides broadcasting their own information, users help each other by relaying their partner's information.

Network coding [4], [5], a technique originally proposed to attain maximal information flow in lossless networks, has recently been applied to wireless networks to improve their reliability. This method allows the users to perform simple linear coding operations before transmitting packets to subsequent nodes. An important scenario is when several users have independent information packets to send to a common destination. Since a user node can decode the packets from its neighbors, a linear combination of packets transmitted to destination may be interpreted as a parity-check packet of a distributed block code. So, a key observation is that the system transfer matrix can be viewed as a generator matrix of a linear block code. Recent works have expanded concepts from the classical coding theory to network coding [6]–[10].

In [9], Bao and Li have proposed the so-called *Adaptive Network Coded Cooperation* (ANCC) scheme, in which the (instantaneous) network topology is viewed as a bipartite graph. When the network is composed of a large number os users, by restricting the number of packets in the linear combination performed by each user when acting as relay, a sparse graph (sparse matrix) of a LDPC-like code is created. In the ANCC scheme, each user sends an information packet of its own and then, in the cooperative phase, each user sends one parity-check packet regardless of its forward channel quality. Therefore, the code rate is fixed.

In this paper, we elaborate on the ANCC scheme by considering that the BS is able to feedback only one bit after each received information packet. This feedback bit serves to indicate whether a user will or will not participate in the cooperative phase, based on its instantaneous channel conditions. The benefits are two-fold. First, since only users with sufficiently good forward channels participate as relays in the cooperative phase, the error performance may be improved under certain conditions. Second, as fewer parity-check packets are sent, an increase in the transmission rate is obtained. The selection of users is made by comparing the users channel gains to a threshold.

With the proposed scheme, called *Feedback-Assisted Adaptive Network Coded Cooperation* (FA-ANCC), a considerable improvement in the system overall rate can be achieved (over ANCC) without compromising the system performance.

We present two different approaches for the FA-ANCC. In the first approach, we consider a fixed threshold. In the second approach, the threshold is made to vary according to the signal-to-noise ratio (SNR). We show through computer simulations that FA-ANCC significantly improves the average rate without compromising the bit error rate (BER) performance. The improvement is larger with the second approach.

The remainder of this paper is organized as follows. The next section presents the system model and the ANCC scheme [9]. The motivation for FA-ANCC is presented in Section III. Section IV presents the two approaches for the proposed FA-ANCC scheme. Simulations results are presented in Section V. Finally, Section VI presents our conclusions and final comments.

II. Preliminaries

A. System Model

We consider a system in which multiple users ($M \geq 2$) have different information to send to a common base station (BS). One time slot (TS) is defined as the time period in which all the M users realize a single transmission (through orthogonal channels, either in time, frequency or code), that is, one TS corresponds to M transmissions. One cooperation round block T is defined as the sum of the broadcast phase time (one TS) and the cooperation phase time (up to one TS, as will be explained later).

The received baseband codeword at user i at time slot t is given by

$$y_{j,i,t} = h_{j,i,t}x_{j,i,t} + n_{j,i,t}, \quad (1)$$

where $j \in \{1, \cdots, M\}$ represents the transmit user index and $i \in \{0, 1, \cdots, M\}$ the receive user index (0 corresponds to the BS). The index t denotes the time slot. $x_{j,i,t}$ and $y_{j,i,t}$ are the transmitted and the received codewords, respectively. $n_{j,i,t}$ is the zero-mean additive white Gaussian noise with variance $N_0/2$ per dimension. The dependence on j in $n_{i,j,t}$ reflects the M orthogonal transmissions within time slot t. The channel gain due to multipath fading is denoted by $h_{j,i,t}$, and it is assumed to have independent identically distributed (i.i.d.) (across space and time) Rayleigh distribution with unit variance.

Assuming the $x_{j,i,t}$'s to be i.i.d. Gaussian random variables and considering all the channels with the same average SNR γ, the mutual information $I_{j,i,t}$ between $x_{j,i,t}$ and $y_{j,i,t}$ is

$$I_{j,i,t} = \log(1 + \gamma|h_{j,i,t}|^2). \quad (2)$$

Assuming powerful enough channel codes, $x_{j,i,t}$ can be correctly decoded if $I_{j,i,t} > r_{j,i,t}$, where $r_{j,i,t}$ is the information rate from user j to user i in the time slot t. Considering that all the users have the same rate, the index of r can be dropped. Thus, $x_{j,i,t}$ cannot be correctly decoded if

$$\gamma|h_{j,i,t}|^2 < \lambda, \quad (3)$$

where $\lambda = 2^r - 1$. The probability that such an event happens is called the *outage probability*. For Rayleigh fading, the outage probability is calculated as [2], [11]

$$P_e = \Pr\left\{\gamma|h_{j,i,t}|^2 < \lambda\right\} = 1 - e^{-\frac{\lambda}{\gamma}} \approx \frac{\lambda}{\gamma}, \quad (4)$$

where the approximation holds for a high SNR region.

