

A Frequency Domain Resource Allocation Technique with Reduced Complexity for PLC System

Guilherme R. Colen[†] and Thiago M. Peixoto[†] and Victor Fernandes[†] and Moises V. Ribeiro^{†,*}

[†]UFJF - Federal University of Juiz de Fora, Brazil.

*Smarti9 Ltda. - Juiz de Fora, Brazil.

Abstract—The present work aims to suggest a resource allocation approach to reduce computational complexity of bit-loading techniques applied to power line communication (PLC) based on multi-carrier modulation. This approach is based on grouping adjacent subcarriers and using the same resource allocation in this group, but it results in a data-rate degradation. Thus, there is a trade-off between computational complexity reduction and data-rate degradation driven by the group length. With in-home power line channels data, obtained from a measure campaign covering the frequency bands 1.7-30, 1.7-50 and 1.7-100 MHz, we have shown the trade-off performance. Our results show that the suggested technique is capable of giving priority to computational complexity in exchange for a little data-rate degradation.

Keywords—Bit-loading algorithm, resource allocation technique, power line communication, orthogonal frequency-division multiplexing.

I. INTRODUCTION

Power line communication (PLC) is a promising telecommunication technology to attend smart grid communication and digital divide demands, once the electrical grids are present in most places. Thus, the investigation of PLC technologies are quickly growing and, in general, they are based on multicarrier modulation, such as orthogonal frequency-division multiplexing (OFDM) [1]–[3], due to the channel frequency selective behavior, fading, and the presence of impulsive noise. Thereby, resource allocations techniques are of paramount importance in order to increase the transmission data-rate [1]–[4]. These techniques, using bit loading algorithms, are able to optimally define the number of bits that will be transmitted by each subcarrier and the power used for it. The optimal resource allocation technique is known in the literature [3], but it demands a high computational complexity, mainly when the number of subcarriers is high (the computational complexity of bit-loading algorithms are proportional to the number of subcarriers) and the channel is linear periodically time varying (LPTV) - the bit-loading algorithm must be executed periodically - as the in-home PLC channels are. All these make the optimal resource allocation impractical.

Some examples of resource allocation research for PLC systems are shown as follows. The influence of the channel estimation, using a least mean square estimator, on the resource allocation techniques for broadband PLC systems has been examined in [1]. Also, [2] compares four different bit-loading algorithms, in terms of the number of bits and the power allocated to each subcarrier. The problem of optimal resource allocation in OFDM-based PLC systems, is discussed in [3], under the assumption that the total power used during one

cycle of the mains signal is optimally distributed among the so-called microslots — but this technique is tough intensive as far as computational complexity is considered. The problem of fair resource allocation in PLC systems with multiple users is addressed in [4].

This work aims to reduce the computational complexity of resource allocation problem in PLC systems by grouping adjacent subcarriers and, thus, solves an allocation problem with lower dimensionality than the original one. This approach is used in some standards as [5], however, to the best of our knowledge, the analysis of the data-rate degradation is still an open issue in the literature. Thus, this paper studies the trade-off between computational complexity reduction and data-rate degradation, yielded by the proposed technique, in order to know the best number of adjacent subcarriers that should be grouped. Numerical results are obtained for three different frequency bands: 1.7-30 MHz, 1.7-50 MHz and 1.7-100 MHz. These frequency bands are chosen once the first and the second frequency bands comply with European and Brazilian regulations for PLC systems, respectively, while the latter is a possibility for future regulatory and standardization of PLC systems.

This paper is organized as follows. Section II formulates the resource allocation problem in PLC systems, while Section III proposes a technique to reduce the computational complexity in resource allocation techniques, with a data-rate degradation consequence. This trade-off, between computational complexity and data-rate degradation, is studied in Section IV, which presents the proposed technique results. Finally, Section V addresses the conclusions of this study.

