Delay Analysis of SHARP – Spectrum Harvesting with ARQ Retransmission and Probing in Cognitive Radio

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***Abstract* — This paper proposes a new metric for performance analysis of the method entitled Spectrum Harvesting with ARQ Retransmission and Probing (SHARP), an underlay cognitive radio network (CRN) technique, i.e. the delay within the primary user transmissions. This variable depends on the operating region, channel utilization and data rate. The influence in primary transmissions is analyzed and mathematical results present conclusions about the impact in the effective primary user transmission time.**

Keywords*— *ARQ, cognitive radio, outage probability, spectrum sharing, SHARP, CRN, delay.

I. INTRODUCTION

Cognitive radio techniques have become an important solution for solving spectral congestion problems. The literature involving the coexistence of primary and secondary users grows with interest of increasing spectrum efficiency [1].

A fundamental requirement of a cognitive radio is to be aware of the primary users (PU) in the environment, inflicting minimal interference on them. The theoretical and practical problems involved in cognitive radio have proved to be very challenging. Zhao and Sadler [2] summarizes some results in dynamic spectrum sharing.

Several studies present possible improvements in CR techniques and its viability. Chandwani, Jain, and Vyava [3] discuss the primary user signal and channel noise conditions in a throughput comparison. Park, Kim, Lim, and Song [4] explore CR techniques by searching for new relevant channels for communication. Barnawi [5] focused on a novel approach by sensing the radio environment using a wideband chirp signal. Hattab [6] shows the advances in multiband spectrum sensing techniques for cognitive radios networks (MB-CRNs).

Furthermore, ARQ retransmission protocols have been exploited in cognitive radio. Li, Zhang, Nosratinia, and Yuan introduce the SHARP scheme [7], a method of underlay cognitive radio where the secondary users (SU) listen to the primary ARQ feedback to glean information about the primary channel. More recently, Bahgat presents a design of a low cost cognitive radio platform for demonstration and testing purposes [8], useful for validating the theory study.

This paper aims to extend the performance analysis of the SHARP scheme previously presented in [7]. A new variable, i.e. the delay in primary transmission, is introduced. The main goal is to extend the analysis in order to verify the delay influence in the performance of the scheme and achieve some insight about its effect in the system, realizing the impacts in primary transmissions.

The remainder of this paper is organized as follows. Section 2 summarizes the main aspects of the system model, operating regions and flow chart [7]. In Section 3, main aspects of the delay variable are analyzed and explained. Section 4 presents the delay for every operating region and the total system delay. Finally, numerical results and conclusion remarks in Sections 5 and 6, respectively.

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II. SYSTEM MODEL, OPERATING REGIONS AND FLOW CHART

A. System Model

In the SHARP method, the primary transmitter occupies the channel continuously; meanwhile the secondary transmitter uses the channel only through spectrum sharing. The primary transmitter operates at a constant power P_p and the nominal spectral efficiency of R_p bits/sec/Hz. If the first transmission at this rate and power is not successful (indicated by a NACK from the primary receiver), the same packet is retransmitted at the same power. The receiver combines the two packets and, if there is a failure in decoding after two transmissions, outage is declared. The secondary transmitter has a power constraint of P_s and a nominal spectral efficiency of R_s bits/sec/Hz. The secondary receiver does not generate ARQ feedback. It keeps transmitting as long as it is allowed.

Figure 1 illustrates the system model as explained in [7]. The channel gains are g_{ij} from a transmitter i to a receiver j, where the subscript value 1 denotes the primary and 2 denotes the secondary. Channel gains obey an exponential distribution with mean γ, which will influence in the operating regions probabilities. A slow fading scenario is considered, where the channel gain is assumed to be approximately constant over several transmissions, but is subject to change over large time scales, and all channels (primary, secondary and the cross-interfering links) are assumed to follow the Rayleigh distribution.

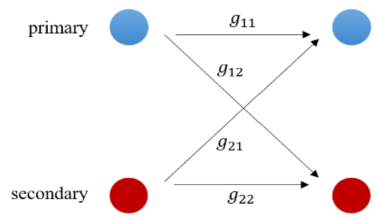

Fig. 1. ARQ-based spectrum sharing transmission system.

