Non-Parametric VSS-NLMS Algorithm With Control Parameter Based on the Error Correlation

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***Abstract*—The behavior of variable step-size least-mean-square (VSSLMS) algorithms is strongly affected by measurement noise. Thereby, aiming to maintain an adequate performance of these algorithms, their parameters must be adjusted whenever changes occur in the signal-to-noise ratio (SNR) of the adaptive system. A well-known VSSLMS algorithm based on error correlation provides a performance enhancement for low SNR environment; however, its immunity to measurement noise changes is still poor. This paper presents a new non-parametric variable step-size normalized LMS (VSS-NLMS) algorithm with control parameter based on error correlation. This approach is very robust to measurement noise changes, not requiring any manual adjustment of algorithm parameters. Numerical simulation results confirm the effectiveness of the proposed algorithm, considering a system identification problem.**

***Keywords*—Adaptive filters, adaptive signal processing, variable step-size least-mean-square (VSSLMS) algorithm.**

I. INTRODUCTION

The least-mean-square (LMS) algorithm is one of the most popular and used adaptive algorithms in widespread applications. This strength is due to its low computational complexity, robustness, and very good stability characteristic, among other attributes [1]. Such advantages have made the LMS algorithm adequate for different applications, such as system identification, active noise/vibration control, echo cancellation, channel equalization, among others [2]. The standard LMS algorithm uses a fixed step size, which is determined by allowing for a trade-off between convergence speed and misadjustment error. A large step-size value leads to a faster convergence (providing the maximum value to guarantee algorithm stability is not violated) along with a larger misadjustment. Conversely, a small step size provides a small misadjustment at the expense of a slower convergence. To overcome this trade-off, variable step-size LMS (VSSLMS) algorithms have been proposed in the literature. All existing VSSLMS algorithms have as common feature to use a large step size at the beginning of the convergence process and reducing it as the steady-state convergence is approached [3]-[22]. The adjusting law of the variable step size can be based on different criteria. For instance, in [3], the proposed VSSLMS algorithm has its step size adjusted according to the gradient of the square-error signal. The central idea of this approach is that the higher the value of the gradient the larger the distance from the minimum MSE (MMSE). Therefore, the step size must be larger to speed up convergence. Conversely, for a lower value of the gradient, the smaller should be the step size for reducing the final misadjustment error. Other algorithms based on gradient can be found in [4]-[10]. In general, such a class of algorithms is harmed by the presence of gradient noise, leading to an increase of the steady-state excess MSE.

Another strategy, proposed in [11], considers the instantaneous square error for adjusting the step size in VSSLMS algorithms. Variations of this method are presented in [12]-[13]. Algorithms of this class undergo a straightforward interference of measurement noise, since the variance of this noise is inserted into the squared error signal. A third method to update the step size is based on the error autocorrelation function [14]. Such a function is a reliable measure of nearness to the MMSE, allowing the algorithm to effectively achieve the step-size adjustment. Further strategies of VSSLMS algorithms consider the absolute adaptation error [15]-[16], error vector normalization [17], absolute values of the weight vector coefficients [18]-[20] and other methods [21]-[22]. In most of such approaches, the presence of measurement noise degrades the performance of the algorithms.

As a general matter, a good number of VSSLMS approaches require the adjustment of several algorithm parameters. In most cases, this is made by trial-and-error procedure. Another fact to consider is the dependence of the step-size adjustment on the measurement noise. Thereby, whenever the noise changes, the algorithm parameters should be readjusted. If we take into account that the variance of measurement noise is generally unknown and variable along time in practical applications, this procedure reveals to be ineffective.

In [11], a VSSLMS algorithm based on the instantaneous square error is discussed. This algorithm has been widely used, presenting a satisfactory performance in most applications. However, for low SNR environment, the algorithm performance degrades, since both the step-size adjustment and the misadjustment error are seriously affected by the measurement noise. A modification of this algorithm is proposed in [14], enhancing its performance for both low SNR and white measurement noise. Although this algorithm presents a better performance under certain operating conditions, it still has not improved its immunity to measurement noise (which implies that changes in the algorithm performance occur whenever the measurement noise variance changes, unless a new parameter setting is made).

