Dynamic Bayesian Approach Applied to Link Adaptation for 5G Wireless Systems

Hitalo J.B. Nascimento, Francisco R. P. Cavalcanti, André L. F. de Almeida and Mateus P. Mota.

***Abstract*— With technological development, wireless communication has been one of the fastest growing fields of Computing and Engineering in recent years. This fact requires that new approaches be developed to ensure better performance and reliability in wireless communication. In this paper a new approach has been proposed as a solution to the problem of adaptive modulation and coding (AMC), through the development of an extension of the method naive Bayesian classifier, known as dynamic naive Bayesian classifier, to maximize spectral efficiency. The proposed approach exhibits a better performance than k-nearest neighbours algorithm and the traditional Look-Up table solution, with average classification error 2.85%, which represents approximately 10% with respect to the most similar method.**

***Keywords*—** Adaptive modulation and coding, Link adaptation, Bayesian network.

I. Introduction

With the rapid development of technology, wireless communication has been one of the fastest growing fields of Computing and Engineering in recent years, and this is motivated mainly by the great demand for services based on multimedia contents. In the past decade, adaptive modulation and coding (AMC), based on machine learning has attracted much attention of researchers, and reason for this is that traditional solutions to the AMC problem such as Look-Up table (LUT) are not obtained in real time, they may require a great amount of memory in order to be stored, and they do not reflect the unique radio-frequency characteristics of each device [1], [2], [3]. In this sense, it is necessary to develop other tools or techniques in order to ensure wireless system to choose the highest order modulation schemes depending on the channel conditions to achieve higher system throughput's of the particular user based on the received signal quality by minimizing the BLER, noise and interference. A survey of the main implementation techniques shows that more common machine learning applied in AMC problem are: k nearest neighbours (k-NN), neural networks (NN), support vector machine (SVM) and random forest (RF). In this context, k-NN algorithm are discussed in [1] and [4], whereas in [2], a comparison between NN and k-NN to the problem was proposed. In [5], three supervised learning techniques, k-NN, SVM and RF, are applied. In [6], is proposed a mapping between the channel state information (CSI) and parameters such as rank indicator (RI) and channel quality indicator (CQI) feedback. The authors first propose an unsupervised artificial neural network called autoencoder and multi-class SVM to select MCS through SNRs (Signal-to-noise ratio) for MIMO-OFDM systems. A scheme based on matrix channel is also proposed to select spatial mode and MCS for MIMO systems, where autoencoder is used to extract features from CSI. [7] proposes a framework using k-NN based on the singular value decomposition (SVD). The experiments reported show that the proposed framework can successfully classify each MCS and perform perfect selection of MCS for frequency flat fading channels. In [8], was proposed an approach for link adaptation using fuzzy rule-based system for packet-based wideband networking waveform of software defined radio. According to the authors, this system selects an optimum pair of modulation and multicode indices to provide possible maximum or desired throughput, depending upon the throughput required by the user or application. The reported result shown that the proposed scheme reaches better throughput, reducing the packet retransmissions overhead. [9] propose the application of machine learning technique for channel-type identification in IEEE 802.11ac systems. According to the authors, the benefit of channel-type identification for the link adaption in 802.11ac systems is demonstrated, with up to 1.6 dB gain achieved at high SNR and classification accuracy of more than 94%. In [10], is proposed an AMC method based on a new on a simplified distributed space-time block coding scheme for a cooperative network with single antenna source and relay nodes. The results reported revealed that, relative to k-NN, the proposed solution enjoys better precision and robustness, and allows achieving higher objective performance.

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In this paper a new approach known as dynamic naive Bayesian classifier (DNBC) has been proposed as a solution to the problem of modulation and adaptive coding scheme, in order to optimize the precision of the system, ensuring spectral efficiency. In this sense, our research contributions can be summarized as follows:

- We propose a design and implementation of an approach with high precision, that ensures a given BLER while maximizing the throughput. The experimental results show that the proposed solution has an average classification error 2.85%;
- We investigate if the SNR observations in each state are normally distributed through of the Kolmogorov Smirnov test, this is because many of the statistical procedures, including correlation, regression, t tests, and analysis of variance, are based on the assumption that the data follows a normal distribution [11];
- In addition to the comparison between the proposed

method and k-NN, one of the most popular machine learning algorithms, we also provide a comparison with the baseline solution LUT.

