

A Full-Diversity Scheme for Four Transmit Antennas and Quantized Feedback Channel

Samuel T. Valduga, José C. Menezes Jr., Renato Machado, Natanael R. Gomes, and Andrei P. Legg

Resumo—Neste artigo, propõe-se um esquema de diversidade espacial para sistemas de comunicação sem fio com quatro antenas transmissoras e canal de realimentação quantizado. Apresenta-se uma técnica de pré-processamento que permite ao esquema obter um grau de diversidade espacial completo bem como um ganho de codificação. Uma análise da realimentação quantizada para canais de comunicação sem fio com desvanecimento Rayleigh, plano e quase estático é apresentada também. Através de resultados de simulações, mostra-se que o esquema proposto apresenta uma melhora no desempenho em torno de 1,6dB e 2,0dB em relação ao esquema ACBS para os casos em que ocorrem o uso de 3 e 4 bits de realimentação, respectivamente.

Palavras-Chave—Ganho de codificação, sistemas MIMO, realimentação quantizada, diversidade de transmissão.

Abstract—In this paper we propose a spatial diversity scheme for four transmit antennas and quantized feedback channel systems. We present a preprocessor design that enables this scheme to achieve full diversity order as well as a coding advantage. A quantized feedback analysis of the proposed scheme for quasi-static flat Rayleigh fading channels is also performed. Through simulation results, we show that the proposed scheme has a performance improvement of about 1.6dB and 2dB over ACBS scheme for 3 and 4 feedback bits, respectively.

Keywords—Coding gain, MIMO systems, quantized feedback, transmit diversity.

I. INTRODUCTION

It is well known that multiple-input multiple-output (MIMO) wireless communication systems can exploit the spatial dimension to improve capacity and reduce sensitivity to fading.

The use of space-time codes are an effective way to exploit spatial diversity in MIMO wireless communication systems and since the work of Alamouti [1], the spatial diversity could be obtained in multiple-input single-output wireless communication systems. The Alamouti code is also the unique full-rate orthogonal space-time block code (OSTBC) that exists for any arbitrary complex symbol constellation. Moreover, if the number of transmit antennas is increased the data rate must then be sacrificed.

OSTBCs require no knowledge of the channel state information (CSI) at the transmitter to provide full diversity

gain. While the orthogonality constraint is required in order to achieve maximum diversity advantage, it has been shown [3], [4] that by relaxing the orthogonality requirement, higher transmission rates can be obtained at the cost of losing some degree of diversity order.

MIMO communication systems can obtain significant performance improvements if a feedback channel exists so that the CSI is known at the transmitter [5], [6]. Considering this scenario, Machado and Uchôa-Filho [7] have proposed a hybrid transmit antenna/code selection scheme that chooses from a list of space-time block codes the best code to be used with a subset of transmit antennas. The code selection is based on the instantaneous error probability minimization criterion. This idea has been refined later in [8], [9].

The transmission methods for MIMO channels using a quantized feedback channel is a very interesting area of research and several papers on this area can be found [10]–[17].

In [17], Choi *et al.* have proposed an interesting phase-feedback-assisted scheme with four transmit antennas which uses a preprocessor for combining two Alamouti codes in terms of Frobenius norm maximization. They have shown that full diversity is achieved by the combining effect at the preprocessor. Also the phase feedback is utilized to increase a coding gain. Herein we refer to this scheme as Alamouti-code based scheme (ACBS).

Following these ideas, in this paper, we propose a four-antenna transmit diversity scheme with quantized feedback. We design a preprocessor which combines four “trivial”¹ codes. The preprocessing of the proposed scheme is based on the designs presented in [9], [15], [17]. As a result, we observe an increase of the coding gain. A compromise of this proposal is that the receiver needs to feed, at least, 3 bits back to the emitter to ensure a full spatial transmit diversity order.

In this paper, we also present a procedure for determining the feedback information which maximizes the signal-to-noise ratio (SNR) at the receiver. Another important feature of the proposed scheme is that the maximum likelihood detector is based on linear processing, which leads to a very simple receiver. Simulation results reveal that the proposed scheme yields the maximum diversity advantage and also outperforms the ACBS scheme.

