

# On the Use of Precoding in FBMC/OQAM Systems

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**Abstract**—Linear precoding can be applied to multicarrier systems to minimize the effect that a deep fade in one of the subchannels can have on the overall bit error ratio when the transmitter does not have channel state information (CSI). But for filterbank multicarrier (FBMC) systems, which were proposed to overcome the poor spectral characteristics and bandwidth/energy waste caused by the use of the cyclic prefix, there are very few proposals of precoded systems in the literature. This paper proposes a precoded SISO (single-input single-output) filterbank multicarrier system with zero-forcing (ZF) and minimum mean square error (MMSE) equalization. Results show that in a well-equalized system, the Monte Carlo simulation results match the ones provided by the BER approximation presented in this paper. However, in systems with residual interference, the Monte Carlo results drift from the approximation at high SNR values, with the error performance from precoded FBMC systems being more affected by the unequalized interference.

## I. INTRODUCTION

Modern wireless communication systems require ever increasing transmission rates; these systems have to transmit and receive data through heavily multipath communication channels, which cause severe intersymbol interference (ISI) in the received data stream. Multicarrier systems have been proposed to overcome the inherent difficulty of equalizing the received data through these channels. If the transmitter has knowledge of the channel state information (CSI) in a multicarrier system, adaptive coding and modulation can be employed to transmit more data on the subcarriers with the highest SNR, discarding the ones who are not fit to transmit data with a low error probability. However, this knowledge can be hard to acquire precisely, since the channel in a wireless environment changes rapidly. For the situations when the transmitter does not have the CSI, linear precoding [1] can be employed to minimize the effect that a deep fade in one of the subchannels can have on the overall bit error ratio.

Precoding proposals for wireless multicarrier systems with only one antenna in the transmitter and in the receiver (SISO) can be found in [2], [3], but they need CSI at the transmitter. The single carrier system with cyclic prefix (SC-CP) [4] is a DFT-based transceiver with a channel independent unitary linear precoding matrix. It can be seen as a precoded OFDM system with the DFT matrix as a precoder. Precoded multicarrier systems are also being used for the uplink in multiuser systems (as a modified form of OFDMA, called SC-FDMA

[5]), due to its lower PAPR properties and lower sensibility for (sub)carrier frequency offset.

OFDM/QAM systems, which are the widely used multicarrier ones, use a rectangular window to separate their subchannels. This window has poor spectral and time characteristics, often requiring post-filtering to conform to a spectral mask and making the use of a cyclic prefix mandatory to eliminate the ISI. These drawbacks can be eliminated with the usage of pulses better localized in time and in frequency to separate the subchannels. The system using these filters, called Filterbank Multicarrier with Offset QAM (FBMC/OQAM) [6] is more efficient spectrally and can discard the cyclic prefix, gaining more efficiency in bandwidth and power.

The only proposal so far in the literature for a precoded FBMC system in the literature can be found in [7]. In this proposal, classical multicarrier, precoded multicarrier and pure single carrier transmissions can be done simultaneously, each in its group of subchannels, due to the high subchannel selectivity inherent to the FBMC systems.

This paper presents a precoded SISO filterbank multicarrier system, using linear zero-forcing (ZF) and minimum mean square error (MMSE) equalization. We provide an BER approximation for the best-case scenario and compare it with Monte Carlo simulations from FBMC systems in different situations. The paper is divided in the following sections. Section II presents an introduction to filterbank multicarrier systems. The following section presents an analysis on the error behaviour for precoded multicarrier systems. Simulation results for FBMC systems and a comparison with the BER approximation provided earlier in the paper are presented in Section IV and the concluding remarks in Section V.

## II. FILTERBANK MULTICARRIER SYSTEMS

A computationally efficient way to implement a large number of well conformed in time and frequency filters to separate the subchannels is the usage of a filterbank, which can be implemented through the filters' polyphasic decomposition associated with the fast Fourier transform (FFT). This way, the synthesis (SFB) and the analysis (AFB) filterbanks implement the systems' modulator and the demodulator, respectively. The synthesis filterbank associates the polyphase network (PPN), which is the set of the so-called prototype filters, with the inverse fast Fourier transform (IFFT) to divide the bandwidth in  $M$  subchannels for transmission. The inverse operation in reversed order is realized in the receiver by the analysis filterbank to recover the transmitted data.