B. *Operating Regions*

The premise of SHARP method is to allow a secondary transmitter to share the primary channel without needing much channel-state information. It uses the ARQ transmissions from primary receiver to exploit opportunities whenever possible. The resulting interference may not drive the primary into outage. The channel states are not directly available to the secondary due to practical reasons. It can only observe the ACK/NACK from the primary receiver. Since it also knows its own transmissions, it is possible to know whether the ACK/NACK from the primary was under secondary interference or not.

Secondary transmissions are allowed depending on which region the system is operating, defined by the relative strength of the direct channel gain, g_{11}, and the cross channel gain, g_{21}. From the Shannon Capacity Theorem, the signal-to-noise ratio in the primary and secondary links are, respectively, $\gamma_p \triangleq 2^{R_p} - 1$ and $\gamma_s \triangleq 2^{R_s} - 1$. These parameters are the thresholds used to characterize the six operating regions (S_1-S_6) as in [7], which are defined according to each possible scenario of primary transmission success. Their corresponding probabilities are calculated in [7], considering a slow fading scenario and Rayleigh distribution.

C. *Flow Chart*

In some conditions, the SU may remain silent to avoid pushing the PU into outage or it can transmit without causing any outage, but decreasing the effective PU throughput due to retransmissions. In the *aggressive* SHARP, the secondary may transmit whenever possible, whereas in the *conservative* SHARP, it can only transmit when there is no effect on the primary. The probing and discovery mechanism are characterized by secondary transmission decisions. The transmission modes are as follows:

T_0 = {primary transmits new packet; secondary keeps silent}
T_1 = {primary repeats old packet; secondary keeps silent}
T_2 = {primary transmits new packet; secondary transmits}
T_3 = {primary repeats old packet; secondary transmits} .

Using the above notation, the mechanism is systematically implemented for aggressive and conservative SHARP as illustrated in Figure 2 and 3. Starting from the root of the tree, the secondary stays silent for the first transmission and observes the primary ACK/NACK. Each detection traces a path from the top of the tree to one of the six possible leaves. When a region is found, a transmission loop starts and continues until the channel conditions change. Occasionally, due to channel fading, the secondary may redo the probing and restart the process. For delay calculation purposes, it is defined that once detected a region, the system will stay there indefinitely and redo the channel probing for every transmission. It is important to note that neither aggressive nor conservative SHARP produce any primary outage. The main difference resides in the fact that aggressive SHARP may occasionally slow down the primary by forcing it in some occasions to retransmit, causing some degradation compared to conservative SHARP.

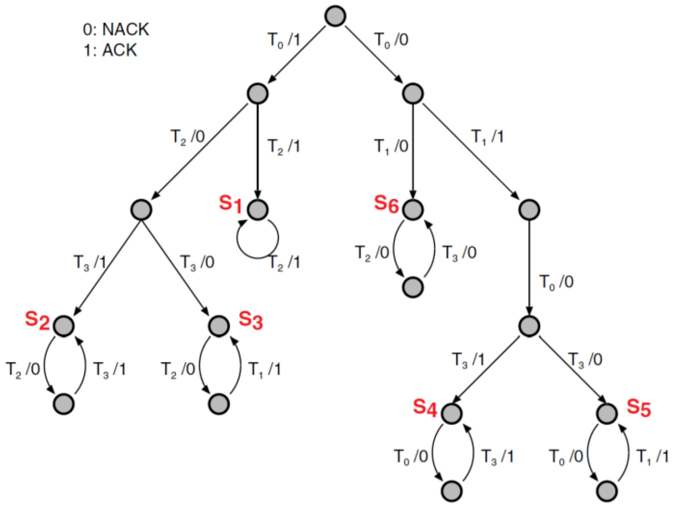

Fig. 2. Flow chart for the aggressive SHARP [7].

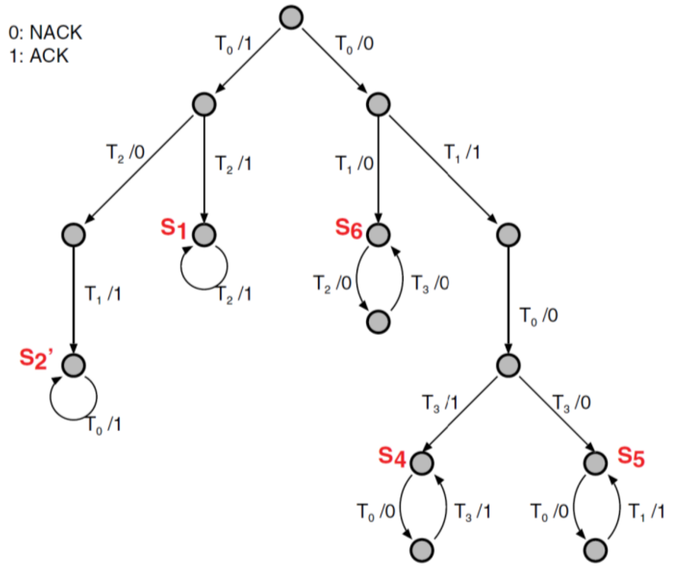

Fig. 3. Flow chart for the conservative SHARP [7].