In this work, block fading means that fading coefficients are i.i.d. random variables for different round blocks but constant during the same cooperation round block.

B. Adaptive Network Coded Cooperation (ANCC)

In the ANCC scheme [9], besides broadcasting their own information, users help each other by transmitting to the BS linear combinations (over the binary field) of their own information packets and the information packets from their partners received during the broadcast phase. The ANCC protocol is illustrated in Fig. 1. Due to the random nature of the wireless channel, the inter-user channels may be subject to failures, and consequently not all users overhear all the others all the time. In a specific cooperation phase, let deliver-set $\mathcal{D}(i)$ denote the set of users that have overheard User i's broadcast. Let also the retrieval-set $\mathcal{R}(i)$ denote the set of terminals whose User i has successfully recovered.

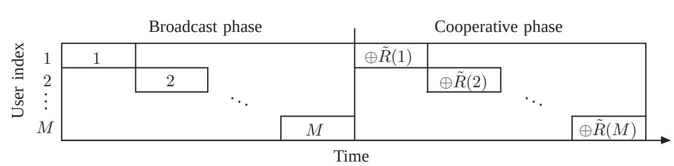

Fig. 1. Adaptive network coded cooperation (ANCC) protocol. $\oplus\tilde{R}(i)$ denotes the check sum of a subset of symbols from $\mathcal{R}(i)$, where $\mathcal{R}(i)$ denotes the retrieval-set. (Adapted from [9].)

The idea presented in [9] is that, instead of transmitting linear combinations of all the information available in the cooperative phase, each user randomly selects a small number of users (say D) from the set $\mathcal{R}(i)$ and forms a linear combination of their information. By doing so, the main contribution of ANCC is to match network-on-graph, *i.e.* instantaneous network topologies described in graphs, with code-on-graph, such as LDPC codes [12]. This procedure is illustrated in Fig. 2 for a 5-user network, where the box symbol (⊞) stands for the check-nodes and the black/white circles (● and ○) represent the variable-nodes. The bipartite code

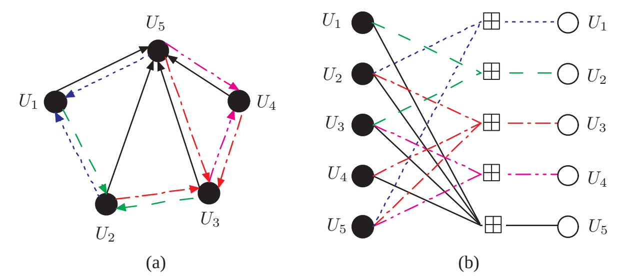

Fig. 2. Generating a code-on-graph from a network-on-graph. (a) A network graph used to describe the network topology (for convenience, the BS is omitted)(b) A bipartite code graph derived from the network graph. Adapted from [9].

graph is generated from the network graph by connecting each variable-node (i.e. black circle) to a check-node (box) if there is a connection between the corresponding users in the network graph. We can see in Fig. 2(b) that a (10,5) systematic LDPC-like code is obtained. Then, the code may be decoded by a properly algorithm, such as the message passing algorithm [12].

The ANCC scheme considers a fixed and restrictive code rate equal to $\frac{M}{2M} = 1/2$, that is, all the users must participate in the cooperative phase.

III. Motivation for this Work

In the ANCC scheme, as mentioned before, all the users must participate in the cooperative phase. Under the assumption of block fading channels as defined in Section II-A, a small quantity of feedback from the BS could help the avoidance of two detrimental effects on the system rate inherent to the ANCC proposal:

1) First, in the low SNR region, a large number of users may have their forward channels in poor conditions when transmitting their parity-check packets, which will probably not be correctly decoded, wasting resources such as rate and transmit power. Moreover, these unreliable packets may have a harmful influence in the decoding process, since they are a source of uncertainty.

2) Second, in the high SNR region, the BS may be able to recover all the information packets from a small number of parity-check packets. In this situation, the transmission of unnecessary parity-check packets yields a reduction in the system rate.