This work has been partially sponsored by National Council for Scientific and Technological Development (CNPq) and CAPES/COFECUB project (544/07).

However, the usage of these efficient filters to separate the subchannels instead of the rectangular window imposes the use of a real modulation to maintain the orthogonality between subcarriers, since the transmultiplexer's impulse response is non-unitary due to interference from the neighboring subchannels and time instants. Thus, the FBMC system has to transmit a real symbol every half OFDM symbol duration, yielding to the so-called FBMC/OQAM system.

The baseband discrete signal at a time instant  $k$  at the output of a synthesis filterbank in a FBMC/OQAM system is expressed by

$$s[k] = \sum_{m=0}^{M-1} \sum_n a_{m,n} q_{m,n}[k], \quad (1)$$

where  $a_{m,n}$  are real OQAM symbols with energy  $E_s$  and

$$q_{m,n}[k] = q \left[ k - n \frac{M}{2} \right] e^{-j \frac{2\pi}{M} n (k - \frac{L-1}{2})} e^{-j \phi_{m,n}}, \quad (2)$$

with  $q[k]$  being the impulse response of a real and symmetric prototype filter (with unit energy and length  $L$ ),  $M$  the number of subchannels,  $m$  the subchannel index,  $n$  the time index for the OQAM symbol and  $\phi_{m,n} = \phi_0 + \frac{\pi}{2}(m+n) \pmod{\pi}$ , with an arbitrary  $\phi_0$ . The filter length  $L$  can be expressed by  $KM$ , where  $K$  is the overlapping factor [8].

The filters  $q_{m,n}$  are only orthogonal in the real field, that is

$$\Re \left\{ \sum_k q_{m,n}[k] q_{m',n'}^*[k] \right\} = \delta_{m,m'} \delta_{n,n'}, \quad (3)$$

where  $\delta_{i,j}$  is the Kronecker delta (1 if  $i = j$  and 0 otherwise). This condition implies that, even in a transmission with a perfectly equalized channel and no channel noise, there will be residual imaginary intercarrier/intersymbol interference (ICI/ISI).

In OFDM/QAM systems, a cyclic prefix with length greater than the channel impulse response guarantees that a one-tap per subchannel equalizer will completely eliminate the ISI. Since the FBMC/OQAM systems do not use the cyclic prefix, if the transmission channel is highly selective in frequency equalizers with multiple taps per subchannel may be necessary. These equalizers are fractionally spaced (because the system operates at twice the symbol rate) and can deal with residual misalignments and severe channel impairments [9].

### III. BER ANALYSIS FOR PRECODED FILTERBANK MULTICARRIER SYSTEMS

The block diagram of a precoded FBMC/OQAM system is presented in Figure 1. In this diagram,  $\mathbf{W}$  is the normalized  $M \times M$  DFT matrix,  $\mathbf{T}$  is the precoding matrix,  $\eta[k]$  is the AWG noise with variance  $\sigma_\eta^2$  and  $\mathbf{H}$  is the channel frequency response matrix, with  $H_0, H_1, \dots, H_{M-1}$  being the diagonal elements of the matrix, corresponding to the channel frequency response in each subchannel. The corresponding impulse response from the channel is expressed by  $h[k]$ .

We can write the received signal  $r[k]$  as

$$r[k] = h[k] * s[k] + \eta[k] \quad (4)$$

$$= v[k] + \eta[k], \quad (5)$$

where  $s[k]$  is the desired part of the received signal and  $\eta[k]$  is the noise. After the demodulation the received signal can be expressed as

$$r_{m,n} = \Re \left\{ \sum_{k=1}^L q_{m,n}^*[k] v[k] + \eta_{m,n} \right\} \quad (6)$$

$$= \hat{r}_{m,n} + \Re \{ \eta_{m,n} \}, \quad (7)$$

with  $\hat{r}_{m,n}$  being the useful signal and

$$\eta_{m,n} = \sum_k \eta[k] q \left[ k - n \frac{M}{2} \right] e^{j \frac{2\pi}{M} m (k - \frac{L-1}{2})} e^{j \phi_{m,n}}. \quad (8)$$