III. Delay Analysis

In this section, the delay to transmit a correct package in the primary network is computed.

By observing the system operation, many variables need to be considered for studying the delay in primary transmissions, i.e. packet transmission time, ACK packet transmission time, region identification time and queueing time.

A. *Data packet transmission time*

The data packet transmission time (T_{TX}) is simply defined by the packet length divided by the throughput.

$$T_{TX} = \frac{l_p}{B * R_p} \tag{1}$$

where l_p is the packet length and B is the transmission bandwidth.

B. ACK packet transmission time

Similarly, the ACK packet transmission time (T_{ACK}) is defined as:

$$T_{ACK} = \frac{l_{ACK}}{B * R_p} \quad (2)$$

C. Retransmission time

In case of an incorrect packet transmission, a retransmission is necessary, adding the corresponding time of a new transmission.

$$T_{RTX} = T_{TX} + T_{ACK} \quad (3)$$

D. Region identification time

The region identification time (T_{ID}) reflects the necessary period for the system to reach the stationary operating region. It follows according to the flow chart presented (aggressive and conservative). Below is summarized the necessary time to get to each operating region:

$$S_1 \text{ and } S_6 \rightarrow T_{ID} = 2(T_{TX} + T_{ACK}) \quad (4)$$

$$S_2 \text{ and } S_3 \rightarrow T_{ID} = 3(T_{TX} + T_{ACK}) \quad (5)$$

$$S_4 \text{ and } S_5 \rightarrow T_{ID} = 4(T_{TX} + T_{ACK}) \quad (6)$$

$$S_2{}' \rightarrow T_{ID} = 3(T_{TX} + T_{ACK}) \quad (7)$$

E. Queueing time

Primary stations are considered to have a buffer and generate packets following a Poisson distribution at a rate λ_1 packets/second. The queueing system follows an M/G/1 queue [9]. The queueing time (T_Q) is computed by:

$$T_Q = \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \quad (8)$$

where ρ_1 is the channel utilization, X_1 is a random variable representing the necessary time for a correct packet transmission and $E[X_1{}^2]$ is its second moment.

It is also valid to define the first moment of X_1 ($E[X_1]$), which represents the average correct packet transmission time (including ACK time and retransmission time) and may vary depending on the operating region (channel conditions) and number of retransmissions. The expressions for computing $E[X_1]$ and $E[X_1{}^2]$ are:

$$E[X_1] = \left\{\sum_{N_r=0}^{N_{r_{max}}} (N_r+1)T_{TX}(PER)^{N_r}(1-PER)\right\} + \left\{(N_{r_{max}}+1)T_{TX}\left[1-\sum_{N_r=0}^{N_{r_{max}}}(PER)^{N_r}(1-PER)\right]\right\} \quad (9)$$

$$E[X_1{}^2] = \left\{\sum_{N_r=0}^{N_{r_{max}}} [(N_r+1)T_{TX}]^2(PER)^{N_r}(1-PER)\right\} + \left\{\left[(N_{r_{max}}+1)T_{TX}\right]^2\left[1-\sum_{N_r=0}^{N_{r_{max}}}(PER)^{N_r}(1-PER)\right]\right\} \quad (10)$$

where PER refers to the packet error rate. In this system model, the operating region itself implicitly defines the PER and $E[X_1]$ by telling whether the primary user send the packet successfully in one or two transmissions.

F. Time delay in primary transmission

The time delay for primary transmissions may be the junction of all presented variables.

$$T_D = T_{ID} + \underbrace{T_{TX} + T_{ACK} + T_{RTX} + T_Q}_{W_1} \quad (11)$$

where W_1 is the average delay in primary network, excluding the probing channel time and may be calculated by:

$$W_1 = E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \quad (12)$$

Important aspects that may direct influence the time delay in primary transmissions are whether the system is aggressive or conservative SHARP and the operating region.