In this work, we elaborate on the ANCC scheme by allowing the BS to select a subset of users that will participate in the cooperative phase, based on their channels conditions. The BS will then inform each user whether or not it has been selected. Only one bit per user of feedback is sufficient. With $D < M$ parity-check packets transmitted in the cooperative phase, a considerable increase in the system rate can be achieved. In spite of the reduced redundancy, the BER performance can be made not to deteriorate with an appropriate choice of D, because of the high quality of the selected channels.

IV. Feedback-Assisted Adaptive Network Coded Cooperation (FA-ANCC)

From (3), we can see that if the condition $|h_{j,0,t}|^2 \geq \lambda/\gamma$ is not satisfied, the link between the jth user and the BS will be in outage, and the packets transmitted through this link will not be correctly decoded. In what follows we propose two approaches to increase the system rate based on the BS feedback.

A. Approach 1

In this first approach, after the broadcast phase, the BS just sends back a packet (composed of M bits) informing the users if their individual forward channels are either above or below the threshold λ/γ. Upon receiving the feedback, only the qualified users will participate in the cooperation phase. This proposal is illustrated in Fig. 3.

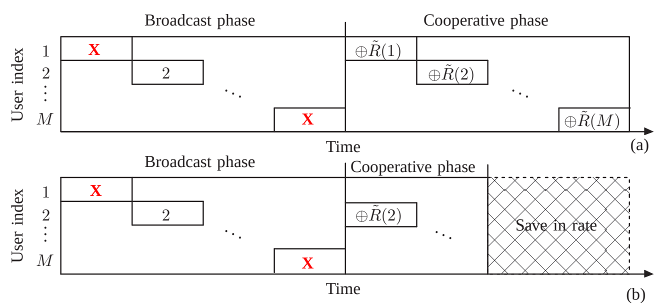

Fig. 3. (a) ANCC protocol, according to Fig. 1 (b) Proposed scheme's protocol. The **X**´s represent the users whose forward channel gain is below a certain threshold λ.

At first this sounds conflicting: How is it possible to increase the code rate and keep (or even improve) its BER performance? The answer for this question resides on how the parity-checks are selected. By allowing only the users whose channels are in good conditions to participate in the cooperative phase, we are disregarding the unreliable parity-check packets that would be received through poor channels. These unreliable packets could disturb the convergence of the iterative decoder.

Let T be the number of transmitted cooperation blocks. The number of transmitted information packets per block is always M. The instantaneous rate is given by

$$R(P_\tau) = \frac{M}{M + P_\tau}, \tag{5}$$

where P_τ denotes the number of transmitted parity-check packets in block τ, for $\tau = 1, \ldots, T$. According to the description of the proposed system (see Fig. 3(b)), P_τ varies with τ according to the channel conditions. The probability distribution function of P_τ is given by

$$\Pr\{P_\tau = p\} = \binom{M}{p} P_e^{M-p}(1 - P_e)^p, \tag{6}$$

which corresponds to a binomial distribution. The average number of transmitted parity-check packets in a given block τ is

$$E[P_\tau] = M(1 - P_e) \tag{7a}$$

$$= Me^{-\frac{\lambda}{\gamma}}. \tag{7b}$$

The length of block τ is $L_\tau = M + P_\tau$ packets. The overall rate for the T transmitted blocks is given by

$$R_T = \frac{TM}{\sum_{\tau=1}^{T} L_\tau} \tag{8a}$$

$$= \frac{TM}{TM + \sum_{\tau=1}^{T} P_\tau} \tag{8b}$$

$$= \frac{M}{M + \frac{1}{T}\sum_{\tau=1}^{T} P_\tau}. \tag{8c}$$

The average rate is given by

$$\overline{R} = \lim_{T\to\infty} R_T \tag{9a}$$

$$= \frac{M}{M + E[P_\tau]} \tag{9b}$$

$$= \frac{1}{1 + e^{-\frac{\lambda}{\gamma}}}. \tag{9c}$$

As will be seen in the simulation results, this approach solves the first problem presented in Section III, since it prevents a user with poor channel conditions from transmitting parity-check packets, and increases significantly the code rate in the low SNR region. However, when the SNR increases, the number of users above the threshold λ/γ also increases, and the system rate becomes lower, approaching the ANCC rate. Thus, this approach fails in solving the second issue presented in Section III.

In order to overcome this limitation, we propose the next approach.