The rest of the paper is organized as follows: In section 2, we present the problem statement. The proposed solution is described in section 3. Experimental results are described in section 4 and conclusion are presented in section 5.

II. Problem statement

The objective of link adaptation based on AMC, is to select the optimal AMC parameters, such as the modulation order M and convolutional coding rate C, to maximize the throughput R under block error rate (BLER) constraint as given by [12]:

$$MCS(\gamma) = \underset{i \in \mathscr{S}}{argmax}\{\mathscr{R}_i \mid BLER_i(\gamma) \leqslant BLER_{Tar}\} \quad (1)$$

where γ represents SNR feedback from the receiver, i denotes the i^{th} modulation and coding scheme (MCS), $\mathscr{S} = \bigcup_{i=1}^{N} \mathcal{MCS}_i$ is a set of MCSs, $\mathscr{R}_i$ is the instantaneous data rate of $\mathcal{MCS}_i$, $BLER_i(\gamma)$ is BLER of MCS i at SNR (γ) and $BLER_{Tar}$ represents the target of BLER, i.e. the maximum allowable BLER. Consider a MIMO-OFDM system with N_r and N_t representing, the number of receive and transmit antennas respectively. The received symbol of the m-th symbol $m \in \{1, 2, \cdots, N_o\}$ and the n-th subcarrier $\in \{1, 2, \cdots, N\}$, after discrete Fourier transform (DFT) is given by:

$$\mathbf{Y}[m,n] = \sqrt{E_s}\mathbf{H}[n]\mathbf{X}[m,n] + \mathbf{V}[m,n] \quad (2)$$

where $\mathbf{X}[m,n]$ is the transmitted symbols, E_s designates the expected total transmit energy, $\mathbf{H_n} \in \mathbb{C}$ represents the channel matrix, $\mathbf{V}[m,n]$ is the complex additive white Gaussian noise (AWGN), where the real and imaginary components are independent and identically distributed (iid) normal random variables with zero mean and variance N_0 [13].

III. Proposed system

We proposed solution is based on Bayesian network (BN), which is denoted by $\mathscr{B} = \langle G, \theta \rangle$, is a directed acyclic graph. With $G = (V, E)$ defined by a pair composed of vertices (V) that represent a set of random variables, $\mathbf{x} = \{\mathscr{X}_1, \mathscr{X}_2, \cdots, \mathscr{X}_N\}$, and edges (E) represent the dependence between the random variables. θ represents the set of conditional probabilities that are related to each random variable. AMC from this concept is defined in the following form: let $\mathbf{snr} = \{snr_1, snr_2, \cdots, snr_N\}$ be the SNR vector observed and $\mathcal{MCS}_i$, for $i = 1, 2, \cdots, N$, MCSs. Select $\mathcal{MCS}_i$ if $P(\mathcal{MCS}_i|\mathbf{snr}) > P(\mathcal{MCS}_j|\mathbf{snr})$, for $i, j = 1, 2, \cdots, N$ and $i \neq j$. This classification is formally obtained by applying the well-known the Bayes theorem given by

$$\begin{aligned} P(\mathcal{MCS}_i|\mathbf{snr}) &= \frac{P(\mathbf{snr}|\mathcal{MCS}_i)P(\mathcal{MCS}_i)}{P(\mathbf{snr})} \\ &= \frac{P(snr_1, snr_2, \cdots, snr_N|\mathcal{MCS}i)P(\mathcal{MCS}i)}{P(\mathbf{snr})} \end{aligned} \quad (3)$$

Using the general multiplication rule [14], the term $P(snr_1, snr_2, \cdots, snr_N|\mathcal{MCS}i)$, can be decomposed as $P(snr_1, snr_2, \cdots, snr_N|\mathcal{MCS}i) = P(snr_1|snr_2, \cdots, snr_N, \mathcal{MCS}i) \times P(snr_{N-1}|snr_N, \mathcal{MCS}i)P(snr_N|\mathcal{MCS}i)$.

