

On the Impact of the Number of Interferers and Channel Estimation Errors in SC-FDE SIMO IB-DFE Systems

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Abstract—In this work, we analyze the effect of channel estimation errors on the bit error rate (BER) of a novel linear frequency domain iterative equalizer for SIMO single carrier systems with frequency domain equalization under the effect of multiple interferences, which takes the channel estimation errors into account in the equalization. It was found out that the proposed equalizer outperforms the iterative structure which does not take into account the channel estimation error process. Simulation results showed that this equalizer presents a better error performance when compared to its conventional version.

Index Terms—SC-FDE systems, channel estimation errors, SIMO systems, iterative equalization.

I. INTRODUCTION

Single carrier systems using frequency domain equalization (SC-FDE) [1] have the advantage of a reduced peak to average power ratio (PAPR) with respect to Orthogonal Frequency Division Multiplexing (OFDM) systems, while maintaining their low complexity frequency domain equalization. This makes them more suitable when compared to multicarrier systems for mobile uplink transmissions, which require energy efficient power amplifiers in user terminals in order to improve battery life.

To further exploit the available diversity and improve the error performance in terms of bit error rate (BER) of these systems, multiple antennas can be employed in the receiver. This scheme, with an antenna on the transmitter and several on the receiver, which is called a single input multiple output (SIMO), can be suitable for the uplink, where low-cost terminals are the transmitter and, e.g., a base station is the receiver.

In addition to the intersymbol interference (ISI) due to the distorting effects of the channels, in multiuser communication systems co-channel interference (CCI) appears as an additional factor, limiting system performance [2]. This is due to frequency reuse, where the same carrier frequency is used in several neighboring cells [3]. Thus, in addition to the desired signal, other signals in the same frequencies by unwanted transmitters arrive at the receiver and lead to a performance drop [4].

To eliminate the harmful effects of such interference, joint ISI and CCI suppression schemes are required for

good system performance. Iterative block decision feedback equalization (IB-DFE) can accomplish this task. The IB-DFE structure is an iterative DFE equalizer, where both filters are in the frequency domain [5]. In this scheme, the symbols already equalized are used to improve the reliability of the detected ones at each iteration. It was seen that a small number of iterations is needed to reach a considerable performance advantage with respect to standard linear systems and hybrid decision-feedback ones [6]. In [7], the authors propose the use of this iterative structure in SC-FDE systems in order to deal with the co-channel interference. The results showed that the proposed receiver can clearly outperform a conventional IB-DFE structure.

The channel estimator in these systems receiver can often be imprecise, having a known error variance. This error can affect the error performance obtained with the equalizer, by rendering incorrect symbol estimates. The effect of channel estimation errors in SC-FDE systems using linear iterative equalization was dealt with in [8]. It was seen that if the equalizer does not take the channel estimation error variance into account the error performance can be severely degraded; a high irreducible error floor appears at high SNRs in this case.

The goal of this paper is to propose a novel IB-DFE equalizer for SIMO SC-FDE systems that takes into account channel estimation errors in the estimation process. We will also present an analysis of how the number of receiver antennas impact the number of interferences that this system can cope with an acceptable BER. It was found out that the proposed structure outperforms the one that does not take into account the channel estimation error in the equalization process. Moreover, increasing the number of receiver antennas allows the system to operate under the effect of more interferences with a low BER.

The remainder of this paper is organized as follows: Section II presents the system model considered in this paper and the derivation of the SIMO IB-DFE equalizer that takes into account the channel estimation error variance in this process. Simulation results on the effect of these errors and of the number of the interferers and receiver antennas are shown and analysed in Section III. Finally,

the concluding remarks are presented in Section IV.

In this paper, the vectors are represented by bold lowercase letters, while bold capital letters denote matrices. Time domain elements have a tilde appended to them. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ denote, respectively, conjugate, transpose and Hermitian operations. The identity matrix of size $N \times N$ is denoted by \mathbf{I}_N . The trace of a matrix \mathbf{M} is expressed by $\text{Tr}[\mathbf{M}]$. Finally, the mathematical expectation is $E\{\cdot\}$.

II. SYSTEM MODEL

We consider the system model shown in Fig.1. On the transmitter side, the information block $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N]^T$ has size N , where \tilde{s}_i are symbols with power σ_s^2 considered as unity. In order to eliminate interference between blocks, a cyclic prefix of size greater than or equal to the channel length is inserted into the block before transmission.