B. Approach 2

In this second approach, instead of a fixed threshold λ, we use an adaptive threshold $\lambda(\gamma)$ which is a function of the average SNR. In order to avoid unreliable parity-check packets, we increase the threshold value beyond the threshold considered in the Approach 1, that is

$$\lambda(\gamma) \geq \lambda = 2^r - 1. \tag{10}$$

According to the choose of the threshold $\lambda(\gamma)$, we can increase the system's rate even further. This effect is illustrated in Fig. 4, in which the average rate $\overline{R}$ in (9) is plotted against the SNR for different thresholds. The goal of this

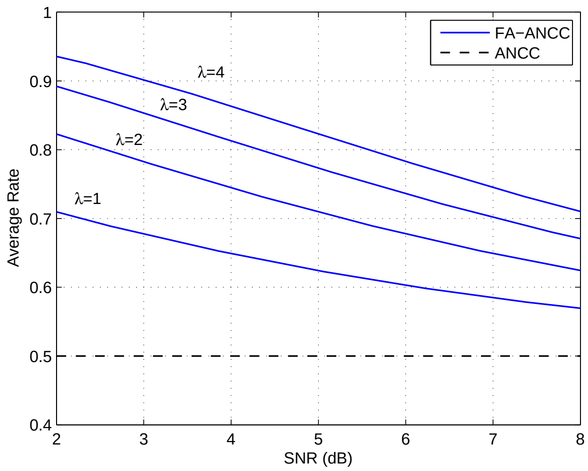

Fig. 4. Average rate $\overline{R}$ versus SNR for the ANCC and the FA-ANCC scheme, the last one with $\lambda = \{1, 2, 3 \text{ and } 4\}$.

second approach is to find the highest threshold possible for a given SNR (and consequently the highest code rate), without sacrificing the system's BER performance.

Due to randomness of the LDPC (LDGM) code, it is hard (if possible) to obtain the optimum value for the parameter $\lambda(\gamma)$ analytically. In the next section, we resort to computer simulations in order to evaluate the efficiency of the proposed scheme.

V. Simulation Results

In this section, we present some simulation results in order to support our proposal and the rate results obtained analytically. Throughout the simulations, we consider that the information rate r is equal to 1, resulting in the threshold $\lambda = 1$ for Approach 1 and threshold $\lambda \geq 1$ for Approach 2, according to (10).

In Fig. 5, the average rate obtained analytically in (9) is plotted versus the SNR and compared with its simulated value, for the FA-ANCC scheme with threshold $\lambda = 1$ (Approach 1). The ANCC rate is also plotted for comparison purposes. We can see that the analytical values matches the analytical ones. The rate of FA-ANCC is considerably higher than the ANCC one.

Besides increasing the average rate, we are interested in keeping (or even improving) the system BER performance. In this regard, we evaluate the error performance through Monte Carlo simulation, adopting the parameters presented in Table I. For each simulated SNR point, packets are transmitted until the occurrence of 5000 errors. The parity part of the generator matrix is randomly generated, following an approach in [9], and the message-passing algorithm [12] is used to decode the LDPC-like code.

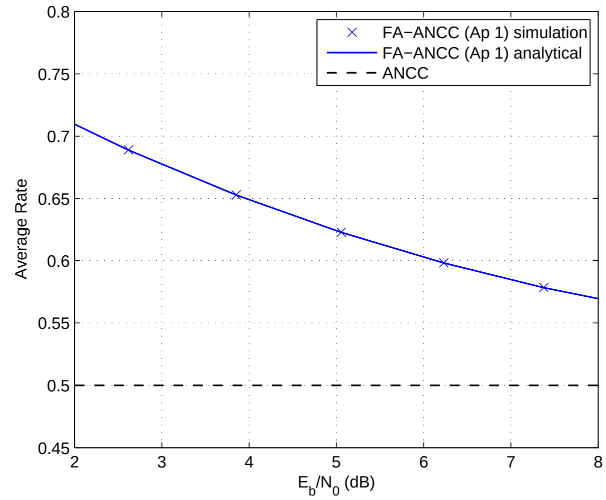

Fig. 5. Average rate $\overline{R}$ versus SNR for the ANCC and threshold-based (with threshold $\lambda = 1$) schemes.

TABLE I
Simulation parameters

Number of users M	1000
Maximum number of iterations	100
Code degree	12
Simulation stopping criterium	5000 errors
Information rate r	1 bits/channel use
Coherence block length	$2M$ packets

Fig. 6 presentes the BER versus E_b/N_0 for: a network with no cooperation, the ANCC scheme, and the FA-ANCC (Approach 1) with $\lambda = 1$. We can see that, for the range of SNR's simulated, the increase in the average rate presented in Fig. 5 can be realized without worsening the BER performance compared with the ANCC scheme. In fact, it can be seen that the performance of the FA-ANCC scheme is a little better.

For each SNR value, we now begin to increase the threshold $\lambda(\gamma)$ up to the point beyond which the BER performance gets worse than the ANCC performance. The BER performance and the average rate obtained for this second approach are presented in Fig. 7 and Fig. 8, respectively.