If $snr_i \perp\!\!\!\perp snr_j|\mathcal{MCS}i, \forall i, j, 1 \leqslant i, j \leqslant N$, then $P(snr_1, snr_2, \cdots, snr_N|\mathcal{MCS}i)$ is given by $\prod_{i=1}^{N} P(\mathbf{snr}|\mathcal{MCS}i)$. Thus, $P(\mathcal{MCS}i|\mathbf{snr})$ $\propto P(\mathcal{MCS}i, snr_1, \cdots, snr_N)$ $\propto \mathcal{MCS}i \times P(snr_1, \cdots, snr_N|\mathcal{MCS}i)$ $\propto P(\mathcal{MCS}i) \times P(snr_1|\mathcal{MCS}i) \times \cdots \times P(snr_N|\mathcal{MCS}i)$, which results in

$$P(\mathcal{MCS}i) \prod_{i=1}^{N} P(\mathbf{snr}|\mathcal{MCS}i) \quad (4)$$

Equation 4 is a special case of BN, known as a naive Bayes (NB). More details on this classifier can be obtained in [15] and [16]. In order to consider the temporal information inherent to the problem, we propose a variation of this algorithm, called the dynamic naive Bayes classifier, which is best suited to real time system modeling. This model is an extension of Hidden Markov model (HMM) and when applied to our research theme, is composed by the set $\mathbf{snr}_M^t = \{snr_M^1, snr_M^2, \cdots, snr_M^T\}$, where snr_M^t, for each $t = 1, 2, \cdots, T$ is a set of M $snr's$ values generated by the process at state $\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_T\}$, in our case, corresponds to set of class MCSs $\mathscr{S}_t$, at each time t. From this concept, we define the joint probability distribution of $(\mathbf{snr}_t, \mathscr{S})$ through following equation $P(\mathbf{snr}, \mathscr{S}) = \prod_{t=1}^{T-1} P(\mathscr{S}_{t+1}|\mathscr{S}_t) \prod_{t=1}^{T} \prod_{j=1}^{M} P(snr_j^t|\mathscr{S}_t)$. The term $P(\mathscr{S}_{t+1}|\mathscr{S}_t)$ represents the transition probability distribution that describes the effects of previous chose MCSs on the recognition of the current MCS. Thus, DNBC is defined by equation 5.

$$\lambda = \{P(\mathcal{MCS}_1), P(\mathcal{MCS}_{t+1}), snr_M^t\} \quad (5)$$

The classification process is given of a sequence of $snr's$ and $\mathcal{L}$ DNBC $\{\lambda_i \mid i = 1, \cdots, \mathcal{L}\}$, each of them trained with samples of a particular $\mathcal{MCS}$ class, where the ideal model is obtained as $\lambda_i = \underset{\lambda_i}{argmax} P(\mathbf{snr} \mid \lambda_\mathbf{i})$. There is no known way to analytically solve the model in question. We can however apply the Baum-Welch algorithm [17], to iteratively improve $P(\mathbf{snr})$ until no significant difference are found in the consecutive likelihoods in the model. Figure 1 and Algorithm 1 summarizes the proposed solution.

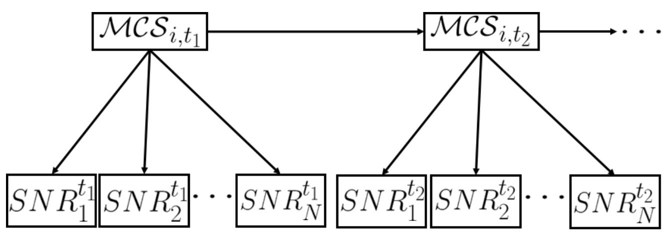

Fig. 1: Graphical representation of a DNBC unrolled 2 times with N attributes.

Algorithm 1 link adaptation based on DNBC.