In this work, a non-parametric variable step-size normalized LMS (VSS-NLMS) algorithm based on the error correlation is proposed. The aim of this algorithm is to enhance the immunity to measurement noise of the algorithm given in [14], without the need for parameter adjustment. Numerical simulation results considering a system identification problem verify the performance of the proposed algorithm.

II. PROPOSED ALGORITHM

This section describes the proposed algorithm which belongs to the class of error-correlation-based algorithms. One of the first algorithms of this class, given in [14], was devised aiming to improve the performance of the algorithm based on instantaneous square error, introduced in [11], for low signal-to-noise ratio (SNR) environment. Such an enhancement can be achieved only through a proper parameter adjustment, which implies to know the variance of additive measurement noise in advance. However, this information is not usually available in practice. Then, to achieve an improved version of the algorithm given in [14], we propose here a new error-correlation-based non-parametric VSS-NLMS algorithm. Such an approach enhances the algorithm robustness against measurement noise variance changes.

Before proceeding, a brief description of the algorithms given in [11] and [14] is presented. For such, let us consider a system identification scheme depicted in Fig. 1.

The unknown system output is

$$d(n) = \mathbf{w}_o^{\mathrm{T}} \mathbf{x}(n) + z(n) \tag{1}$$

where $\mathbf{x}(n) = [x(n)\, x(n-1) \cdots x(n-N+1)]^{\mathrm{T}}$ denotes the input vector with $\{x(n)\}$ being a zero-mean Gaussian process with variance σ_x^2, $z(n)$ is a white Gaussian noise with variance σ_z^2. Vectors $\mathbf{w}_o$ and $\mathbf{w}(n)$ denote, respectively, the unknown plant and adaptive coefficients, both with dimension N.

The error signal is then

$$e(n) = d(n) - \mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n) \tag{2}$$

and the weight update equation [2],

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu(n)e(n)\mathbf{x}(n) \tag{3}$$

where $\mu(n)$ is the variable step size. A necessary condition to guarantee stable operation of the algorithm is given by [1]

$$0 < \mu(n) < \frac{2}{3\mathrm{tr}[\mathbf{R}]} \tag{4}$$

where $\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)]$ is the input autocorrelation matrix.

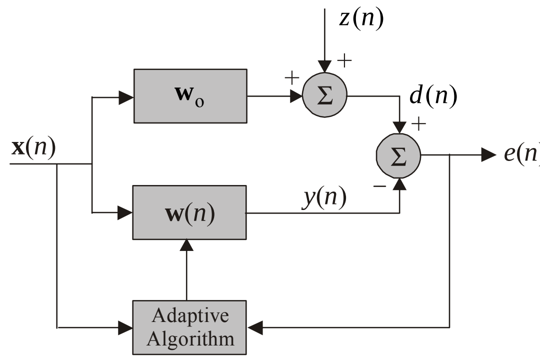


Fig. 1. Block diagram of a system identification problem.

A. Instantaneous Square Error Based Algorithm

This algorithm, described in [11], has the variable step size updated according to the following rule:

$$\mu(n+1) = \alpha\mu(n) + \gamma e^2(n) \tag{5}$$

where α and γ are positive control parameters. In this algorithm, a large error signal increases the step-size value, resulting in a faster convergence; whereas a small one decreases the step size, leading to a smaller misadjustment [11]; however, as the error signal is contaminated by measurement noise, the algorithm becomes strongly dependent on this noise, reducing its performance for a low SNR environment.