This transmitted signal will be corrupted by K independent interference signals represented by $\tilde{\mathbf{s}}_{ik}$ and with power σ_i^2 , which are considered to be parallel cochannel transmissions with the same structure detailed above. Thus, the signal-to-interference ratio (SIR) can be expressed as $\frac{\sigma_s^2}{\sigma_i^2}$. Lastly, uncorrelated additional white Gaussian noise (AWGN) $\tilde{\mathbf{n}}$ with zero mean and variance σ_n^2 also contaminates the transmitted signal.

The transmitted signal will pass through a channel with an impulse response $\tilde{\mathbf{h}} = [\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_L]^T$ with length L in order to reach the j -th antenna of the receiver.

Due to the insertion of the cyclic prefix with length greater than the one from the time domain impulse response of the channel, the channel between the transmitter and each receiver antenna may be modeled as a circulant matrix, with its first column containing the channel impulse response $\tilde{\mathbf{h}}$ appended by $N - L$ zeros. Therefore, this matrix that we call \mathbf{H}' can be decomposed as follows:

$$\mathbf{H}' = \mathbf{W}^H \mathbf{H} \mathbf{W}, \quad (1)$$

where \mathbf{W} is the normalized discrete Fourier transform (DFT) matrix with size $N \times N$ and \mathbf{H} is a $N \times N$ diagonal matrix corresponding to the channel frequency response $\tilde{\mathbf{h}}$. The same procedure is done for the interference channels, resulting in $\mathbf{W}^H \mathbf{H}_{ik} \mathbf{W}$, where \mathbf{H}_{ik} corresponds to the diagonal matrix $N \times N$ of the frequency response of the k -th interfering channel.

In this case we do not consider that the receiver will have perfect channel estimation, i.e., an accurate version of \mathbf{H} . Instead, we consider that after removal of the cyclic prefix the receiver will have a distorted version $\bar{\mathbf{H}}$ as the channel estimate in the frequency domain, which can be expressed as

$$\bar{\mathbf{H}} = \mathbf{H} + \mathbf{H}_e, \quad (2)$$

where \mathbf{H}_e is a matrix representing the estimation error for the desired information channel. \mathbf{H}_e is of size $N \times N$, with its main diagonal represented by uncorrelated circular complex Gaussian random variable with zero mean and a known variance σ_e^2 . The same phenomenon is considered

regarding the imprecise estimation of the interferer channels $\bar{\mathbf{H}}_{ik}$, expressed by

$$\bar{\mathbf{H}}_{ik} = \mathbf{H}_{ik} + \mathbf{H}_{ie}, \quad (3)$$

where \mathbf{H}_{ie} is also a matrix that represents the estimation error of size $N \times N$, but now for the interfering channels. For the sake of simplicity, we consider that the channel estimation error variance for all the interferers is the same, i.e., the main diagonal of \mathbf{H}_{ie} for any k -th interferer channel is represented by uncorrelated Gaussian random variables with zero mean and a known variance σ_{ie}^2 . Thus, since the receiver has an imperfect channel estimate and considering that the receiver uses N_r antennas, the signal after the FFT \mathbf{r} can be expressed as

$$\mathbf{r} = \sum_{j=1}^{N_r} \left[\bar{\mathbf{H}}_j \mathbf{s} - \mathbf{H}_{e,j} \mathbf{s} + \sum_{k=0}^K (\bar{\mathbf{H}}_{ik,j} \mathbf{s}_{ik} - \mathbf{H}_{e,k,j} \mathbf{s}_{ik}) + \mathbf{n}_j \right], \quad (4)$$

where $\mathbf{s} = \mathbf{W}\tilde{\mathbf{s}}$, $\mathbf{s}_{ik} = \mathbf{W}\tilde{\mathbf{s}}_{ik}$ and $\mathbf{n} = \mathbf{W}\tilde{\mathbf{n}}$ are the transmitted information signal, the k -th interfering signal and the noise respectively, all in the frequency domain.