II. PROBLEM FORMULATION

Our discussion will be centered on a communication system for the transmission of bits across an linear time varying (LTV) PLC channel using an OFDM scheme. In this regards, let a LTV PLC channel be taken as linear time-invariant (LTI) channel during a period of time, shorter than the coherence time (T_c) and B_c the LTI PLC channel coherence bandwidth; an OFDM-based PLC system be working in a frequency band from 0 to B Hertz with N subcarriers (subchannel bandwidth of $B_s = 2B/N$); and the channel frequency response (CFR) of the LTI PLC channel be considered as flat in a bandwidth equal to $B_s < B_c$. We can thus write the gain of CFR of the discrete Fourier transform (DFT) in a discrete-time representation of such LTI PLC channel as

$$\mathbf{\Lambda}_{\mathbf{H}^2} = \text{diag}\{|H_0|^2, |H_1|^2, \dots, |H_{N-1}|^2\}, \quad (1)$$

where $\mathbf{H} = [H_0 \ H_1 \ \cdots \ H_{N-1}]^T = (1/\sqrt{N})\mathbf{W}[\mathbf{h}^T \mathbf{0}_{N-L_h}^T]^T$; $\mathbf{h} = [h_0 \ h_1 \ \cdots \ h_{L_h-1}]^T$ is a vector constituted by the coefficients of the channel impulsive response; L_h is the length of the channel impulse response (CIR); $\mathbf{0}_{N-L_h}$ is the $(N-L_h)$ -length column vector composed of zero values; \mathbf{W} denotes the N -size DFT matrix; and $(\cdot)^T$ and $\mathbf{diag}\{\cdot\}$ denote the transposition operator and a diagonal matrix, respectively. Note that the LTV PLC channel becomes LTI once the analysis is carried out into a time period shorter than T_c .

In this work we consider that the noise in the vector model of the LTI PLC channel is modeled as a colored Gaussian random process $\mathbf{V} = [V_0 \ V_1 \ \cdots \ V_{N-1}]^T$. We have, in addition, with $\mathbb{E}\{\cdot\}$ representing the expectation operator, that the frequency domain noise components are uncorrelated, i.e., $\mathbb{E}\{V_k V_j\} = \mathbb{E}\{V_k\}\mathbb{E}\{V_j\}$ for $k \neq j$; $k, j = 0, 1, \dots, N-1$, and, each component V_k is a random variable having zero mean, $\mathbb{E}\{V_k\} = 0$, variance $\sigma_{V_k}^2 = \mathbb{E}\{|V_k|^2\}$ (the majority of works on resource allocation in PLC systems and PLC channel characterization, as in [3] and in [6], assume that the noise is either additive white Gaussian noise (AWGN) or additive colored Gaussian noise).

With the assumption that the power spectral density (PSD) of the additive noise is flat within each subband we can represent the noise PSD as the vector $\mathcal{N}_{\mathbf{V}} = [\mathcal{N}_0 \ \mathcal{N}_1 \ \cdots \ \mathcal{N}_{N-1}]^T$, where \mathcal{N}_k is the PSD at the k^{th} subchannel. For convenience this vector will be arranged as a diagonal matrix

$$\Lambda_{\mathcal{N}_{\mathbf{V}}} = \mathbf{diag}\{\mathcal{N}_0, \mathcal{N}_1, \dots, \mathcal{N}_{N-1}\}. \quad (2)$$

We next introduce $\bar{\gamma}_k = \frac{|H_k|^2}{N_k B_s}$ to denote the normalized signal to noise ratio (nSNR) at the k^{th} subchannel and have the nSNR vector $\bar{\boldsymbol{\gamma}} = [\bar{\gamma}_0 \ \bar{\gamma}_1 \ \cdots \ \bar{\gamma}_{N-1}]^T$. The nSNR vector can be arranged as the nSNR diagonal matrix

$$\begin{aligned} \Lambda_{\bar{\boldsymbol{\gamma}}} &= \mathbf{diag}\{\bar{\gamma}_0, \bar{\gamma}_1, \dots, \bar{\gamma}_{N-1}\} \\ &= \frac{\Lambda_{\mathbf{H}^2} \Lambda_{\mathcal{N}_{\mathbf{V}}}^{-1}}{B_s}. \end{aligned} \quad (3)$$

The number of bits allocated to the subcarriers will vary, according to (3). Also, a solution of the resource allocation problem will be optimal not only in the data-rate sense (properly allocating bits among subcarriers) but also in the power efficiency sense (distributing the available power, optimally, among the subcarriers, according to some criterion).