The remainder of this paper is organized as follows. Section II presents the channel model considered in this work. Section III addresses the full-diversity transmit diversity scheme with

Samuel T. Valduga, José C. Menezes Jr., and Renato Machado are with the Signal Processing and Communications Research Group, Department of Electronics and Computing, Federal University of Santa Maria, Santa Maria, RS, 97105-900 – Brazil. Andrei P. Legg and Natanael R. Gomes are with the Space Science Laboratory of Santa Maria, Federal University of Santa Maria, Santa Maria, RS, 97105-900 – Brazil. Email: {samucatv, jose.c.m.jr}@smaail.ufsm.br, renatomachado@ufsm.br, natanael.rgomes@gmail.com, andrei.legg@gmail.com.

¹Trivial code is the transmission of one information symbol over one symbol transmit period ($\tau = 1$).

quantized feedback. In Section IV, simulation results are presented. Finally, Section V presents some concluding and final remarks.

II. SYSTEM MODEL

Consider a MIMO system with M_T transmit and M_R receive antennas, and that the channels have a flat Rayleigh fading and remains constant over τ symbol intervals. The transmission model consists of *linear processing*, as described in [17]. With some slight modifications, we arrive at

$$\mathbf{Y} = \mathbf{X}\mathbf{H}\mathbf{I}_{SNR} + \mathbf{N}, \quad (1)$$

where \mathbf{Y} is the $\tau \times M_R$ matrix of the received signals and \mathbf{X} is the $\tau \times M_T$ matrix of transmitted signals with unit average energy. Let $\mathcal{CN}(0, \mathbf{R})$ represent the joint p.d.f. (probability density function) of a zero-mean circularly symmetric complex normal random vector with covariance matrix \mathbf{R} . Then, \mathbf{N} is the $\tau \times M_R$ matrix $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{\tau M_R})$ representing the joint p.d.f. of the i.i.d. (independent and identically distributed) additive Gaussian noise samples with unit variance, \mathbf{H} is the $M_T \times M_R$ MIMO channel characterized by the p.d.f. $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_T M_R})$, and \mathbf{I}_{SNR} is the $M_R \times M_R$ identity matrix, where the i -th diagonal element of this matrix is given by $\sqrt{\rho_i}$, with ρ_i the average SNR at the i -th receive antenna.

We assume that there are Q QAM data symbols $\{s_q\}$, $q = 1, \dots, Q$, with unit average energy to be transmitted over τ symbol intervals, and a reliable feedback channel through which b bits can be sent to the emitter.

Throughout this paper, we consider that the receiver has only one receive antenna. Normal letters represent scalar quantities, boldface lowercase letters indicate vectors, and boldface uppercase letters indicate matrices. The superscripts $(\cdot)^T$ and $(\cdot)^*$ represent the transpose and the complex conjugate operation, respectively.

III. FULL-DIVERSITY SCHEME FOR FOUR TRANSMIT ANTENNAS AND QUANTIZED FEEDBACK CHANNEL

In this section, we present how to perform the proposed scheme. The transmitter has four transmit antennas and the data symbol s_1 is preprocessed by \mathbf{p} , resulting in the complex transmitted signal vector \mathbf{x} :

$$\begin{aligned} \mathbf{x} &= \mathbf{p}s_1 \\ &= \left[\mathbf{v}_\varphi \left(\begin{bmatrix} \cos(\theta) & 0 \\ 0 & \sin(\theta) \end{bmatrix} \otimes \mathbf{I}_2 \right) \right] s_1, \\ &= \begin{bmatrix} \cos(\theta) \exp\{j0\} \\ \cos(\theta) \exp\{j\varphi_1\} \\ \sin(\theta) \exp\{j0\} \\ \sin(\theta) \exp\{j\varphi_2\} \end{bmatrix}^T s_1, \\ &= \begin{bmatrix} s_1 \cos(\theta) \\ s_1 \cos(\theta) \exp\{j\varphi_1\} \\ s_1 \sin(\theta) \\ s_1 \sin(\theta) \exp\{j\varphi_2\} \end{bmatrix}^T, \end{aligned} \quad (2)$$

where \otimes is the kronecker product, \mathbf{I}_n is the n -by- n identity matrix, s_1 is the information symbol, and

$$\mathbf{v}_\varphi = [\exp\{j0\} \quad \exp\{j\varphi_1\} \quad \exp\{j0\} \quad \exp\{j\varphi_2\}].$$

From Equations (1) and (2), the received signal y can be written as

$$y = h_e s_1 \sqrt{\rho} + \eta, \quad (3)$$

where

$$\begin{aligned} h_e &= \mathbf{p}\mathbf{h}, \\ &= h_{e1} \cos(\theta) + h_{e2} \sin(\theta), \end{aligned}$$

$\mathbf{h} = [h_1 \dots h_4]^T$, η is the additive white Gaussian noise, h_i denotes the path gain from the i -th transmit antenna to the receive antenna, and h_{e1} and h_{e2} are given by

$$h_{e1} = h_1 + h_2 \exp\{j\varphi_1\},$$

$$h_{e2} = h_3 + h_4 \exp\{j\varphi_2\}.$$

In order to guarantee full diversity order, the receiver needs to inform the appropriate phases, φ_1 , φ_2 , and θ , to the transmitter. Next, we present the expression for determining the phases that provide the maximum instantaneous SNR at the receiver.