Through an iterative structure, equalization and interference suppression is done on the receiver side under the minimum mean square error (MMSE) criterion. The proposed iterative equalizer consists in a feedforward filter and in a feedback one, both operating in the frequency domain. Thus, the symbol estimate vector $\hat{\mathbf{s}}^l$ for the l -th iteration is formed by

$$\hat{\mathbf{s}}^l = \mathbf{y}'^{,l} + \mathbf{y}''^{,l}, \quad (5)$$

where $\mathbf{y}'^{,l}$ is the output of the feedforward filter in the l -th iteration and is given by

$$\mathbf{y}'^{,l} = \mathbf{A}^{l,H} \mathbf{r}, \quad (6)$$

with $\mathbf{A}^{l,H}$ is a matrix corresponding to the feedforward filter of size $N \times N$. The feedforward filter aim is to maximize the SINR of the detected symbols at every iteration. The vector $\mathbf{y}''^{,l}$ is the output of the feedback filter, and it can be expressed as

$$\mathbf{y}''^{,l} = \mathbf{B}^{l,H} \hat{\mathbf{s}}^{l-1}, \quad (7)$$

where $\mathbf{B}^{l,H}$ is the $N \times N$ matrix corresponding to the feedback filter and $\hat{\mathbf{s}}^{l-1}$ is the frequency domain estimated vector after symbol decision. The role of this feedback filter is the removal of the residual ISI.

Both $\hat{\mathbf{s}}^l$ and $\hat{\mathbf{s}}^{l-1}$ are assumed to be independent and identically distributed, with zero mean and no statistical relation to the noise. Thus, to obtain the coefficients for both the feedforward and the feedback filter at the l -th iteration, the goal is to minimize the mean square error (MSE), conditioned on $\bar{\mathbf{H}}$ and $\bar{\mathbf{H}}_{ik}$, in the following way:

$$\begin{aligned} MSE^l &= E[||\mathbf{A}^{l,H} \mathbf{r} + \mathbf{B}^{l,H} \hat{\mathbf{s}}^{l-1} - \mathbf{s}||^2] \\ &= \text{Tr}[\mathbf{A}^{l,H} \mathbf{C}_{rr} \mathbf{A}^l + \mathbf{A}^{l,H} \mathbf{C}_{rs} \mathbf{B}^l - \mathbf{A}^{l,H} \mathbf{C}_{ts} \\ &\quad + \mathbf{B}^{l,H} \mathbf{C}_{sr} \mathbf{A}^l + \mathbf{B}^{l,H} \mathbf{C}_{ss} \mathbf{B}^l - \mathbf{B}^{l,H} \mathbf{C}_{ss} \\ &\quad - \mathbf{C}_{sr} \mathbf{A}^l - \mathbf{C}_{ss} \mathbf{B}^l + \mathbf{I}_N], \end{aligned} \quad (8)$$

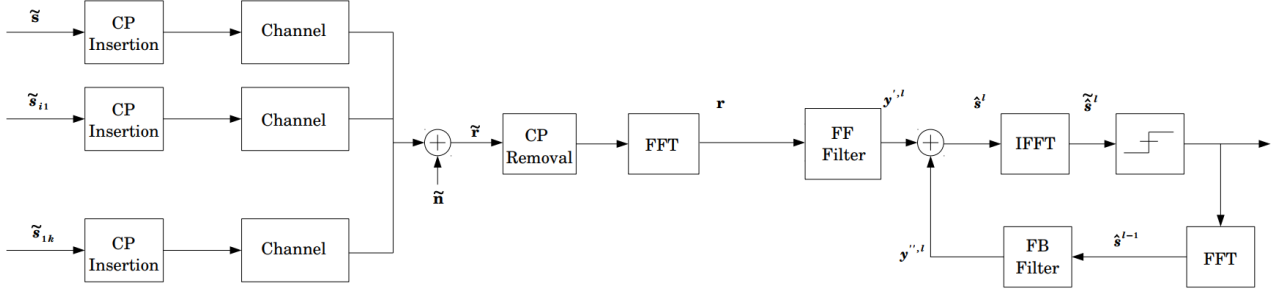


Figure 1. A SIMO SC-FDE system employing IB-DFE equalization under the effect of K interferers.

where

$$\begin{aligned} \mathbf{C}_{\mathbf{r}\mathbf{r}} &= E[\mathbf{r}\mathbf{r}^H] \\ &= \sum_{j=1}^{N_r} \bar{\mathbf{H}}_j \bar{\mathbf{H}}_j^H + \sigma_i^2 \sum_{j=1}^{N_r} \sum_{k=0}^K \bar{\mathbf{H}}_{ik,j} \bar{\mathbf{H}}_{ik,j}^H \\ &\quad + \mathbf{C}_{\mathbf{nn}} + \mathbf{C}_{\mathbf{HH}}(\sigma_i^2 + \mathbf{I}_N) \\ &= \sum_{j=1}^{N_r} \bar{\mathbf{H}}_j \bar{\mathbf{H}}_j^H + \sigma_i^2 \sum_{j=1}^{N_r} \sum_{k=0}^K \bar{\mathbf{H}}_{ik,j} \bar{\mathbf{H}}_{ik,j}^H \\ &\quad + (\sigma_n^2 + \sigma_e^2 + \sigma_i^2 \sigma_{ie}^2) \mathbf{I}_N, \end{aligned} \quad (9)$$