The optimal resource allocation in the sense of data-rate maximization applied to a LTI PLC channel result in $\Lambda_{\mathbf{b}^\circ}$ and $\Lambda_{\mathbf{p}^\circ}$ when we solve the resource allocation problem expressed by

$$\begin{aligned} &\max_{\Lambda_{\mathbf{p}}} [\text{Tr}(\Lambda_{\mathbf{b}})] \\ &\text{Subject to: } \text{Tr}(\Lambda_{\mathbf{p}}) \leq P_t, \end{aligned} \quad (4)$$

where P_t is the total power to be shared among the subcarriers; $\text{Tr}(\cdot)$ is the trace operator; $\Lambda_{\mathbf{b}} = \mathbf{diag}\{b_0, b_1, \dots, b_{N-1}\}$; $\Lambda_{\mathbf{p}} = \mathbf{diag}\{P_0, P_1, \dots, P_{N-1}\}$; b_k and P_k are the number of bits per dimension and power allocated to the k^{th} subcarrier;

$$\text{Tr}(\Lambda_{\mathbf{b}}) = \frac{1}{2} \text{Tr} \left[\log_2 \left(\mathbf{I}_N + \frac{\Lambda_{\boldsymbol{\gamma}}}{\Gamma} \right) \right], \quad (5)$$

in which $\Lambda_{\boldsymbol{\gamma}} = \Lambda_{\mathbf{p}} \Lambda_{\bar{\boldsymbol{\gamma}}}$ is the signal to noise ratio (SNR) matrix associated with the LTI PLC channel during T_c seconds; Γ is a gap that guarantees that the desired bit error rate (BER) will not be violated (e.g., $\Gamma_{dB} = 10 \log_{10}(\Gamma) = 8.8$ dB indicates a maximum BER constraint of 10^{-6} for square quadrature amplitude modulation (QAM)); and \mathbf{I}_N is an $N \times N$ identity matrix. It is worth stating that (4) can be implemented by a bit-loading technique based on water-filling algorithm, greedy algorithm or any variation of them. Water-filling algorithm achieves an optimal data-rate for transmission of $\Lambda_{\mathbf{b}^\circ} \in \mathbb{R}_+^{N \times N}$ bits while greedy algorithm offers an optimal data-rate on the sense of integer number of bits ($\Lambda_{\mathbf{b}^\circ} \in \mathbb{Z}_+^{N \times N}$).

Thereby, the optimal data-rate valid during T_c seconds is, as a result, obtained by

$$R^o = \frac{1}{T_{\text{OFDM}}} \text{Tr}(\Lambda_{\mathbf{b}^\circ}), \quad (6)$$

where $T_{\text{OFDM}} = T_{\text{sym}} + T_{\text{cp}}$ is the OFDM symbol duration, T_{sym} and T_{cp} denote the time intervals for the OFDM symbol and the cyclic prefix, respectively.

The time-varying nature of the PLC channel might require the data-rate and power allocated to each subchannel to be frequently updated (once at every T_c seconds, ideally), thus the computational complexity and the signaling overhead (exchange of control information to convey the channel state information and modulation parameters to all users sharing the same PLC network) increase. That can raise the computational complexity (mainly when the value of T_c is small, as in PLC case) to insurmountable levels. One approach to reduce the computational complexity, in this scenario, is to decrease the order of the optimization problem by grouping adjacent subcarriers and use the same resource allocation for all subcarriers in a group, as detailed in Section III.

III. THE PROPOSED TECHNIQUE

The nSNR in PLC systems has an important characteristic, there is a high connection among nSNR of adjacent subcarriers. Fig. 1 shows an example of nSNR in PLC systems, as can be seen there is a small variation of the nSNR between adjacent subcarriers.

The proposed resource allocation technique exploits this characteristic in order to decrease the order of the resource allocation problem and, thus, decrease the computational complexity of the bit-loading algorithms. However, in exchange, it obtains a suboptimal data-rate. Thus, there is a trade-off between computational complexity reduction and data-rate degradation.

The proposed technique groups adjacent subcarriers in sets and guarantees that all subcarriers in a same set transmit the same number of bits and using the same power. To do this, the proposed technique selects the subcarrier with the minimal nSNR of a group as a reference subcarrier of this group and executes the bit-loading algorithm just to the reference subcarrier. In other words, the bit-loading algorithm is executed for a smaller number of subcarriers and, consequently, it reduces the computational complexity of resource allocation. Finally, the other subcarriers of a group transmit the same number of bits using the same power that the reference subcarrier of