Since we recover  $a_{m,n}$  thanks to the real orthogonality,  $\eta_{m,n}$  can be rewritten as follows

$$\eta_{m,n} = \sum_{k=n \frac{M}{2}}^{L-1+n \frac{M}{2}} \eta[k] q \left[ k - n \frac{M}{2} \right] e^{-j \frac{2\pi}{M} m (k - \frac{L-1}{2})} e^{-j \phi_{m,n}}. \quad (9)$$

$\eta_{m,n}$  is a linear combination of Gaussian random variables. So, it remains a Gaussian random variable with variance

$$\sigma_{\eta_{m,n}}^2 = \sigma_\eta^2 \sum_{k=n \frac{M}{2}}^{L-1+n \frac{M}{2}} q \left[ k - n \frac{M}{2} \right]^2 = \sigma_\eta^2, \quad (10)$$

since the normalization of the prototype filters implies

$$\sum_{k=n \frac{M}{2}}^{L-1+n \frac{M}{2}} q \left[ k - n \frac{M}{2} \right]^2 = 1. \quad (11)$$

Therefore, if there is no ICI and ISI the noise variance will not change neither after the filterbank nor after the FFT, since  $\mathbf{W}$  is an unitary matrix.

For FBMC/OQAM systems, it is often necessary use an equalizer per subchannel with more than one tap in order to compensate the ICI/ISI. However, in the following equations we made the assumption that the channel is sufficiently flat in order to necessitate only a one tap equalizer; the analytical expressions will be approximations when the channel is not flat. With these hypothesis, we can calculate the SNR. The equalizer can be represented by a multiplicative coefficient  $\psi_m$  at the output of the AFB, with  $\psi_m = \frac{1}{H_m}$  for a ZF equalizer and  $\psi_m = \frac{\gamma H_m^*}{1 + \gamma |H_m|^2}$  for a MMSE one.

With a generic linear precoding matrix  $\mathbf{T}$ , the noise variance  $\sigma_m^2$  at the output of the equalizer of the  $m$ -th subchannel is:

$$\sigma_m^2 = \sigma_\eta^2 \sum_{k=0}^{M-1} |t_{k,n}|^2 |\psi_k|^2, \quad (12)$$

where  $t_{m,n}$  denotes the  $(m, n)$ th element of the precoding matrix  $\mathbf{T}$  and  $\gamma = \frac{E_s}{\sigma_\eta^2}$  the SNR.

Thus, the SNR  $\beta = \frac{E_s}{\sigma_m^2}$  at the  $m$ -th subchannel for ZF equalization is:

$$\beta_m = \frac{\gamma}{\sum_{k=0}^{M-1} \frac{|t_{k,n}|^2}{|H_k|^2}}. \quad (13)$$

When using MMSE equalization it is more appropriate to use the signal to interference and noise ratio (SINR), since

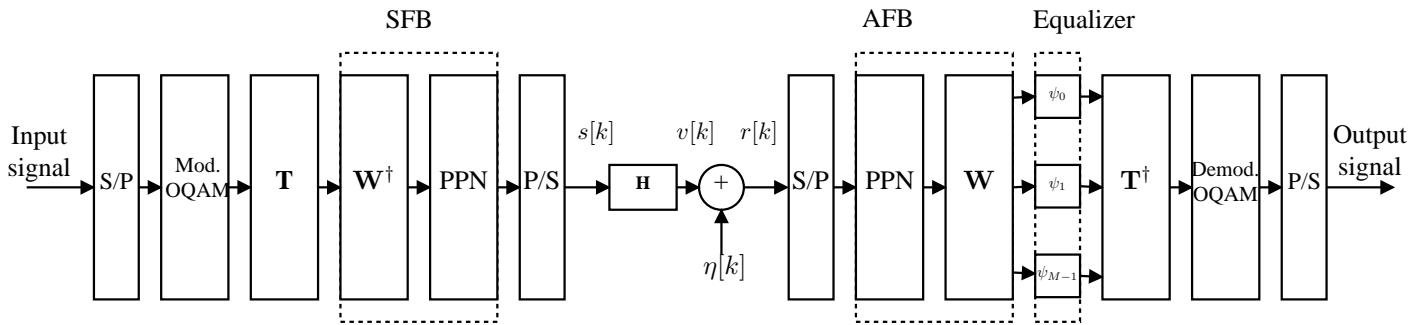


Fig. 1. A precoded FBMC/OQAM system

there is also interference from the other subchannels alongside the noise. It can be expressed by [10]