$$\mathbf{C}_{\mathbf{r}\mathbf{s}} = E[\mathbf{r}\mathbf{s}^H] = \sum_{j=1}^{N_r} \bar{\mathbf{H}}_j \quad (10)$$

$$\mathbf{C}_{\mathbf{s}\mathbf{r}} = E[\mathbf{s}\mathbf{r}^H] = \mathbf{C}_{\mathbf{r}\mathbf{s}}^H, \quad (11)$$

$$\begin{aligned} \mathbf{C}_{\mathbf{r}\hat{\mathbf{s}}} &= E[\mathbf{r}\hat{\mathbf{s}}^{l-1,H}] = E\{\mathbf{r}\hat{\mathbf{s}}^{l-1,H}\} \\ &= \sum_{j=1}^{N_r} \bar{\mathbf{H}}_j E[\hat{\mathbf{s}}^{l-1,H}] \\ &= \sum_{j=1}^{N_r} \bar{\mathbf{H}}_j \mathbf{C}_{\hat{\mathbf{s}}\hat{\mathbf{s}}}, \end{aligned} \quad (12)$$

$$\mathbf{C}_{\hat{\mathbf{s}}\hat{\mathbf{s}}} = E[\hat{\mathbf{s}}^{l-1,H} \hat{\mathbf{s}}^{l-1,H}], \quad (13)$$

$$\mathbf{C}_{\hat{\mathbf{s}}\mathbf{s}} = E[\hat{\mathbf{s}}^{l-1} \mathbf{s}^H] = \mathbf{C}_{\mathbf{s}\hat{\mathbf{s}}}^H, \quad (14)$$

$$\mathbf{C}_{\hat{\mathbf{s}}\hat{\mathbf{s}}} = E[\hat{\mathbf{s}}^{l-1} \hat{\mathbf{s}}^{l-1,H}], \quad (15)$$

$$\begin{aligned} \mathbf{C}_{\hat{\mathbf{s}}\mathbf{r}} &= E[\hat{\mathbf{s}}\mathbf{r}^H] = E\{\hat{\mathbf{s}}^{l-1,H} \mathbf{r}^H\} \\ &= \mathbf{C}_{\mathbf{r}\hat{\mathbf{s}}}^H, \end{aligned} \quad (16)$$

$$\mathbf{C}_{\mathbf{nn}} = E[\mathbf{nn}^H] = \sigma_n^2 \mathbf{I}_N, \quad (17)$$

and

$$\mathbf{C}_{\mathbf{HH}} = E[\mathbf{H}_e \mathbf{H}_e^H] = \sigma_e^2 \mathbf{I}_N. \quad (18)$$

In order to simplify this process we assume, as in [9], that we have ideal feedback after the first iteration. With this, we have to $\mathbf{C}_{\hat{\mathbf{s}}\hat{\mathbf{s}}} = \mathbf{C}_{\hat{\mathbf{s}}\mathbf{s}} = \mathbf{C}_{\mathbf{s}\hat{\mathbf{s}}} = \sigma_s^2 \mathbf{I}_N$, $\mathbf{C}_{\mathbf{r}\hat{\mathbf{s}}} = \mathbf{C}_{\mathbf{r}\mathbf{s}}$ and $\mathbf{C}_{\hat{\mathbf{s}}\mathbf{r}} = \mathbf{C}_{\mathbf{s}\mathbf{r}}$. Thus, by imposing the constraint that the feedback filter removes the ISI but not the desired symbol, by minimizing the MSE expressed by (8) we obtain the

coefficients of the filters \mathbf{A}^l and \mathbf{B}^l , which are given by [10]:

$$\mathbf{A}^l = \mathbf{C}_{\mathbf{r}\mathbf{r}}^{-1}(\mathbf{I}_N - \mathbf{B}^l) \mathbf{C}_{\mathbf{r}\mathbf{s}}, \quad (19)$$

and

$$\mathbf{B}^l = -[\mathbf{C}_{\mathbf{s}\mathbf{r}} \mathbf{A}^l - \beta \mathbf{I}_N], \quad (20)$$

where $\beta = \frac{\text{Tr}[\mathbf{C}_{\mathbf{s}\mathbf{r}} \mathbf{A}^l]}{N}$, $\mathbf{C}_{\mathbf{r}\mathbf{s}} = \bar{\mathbf{H}}$ and $\mathbf{C}_{\mathbf{s}\mathbf{r}} = \bar{\mathbf{H}}^H$. With this, immediately after the first iteration the feedforward filter is switched to the matched filter.