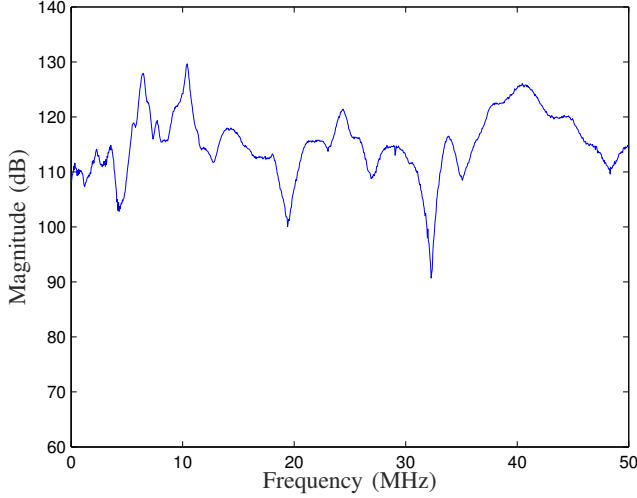


Fig. 1: An example of nSNR in PLC systems.

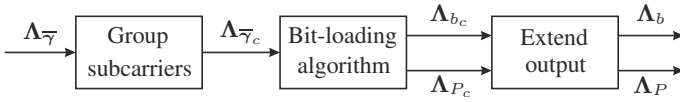


Fig. 2: Block diagram of the proposed technique.

this group. This procedure is illustrated in Fig. 2 and its the mechanism is summarized as follows:

- Step #1 Define L_c as the length of the sets.
- Step #2 Group L_c adjacent subcarriers in sets. Thus, this step generates $N_c = \lceil N/L_c \rceil$ (where $\lceil x \rceil = \min\{m \in \mathbb{Z} | m \geq x\}$). Note that the length of the last one can be smaller than L_c .
- Step #3 Obtain $\Lambda_{\bar{\gamma}_c} = \text{diag}\{\bar{\gamma}_{c,0}, \bar{\gamma}_{c,1}, \dots, \bar{\gamma}_{c,N-1}\}$, where $\bar{\gamma}_{c,i}$ is the minimal nSNR of the i^{th} set.
- Step #4 Solve the resource allocation problem given by

$$\begin{aligned} & \max_{\Lambda_{P_c}} [\text{Tr}(\Lambda_{b_c})] \\ & \text{Subject to: } \text{Tr}(\Lambda_{P_c}) \leq P_{t,c}, \end{aligned} \quad (7)$$

where $P_{t,c} = P_t N_c / N$ is the total power to be shared among the sets; $\Lambda_{b_c} = \text{diag}\{b_{c,0}, b_{c,1}, \dots, b_{c,L_c-1}\}$ and $\Lambda_{P_c} = \text{diag}\{P_{c,0}, P_{c,1}, \dots, P_{c,L_c-1}\}$ are the resource allocation obtained by solving (7); $b_{c,i}$ and $P_{c,i}$ are the number of bits per dimension and power allocated to the i^{th} set;

$$\text{Tr}(\Lambda_{b_c}) = \frac{1}{2} \text{Tr} \left[\log_2 \left(\mathbf{I}_{N_c} + \frac{\Lambda_{\bar{\gamma}_c}}{\Gamma} \right) \right]; \quad (8)$$

$\Lambda_{\bar{\gamma}_c} = \Lambda_{P_c} \Lambda_{\bar{\gamma}_c}$ is the SNR matrix associated with the sets; and \mathbf{I}_{N_c} is an $N_c \times N_c$ identity matrix.

- Step #5 Extend the resource allocation output for all subcarriers of the set. In other words, obtained $\Lambda_b = \text{diag}\{b_0, b_1, \dots, b_{N-1}\}$ and $\Lambda_P = \text{diag}\{P_0, P_1, \dots, P_{N-1}\}$ where b_k and P_k are the number of bits per dimension transmitted on the k^{th} subcarrier and the power used to transmit it. Note that

$b_k = b_{c,i}$ and $P_k = P_{c,i}$ if the k^{th} subcarrier belongs to the i^{th} set.

Note that the data-rate obtained by the use of the proposed technique is

$$\begin{aligned} R_c &= \frac{1}{T_{OFDM}} \text{Tr}(\Lambda_b) \\ &= \frac{L_c}{T_{OFDM}} \text{Tr}(\Lambda_{b_c}), \end{aligned} \quad (9)$$

where L_c is the length of each chunk. It is worth mentioning that L_c drives the trade-off between computational complexity reduction and data-rate degradation, because as high the L_c than more subcarriers are grouped and, consequently, the computational complexity reduces but the data-rate degradation increases. In order to implement this technique, it is important to statistically analyse this trade-off to choose the best L_c that reduces the computational complexity with an acceptable data-rate degradation.