IV. Delay Calculation

The analysis may start computing the average delay (T_D) for each operation region and then finding the total average delay (T_{DELAY}) by the weighted average using their corresponding probabilities.

A. Delay in each operating region

The delay in each operating region is calculated below. The notation may follow the calculated region, followed by aggressive or conservative SHARP. The first and second moment of X_1 may be derived for each operating region by observing the flow chart in Figures 2 and 3.

$$\begin{aligned} T_{D_{S_{1A;C}}} &= T_{ID_{S_1}} + W_{1_{S_1}} \\ &= 2(T_{TX}+T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 2(T_{TX}+T_{ACK}) + (T_{TX}+T_{ACK}) + \frac{\rho_1(T_{TX}+T_{ACK})^2}{2(T_{TX}+T_{ACK})(1-\rho_1)} \\ &= 3(T_{TX}+T_{ACK}) + \frac{\rho_1(T_{TX}+T_{ACK})}{2(1-\rho_1)} \\ &= \left(3+\frac{\rho_1}{2(1-\rho_1)}\right)(T_{TX}+T_{ACK}) \end{aligned} \quad (13)$$

$$\begin{aligned} T_{D_{S_{2A}}} &= T_{ID_{S_2}} + W_{1_{S_2}} \\ &= 3(T_{TX}+T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 3(T_{TX}+T_{ACK}) + 2(T_{TX}+T_{ACK}) + \frac{\rho_1 4(T_{TX}+T_{ACK})^2}{2(T_{TX}+T_{ACK})2(1-\rho_1)} \\ &= 5(T_{TX}+T_{ACK}) + \frac{\rho_1(T_{TX}+T_{ACK})}{(1-\rho_1)} \\ &= \left(5+\frac{\rho_1}{(1-\rho_1)}\right)(T_{TX}+T_{ACK}) \end{aligned} \quad (14)$$

$$\begin{aligned} T_{D_{S_2{}'_C}} &= T_{ID_{S_2{}'}} + W_{1_{S_2{}'}} \\ &= 3(T_{TX}+T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 3(T_{TX}+T_{ACK}) + (T_{TX}+T_{ACK}) + \frac{\rho_1(T_{TX}+T_{ACK})^2}{2(T_{TX}+T_{ACK})(1-\rho_1)} \\ &= 4(T_{TX}+T_{ACK}) + \frac{\rho_1(T_{TX}+T_{ACK})}{2(1-\rho_1)} \\ &= \left(4+\frac{\rho_1}{2(1-\rho_1)}\right)(T_{TX}+T_{ACK}) \end{aligned} \quad (15)$$

$$\begin{aligned} T_{D_{S_{3_A}}} &= T_{ID_{S_3}} + W_{1_{S_3}} \\ &= 3(T_{TX} + T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 3(T_{TX} + T_{ACK}) + 2(T_{TX} + T_{ACK}) \\ &\quad + \frac{\rho_1 4(T_{TX} + T_{ACK})^2}{2(T_{TX} + T_{ACK})2(1-\rho_1)} \\ &= 5(T_{TX} + T_{ACK}) + \frac{\rho_1(T_{TX} + T_{ACK})}{(1-\rho_1)} \\ &= \left(5 + \frac{\rho_1}{(1-\rho_1)}\right)(T_{TX} + T_{ACK}) \end{aligned} \tag{16}$$

$$\begin{aligned} T_{D_{S_{4_{A;C}}}} &= T_{ID_{S_4}} + W_{1_{S_4}} \\ &= 4(T_{TX} + T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 4(T_{TX} + T_{ACK}) + 2(T_{TX} + T_{ACK}) \\ &\quad + \frac{\rho_1 4(T_{TX} + T_{ACK})^2}{2(T_{TX} + T_{ACK})2(1-\rho_1)} \\ &= 6(T_{TX} + T_{ACK}) + \frac{\rho_1(T_{TX} + T_{ACK})}{(1-\rho_1)} \\ &= \left(6 + \frac{\rho_1}{(1-\rho_1)}\right)(T_{TX} + T_{ACK}) \end{aligned} \tag{17}$$