1: Read $\mathbf{snr}_M^t = \left\{snr_M^1, snr_M^2, \cdots, snr_M^T\ \right\}$.
2: **repeat**
3: Apply the algorithm defined in [17], in order to obtain $\lambda_i = \underset{\lambda_i}{argmax} P(\mathbf{snr} \mid \lambda_\mathbf{i})$.
4: **until** no relevant difference in two consecutive likelihoods of the model is found or the maximum number of iterations is reached.
5: Calculate the mean and standard deviation of the prediction variables in each class.
6: **repeat**
7: Calculate $P(\mathbf{snr}|\mathcal{MCS}_i) = \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\left(snr-\mu_{mcs_i}\right)^2/2\sigma_i^2}$
8: **until** all the conditionals probabilities has been calculated.
9: Obtain the likelihood of each MCS and select the most likely MCS i.e. $\arg\max P(\mathcal{MCS}_i \mid \mathbf{snr})$.

IV. RESULTS AND DISCUSSION

In our experiment we considered an omnidirectional antenna model for Base Stations (BS) and User Equipment (UE), with transmit power of 43 dBm and heights given by 15m and 1.5m, for BS and UE, respectively, according [18]. In addition, a bandwidth with a frequency of 28 GHZ has been defined. We also consider an urban macro scenario, with a geometric channel model [19] and [20] given by $\mathbf{H} = \sqrt{N_t N_r} \sum_{l=1}^{L} \alpha_l a_r\left(\theta_l\right) a_t^H\left(\phi_l\right)$, where L is the number of paths, α_l represents the complex path gain of the l-th propagation path, $\theta_l \in \{0, 2\pi\}$ and $\phi_l \in \{0, \pi\}$ are angles of departure (AoD) and angles of arrival (AoA) of the L-th path at transmitter and receiver, respectively. $a_t\left(\theta_l\right)$ and $a_r\left(\phi_l\right)$, denote the array response vectors for transmitting and receiving antennas arrays. The array response of vectors $a_t\left(\cdot\right)$ and $a_r\left(\cdot\right)$, are given respectively by $a_r\left[\theta_l\right] = \frac{1}{\sqrt{N_r}}\left[1, e^{j\frac{2\pi}{\lambda}d\cos\theta_l}, \cdots, e^{j(N_r-1)\frac{2\pi}{\lambda}\cos\theta_l}\right]^T$ and $a_t\left[\phi_l\right] = \frac{1}{\sqrt{N_r}}\left[1, e^{j\frac{2\pi}{\lambda}d\cos\phi_l}, \cdots, e^{j(N_r-1)\frac{2\pi}{\lambda}\cos\phi_l}\right]^T$, where λ is the transmission wave length and d is the antenna spacing. This channel model was developed based non-line-of-sight, in which shadowing was modeled by to a log-normal distribution with standard deviation of 6 dB, according [20]. The simulation starts with the UE moving away from the BS at a speed of 5km/h, with start and end points equal to 20m and 100m from the BS, respectively. After this procedure, the EU returns to its original position through the same path in the reverse direction.

A physical layer model (PHY) in the 5G NR standard was adopted according to [20] and [21]. This standard supports quadrature phase shift keying (QPSK), and three types of modulation of quadrature amplitude modulation (16QAM, 64QAM and 256QAM) for Physical downlink shared channel (PDSCH).

We have implemented a DNBC with five states ergodic probability transition model distribution and trained with expectation–maximization (EM) algorithm, where the stopping criterion is achieved if the absolute difference of log likelihood of two consecutive models in an EM iteration is less than 0.01, according to [22], [23] and [24]. In order to verify that observations in each state are normally distributed, we applied the Kolmogorov Smirnov test. In this test, $\mathbf{snr}_t$ is considered a random sample from an unknown continuous population having the cumulative distribution function $F(x)$. To develop this test, the following hypotheses were considered, with a significance level of $\alpha = 0.05$:

$$\begin{cases} H_0 & : \text{The data} \sim N(\mu;\sigma^2); \\ H_1 & : \text{The data don't} \sim N(\mu;\sigma^2). \end{cases} \tag{6}$$