A. SNR Analysis

Consider the received signal in Equation (3). The following linear processing produces the desired inputs to the maximum-likelihood detection:

$$\begin{aligned} \tilde{s}_1 &= y h_e^* \\ &= (|h_{e1} \cos(\theta)|^2 + |h_{e2} \sin(\theta)|^2 + h_{Re12}) s_1 + h_e^* \eta, \end{aligned} \quad (4)$$

with

$$\begin{aligned} |h_{e1} \cos(\theta)|^2 &= (|h_1|^2 + |h_2|^2 + h_{Re1}) \cos^2(\theta) \\ &= |h_{e1}|^2 \cos^2(\theta), \end{aligned}$$

$$\begin{aligned} |h_{e2} \sin(\theta)|^2 &= (|h_3|^2 + |h_4|^2 + h_{Re2}) \sin^2(\theta) \\ &= |h_{e2}|^2 \sin^2(\theta), \end{aligned}$$

and

$$h_{Re12} = 2\Re\{h_{e1} h_{e2}^*\} \cos(\theta) \sin(\theta)$$

where $|\cdot|^2$ denotes the modulus squared of a complex number, $\Re\{\cdot\}$ its real part,

$$h_{Re1} = 2\Re\{h_1 h_2^* \exp\{-j\varphi_1\}\}$$

and

$$h_{Re2} = 2\Re\{h_3 h_4^* \exp\{-j\varphi_2\}\}.$$

As we can observe, the detection can be performed with low complexity.

It can be shown that the SNR at the receiver is given by

$$\text{SNR} = (|h_{e1}|^2 \cos^2(\theta) + |h_{e2}|^2 \sin^2(\theta) + h_{Re12}) \frac{\sigma_x^2}{\sigma_n^2} \quad (5)$$

where σ_x^2 is the total power of the estimated signal, and σ_n^2 is the noise power at the receiver.

Now, we can determine the optimal phases by differentiation of the Equation (5), yielding a maximum signal-to-noise ratio and full diversity order. Nevertheless, full diversity can also be achieved with quantized feedback. In this regard, the feedback information is used to guarantee that each one of

the terms $|h_{e1}|^2$, $|h_{e2}|^2$, and h_{Re12} , is a positive number in every transmission frame. Moreover, with this feedback, the instantaneous SNR is maximized.

The phases φ_1 , φ_2 , and θ , are the variables taking into account in this optimization process. Since there is dependence among these variables, we need to solve this problem in steps. In this paper, we consider a two-step solution.

First, taking the inner cross terms in $|h_{e1}|^2$ and $|h_{e2}|^2$, i.e., h_{Re1} and h_{Re2} , respectively, we easily verify that these terms are maximized when

$$\varphi_1 = \xi_1 - \xi_2 \quad \text{and} \quad \varphi_2 = \xi_3 - \xi_4$$

where, $h_i = \alpha_i \exp\{j\xi_i\}$.

Second, we differentiate the SNR expression in terms of θ , following a similar procedure adopted in [17]. The first and second differentiations are given by

$$SNR' = 2\kappa (\cos(\theta) \sin(\theta)) + 2\nu (\cos^2(\theta) - \sin^2(\theta)) \quad (6)$$

and

$$SNR'' = 2\kappa (\cos^2(\theta) - \sin^2(\theta)) - 8\nu (\cos(\theta) \sin(\theta)), \quad (7)$$

respectively.

Solving (6) and (7) under the conditions $SNR' = 0$ and $SNR'' < 0$, we obtain the following optimal theta phase² [17]

$$\theta_{opt} = \arctan \left(\frac{\kappa + \sqrt{\kappa^2 + 4\nu^2}}{2\nu} \right), \quad (8)$$

with

$$\kappa = (|h_3|^2 + |h_4|^2 + h_{Re2}) - (|h_1|^2 + |h_2|^2 + h_{Re1})$$

and $\nu = \Re(h_{e1}h_{e2}^*)$.