$$\begin{aligned} T_{D_{S_{5_{A;C}}}} &= T_{ID_{S_5}} + W_{1_{S_5}} \\ &= 4(T_{TX} + T_{ACK}) + E[X_1] + \frac{\lambda_1 E[X_1{}^2]}{2(1-\rho_1)} \\ &= 4(T_{TX} + T_{ACK}) + 2(T_{TX} + T_{ACK}) \\ &\quad + \frac{\rho_1 4(T_{TX} + T_{ACK})^2}{2(T_{TX} + T_{ACK})2(1-\rho_1)} \\ &= 6(T_{TX} + T_{ACK}) + \frac{\rho_1(T_{TX} + T_{ACK})}{(1-\rho_1)} \\ &= \left(6 + \frac{\rho_1}{(1-\rho_1)}\right)(T_{TX} + T_{ACK}) \end{aligned} \tag{18}$$

Since S_6 is the operation region where the primary user is in outage, its delay may not be computed.

$$T_{D_{S_{6_{A;C}}}} = N/A \tag{19}$$

B. Total delay

The total delay depends on the time spent by the primary user in each operating region. Therefore, it is defined as the weighted average using their probabilities calculated in [7].

$$T_{DELAY\,A} = \frac{\left(T_{D_{S_{1_{A;C}}}} P\{S_1\} + T_{D_{S_{2_A}}} P\{S_2\} + T_{D_{S_{3_A}}} P\{S_3\} + T_{D_{S_{4_{A;C}}}} P\{S_4\} + T_{D_{S_{5_{A;C}}}} P\{S_5\}\right)}{1 - P\{S_6\}} \tag{20}$$

$$T_{DELAY\,C} = \frac{T_{D_{S_{1_{A;C}}}} P\{S_1\} + T_{D_{S_2{}'}} P\{S_2{}'\} + T_{D_{S_{4_{A;C}}}} P\{S_4\} + T_{D_{S_{5_{A;C}}}} P\{S_5\}}{1 - P\{S_6\}} \tag{21}$$

V. Numerical Results

Numerical results of different scenarios for delay in primary user transmissions are presented in this section. As stated in Section II, all channels follow the Rayleigh distribution. By default as in [7], the primary user transmission power (P_p) and the secondary user transmission power (P_s), as well as the Gaussian background noise variance (N), are equal to unity. In each case, the channel gain means (defined as γ_{ij} according to the system model in Figure 1) will have specific values. Data packets (l_p) and ACK packets (l_{ACK}) have normalized lengths of 1 and 0.1, respectively. The bandwidth also is normalized to unity for simplification. Unless otherwise stated, the channel utilization (ρ_1) is arbitrarily chosen to be 0.5 and the rate thresholds are set to be $R_p = 1$ (bits/sec/Hz) and $R_s = 0.5$ (bits/sec/Hz) for the primary and secondary user, respectively. The mean of the channel propagation gains shall be $\lambda_{11} = \lambda_{22} = 4$ and $\lambda_{21} = 1$.

The first case shows the increase in total delay by varying the secondary user transmission power (P_s). Notice in Figure 4 that aggressive SHARP schemes may have more delay than conservative ones due to a higher interference from the secondary user. However, it benefits the secondary user by allowing more transmission opportunities. In addition, as interference increases, the necessary time to identify a region expands. The numerical results in total delay are illustrative. They may seem bigger than expected but it is important to observe that the parameters used are normalized. For instance, a 4 seconds result means 4 packet transmission times.

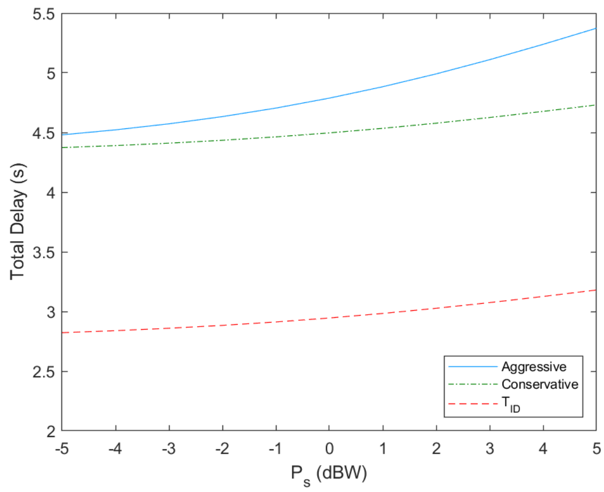

Fig. 4. Total delay vs. P_s.