B. Error Correlation Based Algorithm

This algorithm, discussed in [14], adjusts the variable step size based on the correlation between $e(n)$ and $e(n-1)$, instead of the instantaneous square error $e^2(n)$, as used in [11]. This modification improves the algorithm performance in the presence of uncorrelated measurement noise. In this approach, the variable step-size update equation is given by

$$\mu(n+1) = \alpha\mu(n) + \gamma p^2(n) \tag{6}$$

where α and γ are positive control parameters and $p(n)$ is smoothed error correlation estimated by

$$p(n) = \beta p(n-1) + (1-\beta)e(n)e(n-1) \tag{7}$$

with β a positive control parameter. Note that this algorithm has three control parameters, each of them with different impact on the algorithm behavior.

C. Non-Parametric VSS-NLMS Algorithm

The previous algorithm brings to light a strong dependence on measurement noise, compelling to modify its parameters, particularly γ, whenever the noise level changes. If parameter γ remains constant, the algorithm performance changes substantially with different SNR values. Moreover, the adjustment of γ parameter is usually made by a trial-and-error procedure, being a hard task in most of practical applications.

Aiming to make the algorithm more robust for both measurement noise and input signal power variations, without the need for parameter adjustment, a non-parametric VSS-NLMS algorithm is proposed. Thus, the step-size adjustment rule is given by

$$\mu(n) = \frac{\alpha(n)}{\mathbf{x}^{\mathrm{T}}(n)\mathbf{x}(n)} \tag{8}$$

where $\alpha(n)$ is a variable control parameter, which is given by

$$\alpha(n) = \left[\frac{p(n)}{q(n)}\right]^2 \tag{9}$$

with $p(n)$ denoting the error correlation estimate obtained from a low-pass filter, expressed as

$$p(n) = \beta p(n-1) + (1-\beta)e(n)e(n-1) \tag{10}$$

and $q(n)$ the smoothed squared error signal, given by

$$q(n) = \beta q(n-1) + (1-\beta)e^2(n) \tag{11}$$

where β is a constant close to unity. Note that here β is not a parameter which requires adjustment; it is a constant close to unity, for example, equal to 0.99. Because of this fact, the algorithm is said to be non-parametric.

The adjustment rule (8) works as follows. In the beginning of the convergence process the correlation between $e(n)$ and $e(n-1)$ is approximately equal to $e^2(n)$, making $\alpha(n) \to 1$, thereby, speeding the algorithm convergence. As steady state is approached, the error correlation tends to zero and the signal $e^2(n)$ tends to the variance of the measurement noise, making the parameter $\alpha(n)$ close to zero, reducing thus the final misadjustment error.

III. Simulation Results

This section presents the numerical simulation results comparing the performance of the proposed VSS-NLMS algorithm with the ones presented in [11] and [14]. For such, a system identification problem is considered. Results obtained through the Monte Carlo (MC) method (200 independent runs) show the excess mean-square error (MSE), given by $E\{[e(n)-z(n)]^2\}$, and the step-size evolution $\mu(n)$ for all three algorithms. In addition, the control parameter evolution $\alpha(n)$ of the proposed algorithm is shown.

The plant used has 32 coefficients, given by a scaled Hanning window function, enforcing $\|\mathbf{w}_o\| = 1$. The input data used is a zero-mean Gaussian correlated signal, obtained from an AR(1) process given by $x(n) = ax(n-1) + u(n)$, with $a = 0.9$, $\sigma_x^2 = 1$, and $u(n)$ being a white Gaussian noise with variance $\sigma_u^2 = 0.19$. The eigenvalue spread of the autocorrelation matrix of the input signal is $\chi = 266.5$. The maximum step-size value is limited to 0.003 for all algorithms. The parameter adjustment of the algorithms presented in [11] and [14] is adjusted (manually) only once, considering $\sigma_z^2 = 0.001$ $(\mathrm{SNR} = 30\,\mathrm{dB})$, in such a way that all algorithms have the same final excess MSE equal to $-50\,\mathrm{dB}$. After that, simulations are repeated (without parameter adjustment), considering two different additive noise variances, i.e,. $\sigma_z^2 = 0.00001$ $(\mathrm{SNR} = 50\,\mathrm{dB})$ and $\sigma_z^2 = 0.1$ $(\mathrm{SNR} = 10\,\mathrm{dB})$.