In order to compare the proposed technique performance for different L_c we will use two parameters. The first one is used to quantify the data-rate degradation due to the use of the proposed technique. It is the data-rate loss ratio expressed by

$$\eta = 1 - \frac{R_c}{R^o}, \quad (10)$$

where R^o is the optimal data-rate that can be obtained when $L_c = 1$ (see (6)). The second parameter quantifies the gain in terms of computational complexity that can be attained when is used the proposed technique. It is the computational complexity reduction ratio and is expressed by

$$\rho \triangleq 1 - \frac{1}{L_c}. \quad (11)$$

IV. PERFORMANCE ANALYSES

The purpose of this section is to present the performance evaluations of the proposed technique in terms of data-rate loss ratio (η) and computational complexity reduction ratio (ρ). All analyses are based on measured in-home PLC channels and additive noises that were measured in seven residences, covering houses and apartments. The total number of measured in-home PLC channels is 7,117. This data set was obtained during a measurement campaign carried out in the city of Juiz de Fora, Brazil, by using the measurement setup and methodology discussed in [7] and [8], respectively. For measuring in-home PLC channels, we followed [8] to choose a sampling frequency equal to 200 MHz; frequency band from 1.7 up to 100 MHz; a hermitian symmetric OFDM (HS-OFDM) scheme [9] with lengths of OFDM symbol and cyclic prefix equal to $N = 4096$ and $L_{CP} = 512$, respectively; and frequency resolution equal to 48.8 kHz. All numerical analyses address the frequency bands from 1.7 MHz to 30 MHz, 50 MHz, and 100 MHz, named W_{30} , W_{50} and W_{100} , respectively.

Table I presents a trade-off between data-rate loss ratio (η) and computational complexity reduction ratio (ρ) when the proposed technique is applied for the frequency band W_{30} . This table makes use of the water-filling technique to solve the resource allocation problem and $\Gamma_{dB} = 10 \log_{10} \Gamma = 8.8$ dB. Also, $P(\eta > x)$ indicates the probability of data-rate loss ratio

higher than x . Note that if $L_c = 1$, then the proposed technique is equal to the optimal resource allocation because sub-carrier grouping do not occurs. Note also that L_c drives the trade-off between η and ρ . In other words, as higher L_c , there is a tendency of data-rate loss ratio and computational complexity reduction ratio to grow. As an example, $L_c = 8$, then $P(\eta > 0.05) = 0.424$ and $\rho = 0.88$, but if $L_c = 4$, then $P(\eta > 0.05) = 0.086$ and $\rho = 0.75$.

TABLE I: Computational complexity reduction and data-rate loss ratios achieved by the proposed technique when it is applied at frequency band W_{30} .

L_c	1	2	4	8	16
ρ	0	0.5	0.75	0.88	0.94
$P(\eta > 0.02)$	0	0.108	0.558	0.939	1.0
$P(\eta > 0.05)$	0	0.008	0.086	0.424	0.809
$P(\eta > 0.1)$	0	0	0.008	0.061	0.326

Table II presents a trade-off between η and ρ when the proposed technique is applied for the frequency band W_{50} . This table makes use of the same constraint of Table I. Note that as higher L_c , there is a tendency of data-rate loss ratio and computational complexity reduction ratio to grow. As an example, $L_c = 8$, then $P(\eta > 0.05) = 0.270$ and $\rho = 0.88$, but if $L_c = 4$, then $P(\eta > 0.05) = 0.071$ and $\rho = 0.75$.

TABLE II: Computational complexity reduction and data-rate loss ratios achieved by the proposed technique when it is applied at frequency band W_{50} .

L_c	1	2	4	8	16
ρ	0	0.5	0.75	0.88	0.94
$P(\eta > 0.02)$	0	0.086	0.478	0.895	0.999
$P(\eta > 0.05)$	0	0.011	0.071	0.270	0.765
$P(\eta > 0.1)$	0	0	0.012	0.057	0.182

Table III presents a trade off between η and ρ when the proposed technique is applied for the frequency band W_{100} . This table also makes use of the same constraint of Table I. Once again, as higher L_c , as higher the data-rate loss ratio and the computational complexity reduction ratio are. As an example, $L_c = 8$, then $P(\eta > 0.05) = 0.545$ and $\rho = 0.88$, but if $L_c = 4$, then $P(\eta > 0.05) = 0.135$ and $\rho = 0.75$. When the proposed technique performance is compared between the three frequency bands, we note that L_c drives the trade-off between η and ρ in all frequency bands. Also, although the proposed technique has different results for each frequency band, we noted that $L_c = 8$ (0.4 MHz) results in a good trade-off (for all studied frequency bands) because it presents a reasonable balance between computational complexity reduction ratio and data-rate loss ratio.