The statistic used for the test is $D = \underset{x}{max}\left|F(x) - F_n(x)\right|$, can be interpreted as maximum vertical displacement between $F(x)$ and $F_n(x)$, about the amplitude of the possible values of x. To take a decision about H_0, the test criteria are, reject H_0, if $D_n \leq D_{n,\alpha}$ (tabulated value) [25], otherwise accept H_0. The source code in R [26] in Appendix A, shows the result for the test in question. Since P-value is greater than 0.05, statistically we have enough evidence to conclude with a significance level of 0.05 that the samples obtained in each state are from a normal distribution, in addition, the boxplots in Figure 2, indicates that there is no significant difference between the observations.

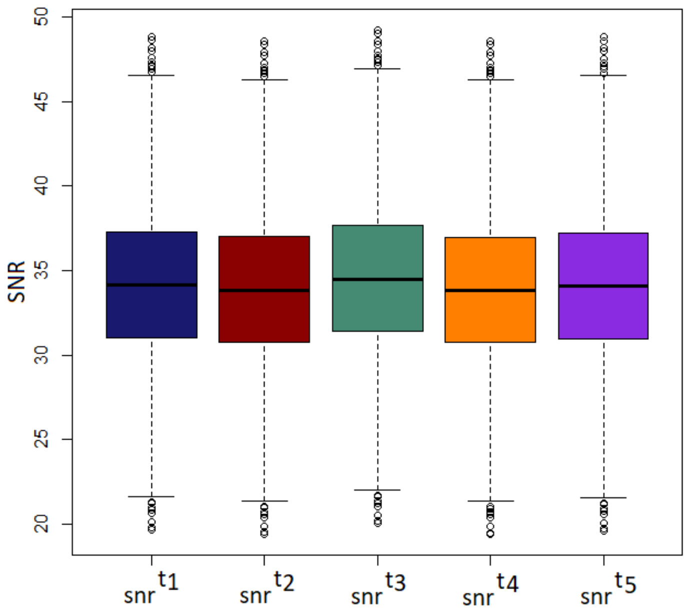

Fig. 2: Boxplots $\mathbf{snr}_t$ samples.

The proposed DNBC method was compared with traditional solutions to the AMC, LUT and the classical supervised algorithm kNN in order to verify the efficiency of the proposed solution. The kNN is a non-parametric algorithm that classifies a new object based on similarity to classes. The classification procedure consists of calculating the distance between the observed object and the nearest k classes. Once the k closest classes are identified, the observation will be classified to the most common class among its neighbours. In the simplest case of this algorithm, when k = 1, called 1 nearest neighbor (1NN), the query point is simply classified into the closest class. This procedure is illustrated in Figure 3. Note that when a new observation is obtained (query point), a circle is implemented around that observation that captures the 6 closest neighbors. Based on this rule, k-NN would then classify the observation as belonging to MCS 1. To implement this algorithm, we use the Euclidean distance. This together with other measures of similarity, including Mahalanobis, Minkowski, and cosine distances, are discussed in [16].

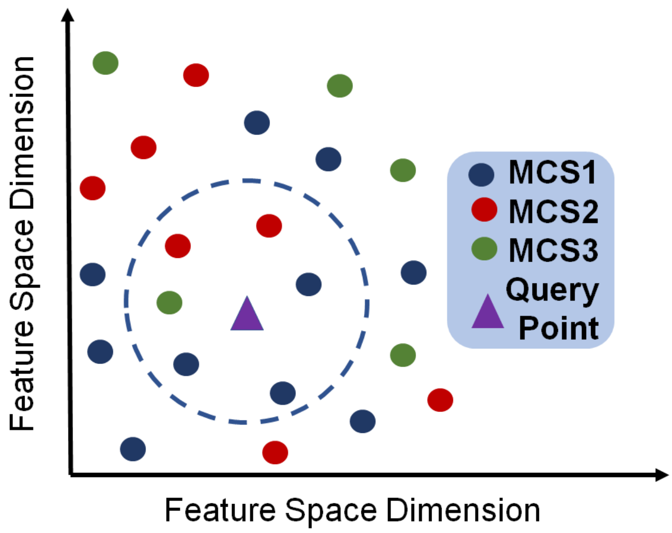

Fig. 3: 6-NN AMC.