Fig. 2 shows the evolution of the excess MSE, variable step size, and control parameter $\alpha(n)$. In this simulation, algorithms are adjusted such that the same error in excess is obtained. Note from this figure that the proposed algorithm performs as well as the ones given in [11] and [14]. The step-size evolution curve points out that the maximum specified value of the step size is reached for the three algorithms. We also observe that the control parameter $\alpha(n)$ is initially close to 1, tending to zero as the MMSE is approached [see Fig. 4(c)].

Fig. 3 shows the excess MSE, variable step size, and control parameter evolution as the SNR is increased by reducing the variance of the additive measurement noise. Note that now the algorithms have no parameter adjustment. Since both reference algorithms ([11] and [14]) use the variance of the additive noise to speed up their convergence, as this parameter is reduced the parameter γ should be increased. If this parameter is maintained constant (fixed), the performance of such algorithms are degraded, as shown in Fig. 3. Note that the performance of the proposed algorithm is much better in this condition.

Fig. 4 shows the excess MSE, variable step-size, and control parameter curves as the SNR is reduced by increasing the variance of the measurement noise. Now, parameter γ should be reduced in both [11] and [14] algorithms, aiming to maintain the same performance as in Fig. 2. Since no parameter is adjusted, these algorithms are forced to work (along the time) with a larger step-size value, leading to a very high steady-state misadjustment. Again, the proposed algorithm shows higher immunity to an increase of the measurement noise variance.

Analyzing presented simulation results, we verify that the proposed algorithm maintains very good performance for all considered scenarios (different measurement noise levels). On the other hand, the algorithms given in [11] and [14] are very sensitive to measurement noise changes.

IV. Conclusions

This paper presented a non-parametric VSS-NLMS algorithm with control parameter based on the error correlation. The new approach improves significantly the algorithm immunity to measurement noise changes, outperforming other algorithms of the literature. By using a system identification problem, the proposed algorithm was assessed for both correlated input signal and different measurement noise levels.