We decide to omit the term $\frac{\sigma_x^2}{\sigma_n^2}$ in (6) and (7), since it is irrelevant to the phase optimization.

B. Quantized Feedback

In this section, we present how to use the proposed scheme with quantized feedback. We assume two uniform phase quantization: φ_{1q} and $\varphi_{2q} \in [0, \pi]$, as adopted in [10], [11], and $\theta_q \in [-\pi/2, \pi/2]$. The receiver needs to send, at least, three feedback bits (three-phase information) to the transmitter (see Table I).

Table I defines the criteria used to choose the quantized phases. Two bits are used to guarantee that the terms h_{Re1} and h_{Re2} are positive. The third bit ensures full diversity order, and the other “extra” ones are used to improve the SNR advantage. In Table I, the symbol $\#$ specifies the “do not care state”. In other words, when $b = 3$ bits, the signal of κ is not taken into consideration for feeding the phase information back to the transmitter.

²Optimal in the sense of maximizing the instantaneous SNR.

TABLE I
QUANTIZED FEEDBACK: CRITERIA FOR THE PHASE SELECTION.

Number of feedback bits	$2\Re\{h_1h_2^*\}$	$2\Re\{h_3h_4^*\}$	$\varphi_{1q} / \varphi_{2q}$
$b = 2$ bits	> 0	> 0	$0 / 0$
	> 0	< 0	$0 / \pi$
	< 0	> 0	$\pi / 0$
	< 0	< 0	π / π
+	ν	κ	θ_q
1 bit	> 0	$\#$	$\frac{\pi}{4}$
	< 0	$\#$	$-\frac{\pi}{4}$
or +	ν	κ	θ_q
2 bits	> 0	> 0	$\frac{2\pi}{6}$
	> 0	< 0	$\frac{\pi}{6}$
	< 0	> 0	$-\frac{\pi}{6}$
	< 0	< 0	$-\frac{2\pi}{6}$
	< 0	< 0	$\frac{2\pi}{6}$

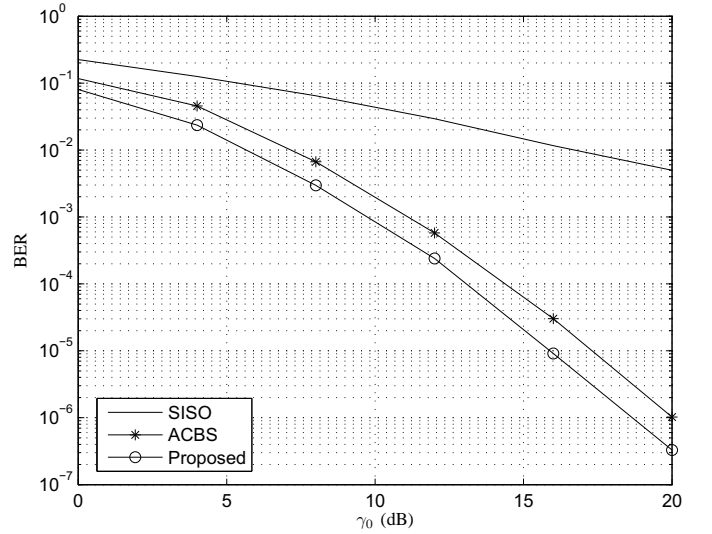


Fig. 1. BER performance of the proposed scheme. $M_T=4$ antennas, $M_R=1$ antenna, and $b=3$ bits.

IV. SIMULATION RESULTS

In this section, we present some simulation results to illustrate the performance of the proposed scheme. In order to assess the coding gain of the proposed scheme, we compare the BER of the proposed scheme to the Alamouti-code based scheme [17]. The performances are compared in terms of bit error rate (BER) versus $\gamma_0 = \frac{\sigma_x^2}{\sigma_n^2}$ over quasi-static flat Rayleigh fading channels.

In Figures 1 – 3, the results are given for $M_T = 4$ transmit antennas, $M_R = 1$ receive antenna, 4-QAM, 130 symbols per frame, and considered as stopping criterion the occurrence of 300 symbol errors per each SNR. The proposed and ABCS schemes are full-rate, i.e., we compare two unitary spatial transmit rate schemes ($R = Q/\tau = 1$). The BER for the no-diversity scenario (SISO) is also plotted, used as a reference.