$$\beta_m = \frac{\sum_{k=0}^{M-1} |t_{k,n}|^2 \gamma |H_k|^2}{\sum_{k=0}^{M-1} |t_{k,n}|^2}. \quad (14)$$

So, the BER  $P$  at the  $m$ -th subchannel is

$$P_m = Q\left(\sqrt{\beta_m}\right), \quad (15)$$

where  $Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-\frac{t^2}{2}} dt$ , and the overall BER for the multicarrier system can be expressed as:

$$P = \frac{1}{M} \sum_{k=0}^{M-1} Q\left(\sqrt{\beta_k}\right). \quad (16)$$

Lin and Phoong [11] state that a channel independent precoding matrix has to satisfy the following condition so that the noise variance is the same in every subchannel:

$$|t_{m,n}| = \frac{1}{\sqrt{M}}, 0 \leq m, n \leq M-1. \quad (17)$$

To satisfy the condition imposed by Equation (17), we can use the DFT matrix  $\mathbf{W}$  or the Hadamard matrix  $\mathbf{H}$ . The Hadamard matrix as a precoder in a multicarrier system is attractive because its implementation requires no multiplications; only additions and subtractions, increasing only slightly the overall computational complexity.

With the same noise variance in every subchannel, the subchannel BERs are also the same. For Hadamard or DFT precoded systems ( $\mathbf{T} = \mathbf{H}$  or  $\mathbf{W}$ ), since the noise variance is the same for every subchannel, the SNR  $\beta^{Pre}$  for each subchannel in a precoded system also can be considered the same, and it can be expressed as:

$$\beta^{Pre} = \frac{1}{\frac{1}{M} \sum_{m=0}^{M-1} \frac{1}{\gamma |\psi_m|^2}}, \quad (18)$$

and the overall uncoded BER  $P^{Pre}$  for the precoded multicarrier system is:

$$P^{Pre} = Q\left(\sqrt{\beta^{Pre}}\right). \quad (19)$$

Let us remember that, when using a linear precoding transform in a multicarrier system, each subcarrier transmits a combination of all the others, and this combination is only undone in the receiver after equalization. As we know, the transmission

of this combination makes the system more resistant to deep fading in the transmission channel. But when there is a null in the channel and/or excess ISI/ICI with ZF equalization, the noise will be amplified; and since all the subcarriers' contents are still combined, this noise amplification will affect all subcarriers, causing a drop in the error performance. Only when the SNR is high the benefits of the subcarrier spreading will appear when using ZF equalization (in a completely equalized system). On the other hand, when using MMSE equalization, the noise amplification effect will not appear, and the gain provided by spreading happens throughout the entire SNR range.

Again, this analysis does not take into account the imperfections that are often present in a FBMC system, such as ICI and ISI. The next section will present results to verify if the approximation presented here can be used to FBMC systems transmitting through time/frequency-selective channels.

#### IV. SIMULATION RESULTS

In this section, simulation results of the error performance from precoded filterbank multicarrier systems are presented for uncoded systems and different number of coefficients in the subchannel equalizers. For all the simulations, the ITU-T Vehicular A channel model is used. Symbols are drawn from a OQPSK constellation. Channel estimation is assumed to be perfect, channel fading is considered to be quasistatic (time-invariant during each transmitted frame) and other system imperfections are not taken into account. Results were averaged over 4000 frames (each with 53 symbols), with a independent channel realization for each one. The prototype filter proposed in [8] is used for the analysis and synthesis filterbanks. The multiple-tap per subchannel equalizers are the ones presented in [12], whose project is based on the frequency sampling approach, interpolation and the IFFT to calculate the equalizers' coefficients. The precoding matrix  $\mathbf{T}$  employed in the systems using precoding is the Hadamard one.