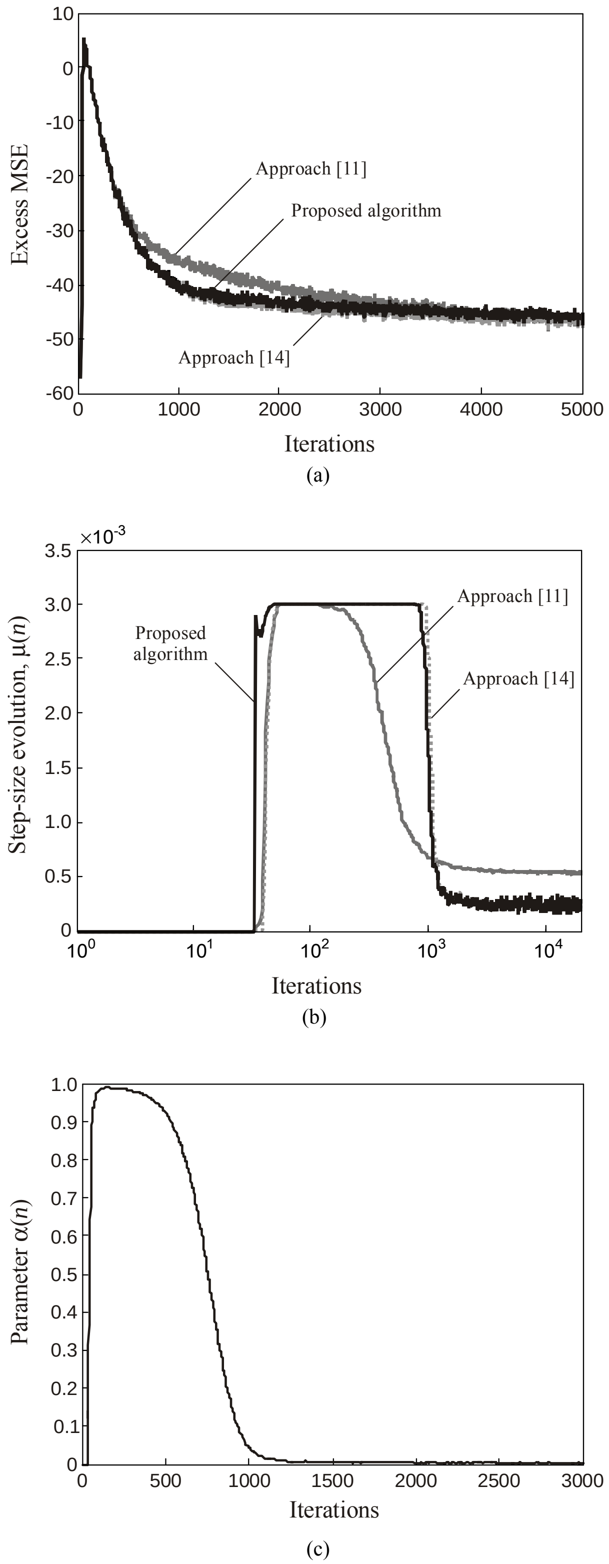


Fig. 2. Simulation results for $\sigma_z^2 = 0.001$ (SNR = 30 dB). (a) Excess MSE curve. (b) Variable step-size evolution. (c) Control parameter evolution.