TABLE III: Computational complexity reduction and data-rate loss ratios achieved by the proposed technique when it is applied at frequency band W_{100} .

L_c	1	2	4	8	16
ρ	0	0.5	0.75	0.88	0.94
$P(\eta > 0.02)$	0	0.242	0.748	0.999	1.0
$P(\eta > 0.05)$	0	0.065	0.135	0.545	0.907
$P(\eta > 0.1)$	0	0.001	0.063	0.101	0.408

V. CONCLUSION

This work investigated the grouping of adjacent subcarriers in order to reduce the computational complexity of resource allocations techniques in OFDM-based PLC systems. The proposed technique was presented and numerical results of the trade-off between computational complexity reduction and data-rate degradation were showed. The discussed numerical analyses were based on a data set obtained in a measurement campaign carried out in in-home PLC channels by taking into account the frequency bands 1.7-30, 1.7-50, and 1.7-100 MHz.

Additionally, we suggested that grouping eight adjacent subcarriers with the same resource allocation is a good practice because it presents a reasonable balance between computational complexity reduction and data-rate degradation. Overall, we showed that the proposed technique can easily trade computational complexity reduction with data-rate degradation and, as a consequence, it can be useful to design low-cost OFDM-based PLC transceivers.

ACKNOWLEDGMENT

The authors would like to thanks FINEP, INERGE, CNPq, CAPES, FAPEMIG, CEMIG, P&D ANEEL, and Smarti9 for their financial support.

REFERENCES

- [1] S. Morosi, D. Marabissi, E. Del Re, R. Fantacci, and N. Del Santo, "A rate adaptive bit-loading algorithm for in-building power-line communications based on DMT-modulated systems," *IEEE Trans. on Power Delivery*, vol. 21, no. 4, pp. 1892–1897, Oct. 2006.
- [2] I. Isha, P. Rana, and R. Saini, "Performance of different bit loading algorithms for OFDM at PLC channel," in *Proc. 2nd International Conference on Advanced Computing Communication Technologies*, Jan. 2012, pp. 486–489.
- [3] M. A. Tunc, E. Perrins, and L. Lampe, "Optimal LPTV-aware bit loading in broadband PLC," *IEEE Trans. on Communications*, vol. 61, no. 12, pp. 5152–5162, Dec. 2013.
- [4] N. Papandreou and T. Antonakopoulos, "Fair resource allocation with improved diversity performance for indoor power-line networks," *IEEE Trans. on Power Delivery*, vol. 22, no. 4, pp. 2575–2576, Oct. 2007.
- [5] *IEEE Standard for Low-Frequency (less than 500 kHz) Narrow-band Power Line Communications for Smart Grid Applications (IEEE Std. 1901.2)*, IEEE Communications Society Std., Oct. 2013.
- [6] A. M. Tonello, F. Versolatto, and A. Pittolo, "In-home power line communication channel: Statistical characterization," *IEEE Trans. on Communications*, vol. 62, no. 6, pp. 2096–2106, Jun. 2014.
- [7] G. R. Colen, C. A. G. Marques, T. R. Oliveira, F. P. V. de Campos, and M. Ribeiro, "Measurement setup for characterizing low-voltage and outdoor electric distribution grids for PLC systems," in *Proc. IEEE PES Conference On Innovative Smart Grid Technologies Latin America*, Apr. 2013, pp. 1–5.
- [8] T. R. Oliveira, C. A. G. Marques, W. A. Finamore, S. L. Netto, and M. V. Ribeiro, "A methodology for estimating frequency responses of electric power grids," *Journal of Control, Automation and Electrical Systems*, vol. 25, no. 6, pp. 720–731, Sept. 2014.
- [9] M. V. Ribeiro, G. R. Colen, F. P. V. de Campos, Z. Quan, and H. V. Poor, "Clustered-orthogonal frequency division multiplexing for power line communication: When is it beneficial?" *IET Communications*, vol. 8, no. 13, pp. 2336–2347, Sept. 2014.