Note that the difference between the equalizer that takes into account the channel estimation errors and the one presented in [10] is the presence of the channel estimation error variances σ_e^2 and σ_{ie}^2 in (9). If both are equal to zero, we have the conventional SIMO IB-DFE equalizer seen in [10], i.e., $\mathbf{H}_e = \mathbf{H}_{ie} = 0_N$. Furthermore, if we only had one iteration ($l = 0$), the feedback filter is zero due to the lack of previous decisions \mathbf{A}^l is also reduced to $\mathbf{A}^l = \mathbf{C}_{\mathbf{r}\mathbf{r}}^{-1} \mathbf{C}_{\mathbf{r}\mathbf{s}}$, that is, the conventional MMSE equalizer [11].

In the next section, we will analyze the error performance of the proposed structure, taking or not into account the channel estimation errors in these equalizers. This analysis will also assume that these errors are in both the desired information channel and/or in the interference channels in order to show the effectiveness of the proposed solution. Unlike in [7], [8], the impact of the number of interferers, receiver antennas and the interferer/estimation error power in the error performance will be analyzed.

III. SIMULATION RESULTS

In this section, we present the results obtained by simulation in order to validate the use of the new iterative equalizer proposed in this paper for different interferer numbers, signal-to-interference values and channel estimation error variances. Using Monte Carlo simulation results, we analyze the behavior of this system under the effect of channel estimation errors.

For these simulations, we performed the transmission of blocks with size $N = 128$, taking into consideration up to K interferers, who transmit equal-sized blocks. The information signal and the interfering signals are from a QPSK constellation and the sampling frequency is 10 MHz. Both signals are affected by a quasi-static Rayleigh fading channel, whose model adopted was the ITU-T Vehicular A one [12]. The length of the CP is the minimum

able to eliminate the interblock interference and the loss of energy introduced by it is taken into account in the SINR calculation. To calculate the final BER in the Monte Carlo simulations, a minimum of 400 errors were considered for each point.

Figures 2, 3 and 4 presents results for $N_r = 1, 2$ and 4 respectively and the iterative equalizer operating with $l = 1$ and 4 iterations, for the receivers taking or not into account the channel estimation errors in the estimation process, for $\sigma_e^2 = 0.06$ and without interferers. For reference, results without channel estimation errors are also presented. It can be seen that with a higher number of receiver antennas the error performance improves; moreover, the added diversity provided by the higher number of antennas reduces the performance gain obtained with more iterations of the equalizer and by taking into account the channel estimation errors in the estimation process. Nevertheless, using the proposed iterative equalizer brings a performance gain for every considered number of receiver antennas.

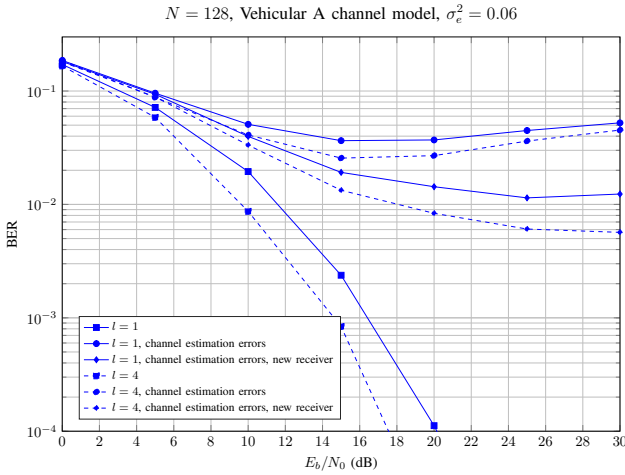


Figure 2: No interferers, 1 receiver antenna and $\sigma_e^2 = 0.06$.