Based on the analysis of the results, the proposed solution has average classification error $(\epsilon_m) = 2.85\%$. This result represents a difference 42% with respect LUT and almost almost 10% with respect to kNN. With respect to the k value of the k-NN algorithm, there is no optimal value for in the literature. [27] suggests that k must be small compared to the total number of observations, whereas [28] recommend $k \approx \sqrt{N}$. In this sense, different values of k were tested through Pearson correlation coefficient between the correct classification and the estimated classification. This experiment suggests that the k-NN method with $k = 3$ is more suitable for classification problem in question. The traditional LUT method, showed worse results compared to other algorithms. Despite its easy implementation and speed, this method is inefficient for modern systems in real time. A comparison of all algorithms, including the (ϵ_m), coefficient of variation (C_v), the first quartile (Q1), and the modal MCS are displayed in Table I, whereas the Figure 4, shows the cumulative distribution function (CDF) of classification algorithms as a function of the spectral efficiency, respectively.

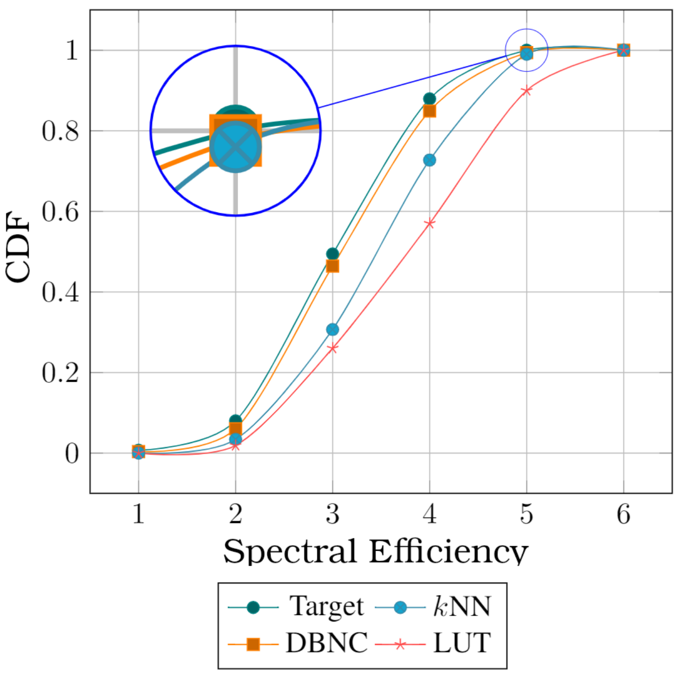

Fig. 4: CDF of the solutions as a function of the spectral efficiency.

TABLE I: Statistics of AMC estimates.

ALGORITHM	ϵ_m	C_v	Q1	M_o
Target	-	3.2%	27	25
DNBC	2.85%	3.3%	27	25
KNN	9.9%	3.5%	20	21
LUT	45.1%	57%	7	13

V. Conclusions

In this paper a new approach has been proposed as a solution to the problem of adaptive modulation and coding (AMC), through the development of an extension of the method naive Bayesian classifier, known as a dynamic naive Bayesian classifier. The proposed algorithm presented better performance than other methods with an average classification error 2.85%. This result represents a difference of almost 10% with respect to the most similar method. The future research includes:

- Application of the proposed solution to multi-layer multi-user MIMO transmission;
- Development of a hybrid system composed of a Bayesian network and a deep neural network to maximize system performance.

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APPENDIX

Result of the Kolmogorov-Smirnov test, implement in the R language.

```
# Kolmogorov-Smirnov test in R.

# Output:

One-sample Kolmogorov-Smirnov test student-t
with df=74.7, location=34.09, scale=4.52

data:  snr_dist[, 12]

D = 0.0099792, p-value = 0.9431

alternative hypothesis: two-sided
```

Listing 1: Result Kolmogorov-Smirnov test in R