It is noteworthy the increase in the average rate achieved with Approach 2, while its BER performance is kept almost the same as the one of Approach 1, both of them slightly outperforming the ANCC scheme's BER performance.

VI. Conclusions and Final Comments

In this work, a feedback-assisted version of adaptive network coded cooperation (ANCC) [9] has been proposed. In the proposed scheme, under the assumption of block fading channels, we have shown that only one bit of feedback for each information packet received at the BS is sufficient for a significant increase in the system rate, without sacrificing the

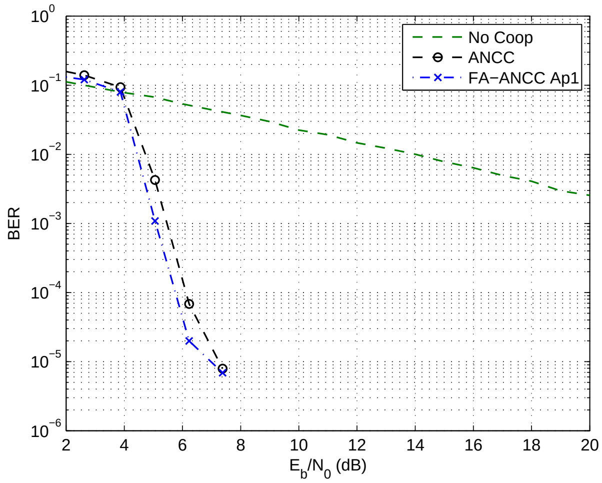

Fig. 6. BER versus E_b/N_0 for a network with $M = 1000$ users, for a network with no cooperation, the ANCC scheme and the proposed scheme (Approach 1) with threshold $\lambda = 1$.

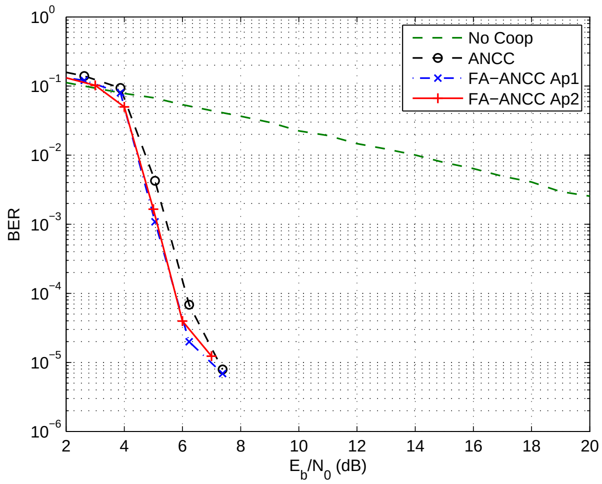

Fig. 7. BER versus E_b/N_0 for a network with $M = 1000$ users, for a network without cooperation, ANCC scheme and the FA-ANCC (Approach 1 and 2).

system error performance.

A rate analysis was developed, and simulation results corroborate the analysis. The BER performance was also evaluated through computer simulations, and we have shown that the rate increase does not affect the system performance.

Finally, it should be mentioned that, in practice, the optimal value of the threshold $\lambda(\gamma)$ for each SNR (or for each small SNR range) can be obtained off-line, and then saved in a memory at the BS.

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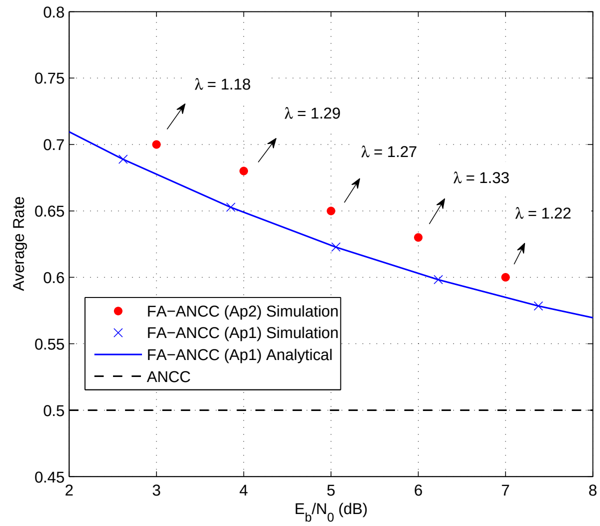

Fig. 8. Average rate $\overline{R}$ versus SNR for the ANCC and FA-ANCC (Approaches 1 and 2) schemes. The threshold values for the Approach 2 are indicated by the arrows.

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