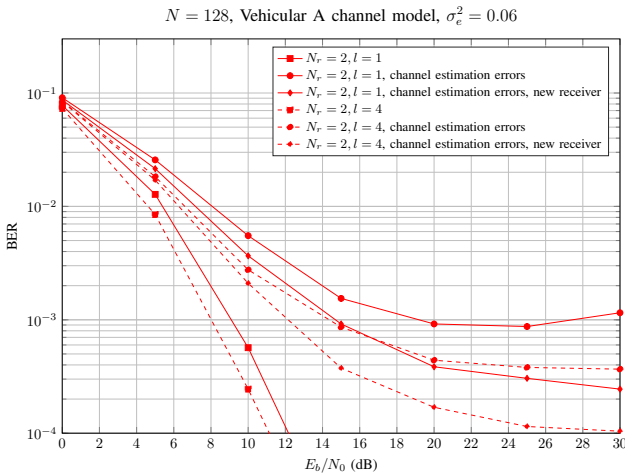


Figure 3: No interferers, 2 receiver antennas and $\sigma_e^2 = 0.06$.

To obtain the results presented in Figure 5 we consider the same scenario, but now we take into account $l = 4$ only and $K = 1, 2, 3$ interferences. It can be seen that by increasing the number of antennas the receiver can reduce

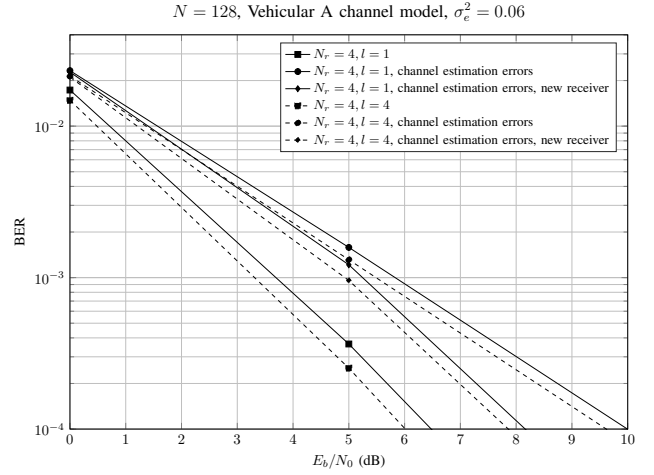


Figure 4: No interferers, 4 receiver antennas and $\sigma_e^2 = 0.06$.

the BER for a given number of interferers, due to the added diversity. Moreover, the impact of an increase of the number of interferers on the BER is reduced with a higher number of receiver antennas, as it can clearly be seen in the case of $N_r = 4$ and $K = 3$. Finally, with channel estimation errors the proposed receiver has a slight performance gain with respect to the version that does not take into account these errors in the estimation process, with this gain increasing with the number of interferers.

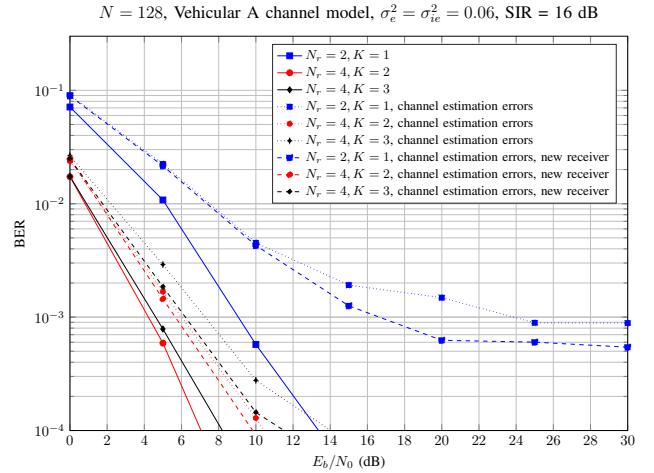


Figure 5: With interferers, $\sigma_e^2 = \sigma_{ie}^2 = 0.06$, SIR = 16 dB.

IV. CONCLUSION

In this paper, we have introduced a new iterative equalizer for SIMO SC-FDE systems under the effect of K co-channel interferences, and considering errors in the channel estimation process. It was found out that the proposed equalizer has a better error performance when compared to its version that does not take into account the channel estimation errors in the process. Also, it was seen that by increasing the number of receiver antennas the receiver can cope with a higher number of interferers, while maintaining a low BER. A suggestion for future work is to carry out a theoretical analysis of the proposed system to analyze the effects of these errors. The use of widely linear processing for interference suppression

and the scenario with soft/turbo decisions in the feedback process can also be explored.

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