Figure 2 presents the simulation results comparing the results obtained from the Monte Carlo simulations to the BER approximation presented in Equation (16) for conventional and precoded systems, both using ZF and MMSE equalization, for a system using 1024 subchannels and a 3-tap per subchannel equalizer. In this case, we can consider the subchannels to have flat fading, and it is possible to see that the Monte Carlo simulation results are very close to the ones provided by the

approximation. We remember here that when the system is not using precoding, the error performance from the systems using ZF and MMSE equalization is the same. However, when precoding is applied, the system using ZF equalization will only outperform the non-precoded system at very high SNR values; on the other hand, the system using MMSE equalization will outperform the non-precoded one throughout the entire SNR range [11].

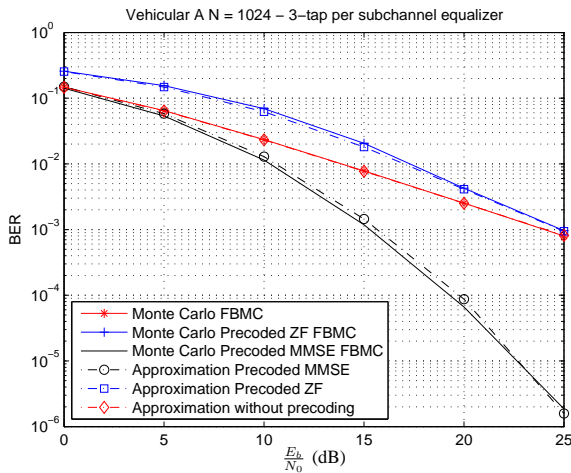


Fig. 2. Comparison between approximation and Monte Carlo simulation results using 1024 subcarriers and the Vehicular A channel model

Figure 3 compares the results from the Monte Carlo simulations to the BER approximation in Equation (16), but now for a system using 128 subchannels. 1-tap and 3-tap per subchannel equalizers are used. For this case, even the multiple-tap equalizer is not enough to completely eliminate the ISI and ICI from the received data stream. It is possible to see that in a low SNR range, the Monte Carlo simulation results are faithful to the approximation; however, in higher signal-to-noise ratios, their results drift from the BER approximation, due to this remaining unequalized interference. The results from the systems using a 1-tap per subchannel equalizer are much farther from the approximation than the ones using a 3-tap per subchannel equalizer, due to its worse equalization performance.

### V. CONCLUDING REMARKS

We have presented a precoded filterbank multicarrier system without channel state information at the transmitter. We have derived analytical BER performances for FBMC systems. These analytical expressions are approximations with quite good precision when the channel is moderately frequency selective, and they are still valid for highly selective channels for a SNR lower than 15 dB. It is possible to see that the residual unequalized ISI and ICI from imperfect equalization causes a larger effect in the error performance in the precoded systems than in the non-precoded ones. Future work will be carried out for non-perfect synchronization and channel estimation cases and in the development of new equalizers to compensate the residual interference.

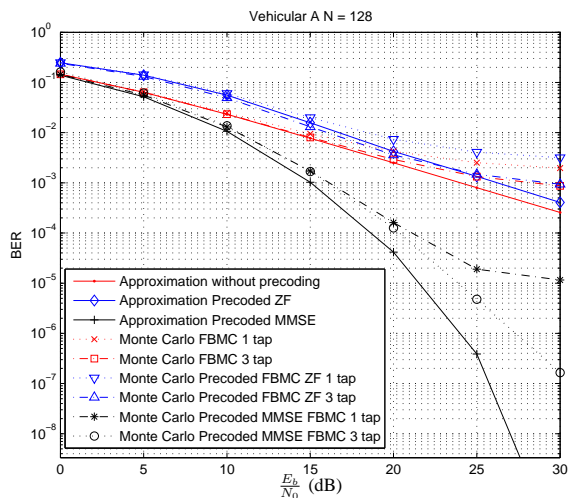


Fig. 3. Comparison between theoretical and Monte Carlo simulation results using 128 subcarriers and the Vehicular A channel model

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