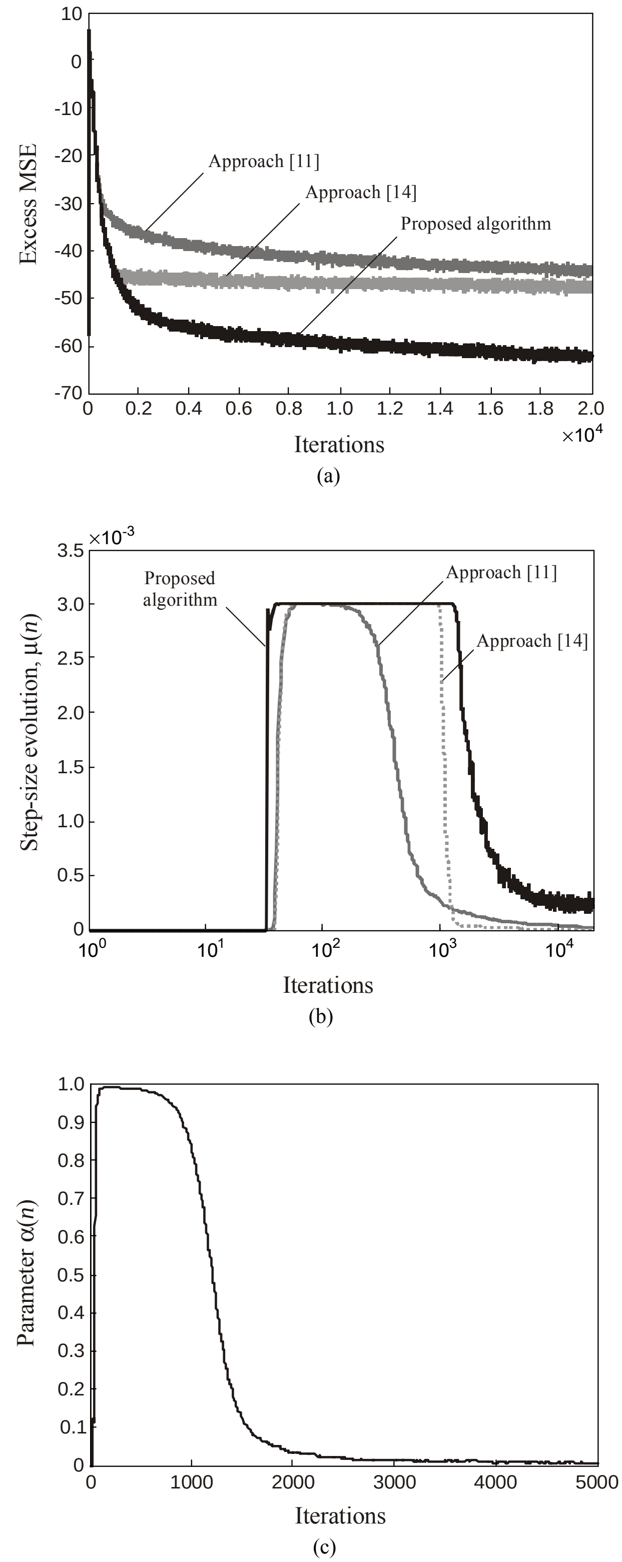
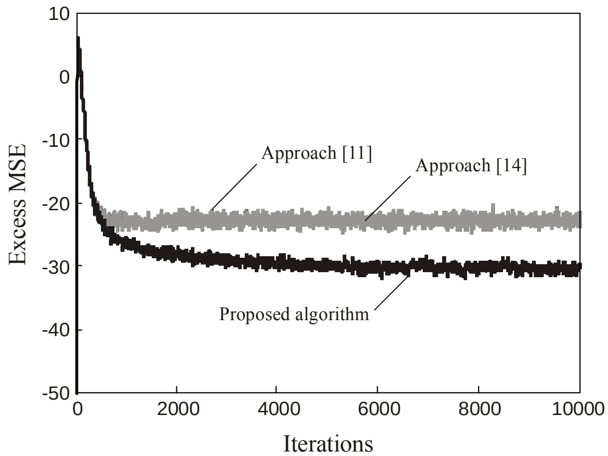
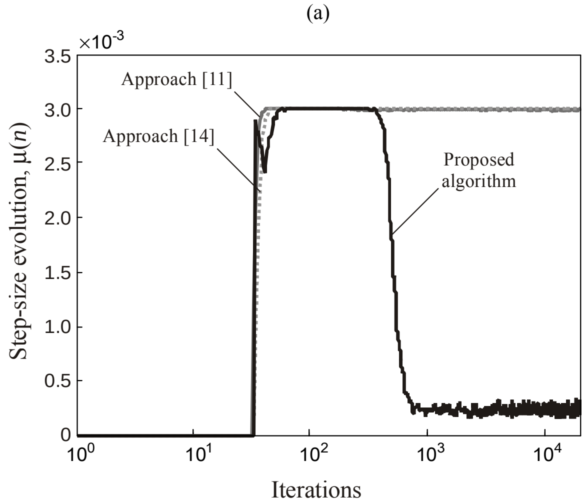


Fig. 3. Simulation results for $\sigma_z^2 = 0.00001$ (SNR = 50 dB). (a) Excess MSE curve. (b) Variable step-size evolution. (c) Control parameter evolution.



(a)



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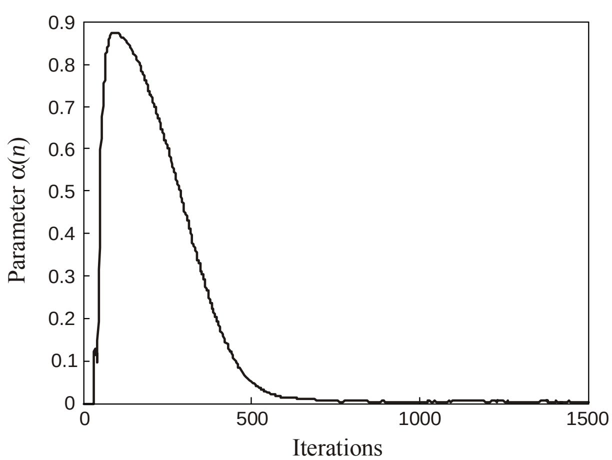

(c)

Fig. 4. Simulation results for $\sigma_z^2 = 0.1$ (SNR = 10 dB). (a) Excess MSE curve. (b) Variable step-size evolution. (c) Control parameter evolution.

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