The BER performance is presented in Figure 1 for $b = 3$ feedback bits, φ_{1q} and $\varphi_{2q} \in \{0, \pi\}$, and $\theta_q \in \{-\pi/4, \pi/4\}$, and in Figure 2, for $b = 4$ feedback bits, φ_{1q} and $\varphi_{2q} \in \{0, \pi\}$, and $\theta_q \in \{-2\pi/6, -\pi/6, \pi/6, 2\pi/6\}$. The quantization adopted in this paper is different from the one presented in [17]. The quantization used here (for the case $b = 4 = 2 + 2$ bits) seems to be slightly better than the one

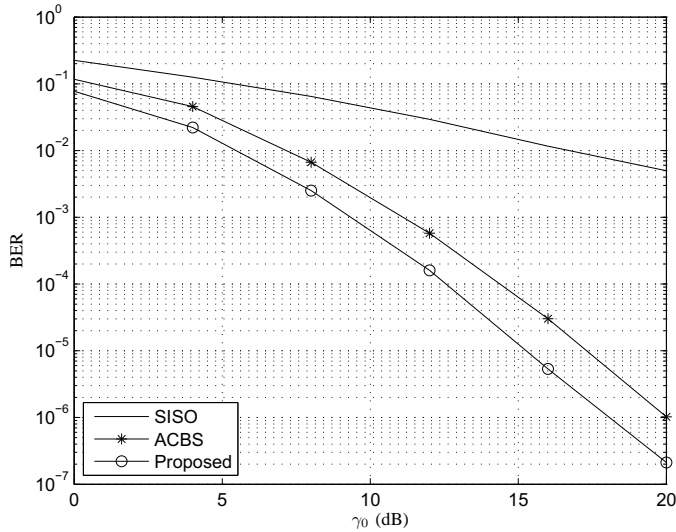


Fig. 2. BER performance of the proposed scheme. $M_T=4$ antennas, $M_R=1$ antenna, and $b=4$ bits.

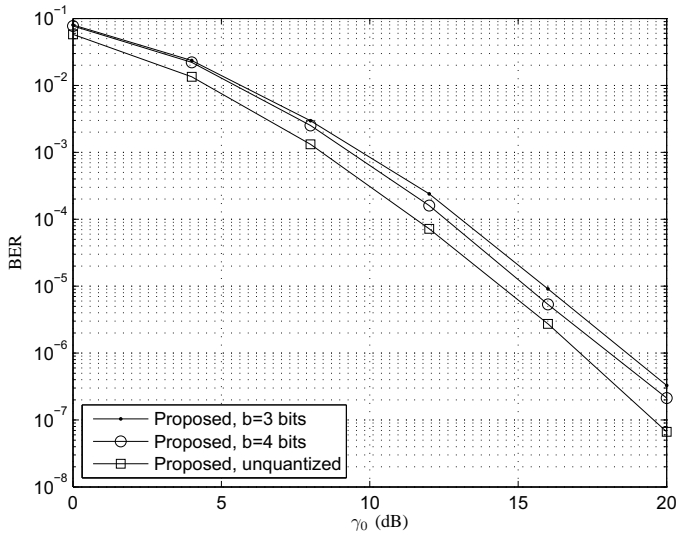


Fig. 3. BER performance of the proposed scheme for different levels of feedback quantization.

presented in [17]. However, it is just an inference and we did not prove it. Figure 3 presents the BER performance of the proposed scheme for different levels of feedback quantization.

In Figure 1, we observe that the proposed scheme has a performance gain of about 1.6dB over ACBS scheme for $b = 3$ feedback bits. Figure 2 illustrates that the proposed scheme has a performance improvement of about 2dB over ACBS scheme for $b = 4$ feedback bits. Figure 3 shows a performance loss due to the feedback quantization.

V. CONCLUSIONS AND FINAL REMARKS

In this paper, a full-diversity scheme for four transmit antennas and quantized feedback channel was proposed. The main idea backing the proposed scheme was to explore the benefits of the transmission schemes [9], [15], and Alamouti-code based scheme [17] in a new transmit diversity strategy. A

quantized feedback analysis for the quasi-static flat Rayleigh fading channels was performed, and their error performance was evaluated through computer simulations. It was observed that the proposed scheme outperforms the ACBS scheme in terms of coding gain.

It is worthy mentioning that the proposed scheme adopts a very simple linear decoding method with a small decoding delay ($\tau = 1$), while the ABCS has a decoding delay equal to two ($\tau = 2$). A compromise of the proposed scheme is that the receiver needs to feed, at least, 3 bits back to the transmitter to ensure full transmit diversity.

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