Next scenario explores how the channel utilization (ρ_1) influences in primary transmissions delay. The mean of the channel propagation gain between primary transmitter and secondary receiver is $\lambda_{12} = 4$. Figure 5 illustrates this case. As expected, increasing ρ_1 close to the limit may cause the system to become unstable since the time delay tends to infinity. The region identification time (T_{ID}) is independent from ρ_1, since it does not interfere in the flow chart, but in the probabilities. Lower channel utilization presents a decrease in total delay.

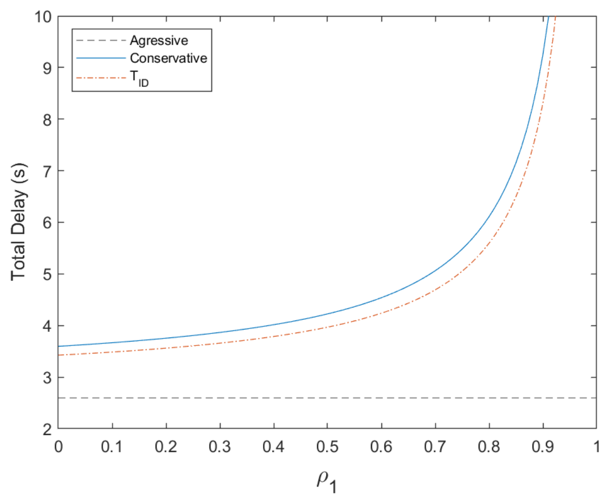

Fig. 5. Total delay vs. ρ_1.

Figure 6 provides the delay evolution with the signal-to-interference ratio at the primary receiver (SIR_{PR}), keeping the same transmission power for both PU and SU, constant primary and secondary link channel gain and varying the interfering channel between them (λ_{21}). In this case, the mean of the channel propagation gain between secondary transmitter and its receiver is $\lambda_{22} = 1$. It can be observed that with a reduced interference scenario for the primary user receiver the delay is decreased, since it is less likely to happen retransmissions. Even the region identification time decreases due to a higher probability of running the probing algorithm faster.

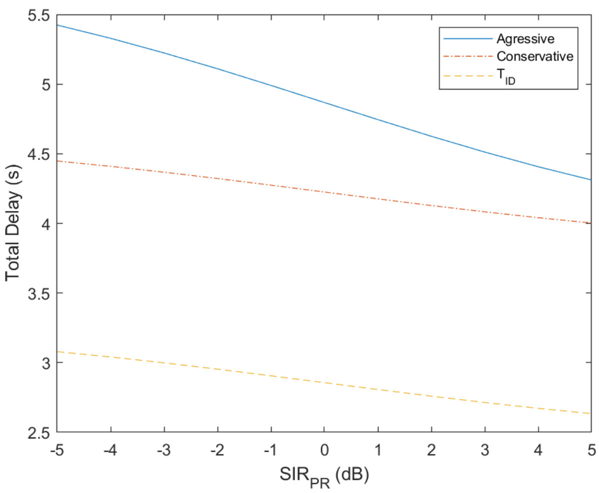

Fig. 6. Total delay vs. SIR_{PR}.

VI. CONCLUSION

This paper aims an extension in the study for underlay cognitive radio named SHARP [7] by considering a new variable, i.e. the delay in primary user transmission. Its influence in the scheme performance was verified and some insight achieved about the effect in the system and the impacts in primary transmissions.

The delay variable was divided in five main parts, i.e. data and ACK packets transmission time, retransmission time, region identification time and queueing time. Each operating region has its own delay time. The total time was computed weighting the region values in respect with their probabilities.

Three different scenarios were studied. The conclusions reside in respect of how the interference increases the delay. As it arises, more retransmissions are needed, slowing the region identification process and the effective packet transmission rates. Primary users in aggressive SHARP may suffer more compared to conservative SHARP because they allow more opportunities for the secondary users to transmit. Channel utilization also is an important limiting factor as it may increase exponentially the queueing time, possibly leading the system to failure. Future studies may compute the effective primary user throughput considering the delay variable and the secondary user delay.

VII. ACKNOWLEDGEMENTS

This work was partially supported by RNP, with resources from MCTIC, Grant No. No 01250.075413/2018-04, under the Radiocommunication Reference Center (Centro de Referência em Radiocomunicações - CRR) project of the National Institute of Telecommunications (Instituto Nacional de Telecomunicações - Inatel), Brazil.

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