Adaptive Beamforming for Antenna Arrays in Cellular Systems Based on a Duality Between Uplink and Downlink Channels

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***Abstract*—This paper presents a beamforming algorithm for antenna arrays attending to both power and signal-to-interference-plus-noise (SNIR) ratio constraints. The proposed adaptive approach makes use of the least-mean-square (LMS) algorithm as well as exploits the duality idea between uplink and downlink channels. Since a reference signal is used, the proposed approach does not require any direction-of-arrival (DoA) algorithms for estimating the angle-of-arrival (AoA) of the signals. The proposed algorithm exhibits both low computational complexity and very good stability conditions. Through numerical simulations the performance of the proposed algorithm is verified.**

***Keywords*—Adaptive antenna arrays, beamforming algorithm, duality approach, LMS-based algorithm.**

I. Introduction

In cellular networks, the use of controlling techniques for the transmission power is an important issue, which provides the support for the interference mitigation and energy management. In particular, code division multiple access (CDMA) systems form a scenario largely limited by interference problems. Thereby, power control procedures allow improving both the capacity and performance of these systems, enhancing some features, such as signal quality, spectral efficiency, and quality of service (QoS), among other parameters [1]-[8]. In addition, the energy saving is a central point looking at the battery lifetime of mobile terminals (MTs).

These aims can be accomplished by using beamforming techniques. Usually, beamforming algorithms are devised for the reception mode, i.e., considering the uplink channel [MT to base station (BS)] [1], [6], [7]. This condition is interesting since working with the uplink channel the direction-of-arrival (DoA) of the signal is known. Then, the downlink channel beamforming can be obtained from the uplink by using the duality concept.

In recent formulations, the use of constrained optimization approach associated with the duality idea between the uplink and downlink channels permits to devise new beamforming algorithms. In such an approach, different goals are focused at the same time, such as beamforming along with power control [8]-[10]. Thus, applying the duality concept, the downlink beamforming problem may be solved by means of an equivalent uplink problem. In [8] and [10], by using this concept, it is shown that the optimal downlink beamforming can be obtained from the optimal uplink by considering a dual uplink channel. In [11], the uplink-downlink duality is used for controlling the signal-to-interference-plus-noise (SNIR) at each mobile, by adjusting pre-filters and transmission powers at BS prior to transmission. In [7], assuming perfect channel knowledge at the transmitter, several transmission schemes under the per-antenna power constraint are investigated. There, it is shown that such a problem can be transformed into a dual uplink problem, generalizing previous uplink-downlink duality results [10], and opening new ways for solving the downlink-beamforming problem [7]-[9]. Another beamforming approach is presented in [12], which is termed improved constrained stochastic gradient (ICSG) algorithm. It exhibits some SINR decrease since the SINR is not controlled by the corresponding algorithm; this point is more evident when one or more interferer angles-of-arrival (AoA) are close to the desired user. Moreover, this algorithm requires performing several vector multiplications, resulting in high computational load.

Thus, likewise in [8] and [9], the proposed algorithm uses the uplink-downlink duality aiming to solve, respectively, the downlink and uplink problems, which are formulated as

$$\min_{\mathbf{w}_i^{\mathrm{d}}} \left(\sum_{i=1}^{M} \left| \mathbf{w}_i^{\mathrm{d}} \right|^2 \right) \quad (1)$$

$$\text{subject to } SINR_i^{\mathrm{d}} \geq \delta_i$$

and

$$\min_{\mathbf{w}_1^{\mathrm{u}},\ldots,\mathbf{w}_M^{\mathrm{u}},\, P_1^{\mathrm{u}},\ldots,P_M^{\mathrm{u}}} \left(\sum_{i=1}^{M} P_i^{\mathrm{u}} \right) \quad (2)$$

$$\text{subject to } SINR_i^{\mathrm{u}} \geq \delta_i$$

where δ_i is a positive parameter representing the minimum achievable SINR value for user i (superscripts 'd' and 'u' denote, respectively, downlink and uplink), M is the number of MTs, and $\mathbf{w}_i^{\mathrm{d}}$ and $\mathbf{w}_i^{\mathrm{u}}$ are, respectively, the complex downlink and uplink beamforming vectors. Variable P_i^{u} denotes the uplink signal power. In contrast to previous research works [3], [6]-[9], and [12], this one has the following specific aims and features:

a) Based on both the duality concept and least-mean-square (LMS) algorithm, the duality constrained LMS (DCLMS) algorithm is proposed.
b) Regarding the use of the LMS algorithm to obtain the beamforming weights, the proposed algorithm exhibits both low computational complexity and good convergence stability.
c) The proposed solution considers a reference signal for performing the beamforming approach, not requiring the use of additional DoA algorithms.

d) The proposed algorithm attends the minimum delivered power with a SINR controlled in both channels (uplink and downlink).

This paper is organized as follows. Section II introduces the basic definitions and main assumptions used in this work. In Section III, the problem statement and the proposed algorithm are presented. Numerical simulation results aiming to verify the performance of the proposed approach are shown and discussed in Section IV. Finally, Section V presents concluding remarks.

II. Definitions

In this section, we state the problem by firstly concentrating on some assumptions, notation, and signal and scenario definitions to assess the proposed solution.

For such, let us consider an antenna array having K elements and a scenario with M (single antenna) users, as depicted in Fig. 1 [13]. Vectors $\mathbf{w}_i^{\mathrm{u}}(n)$, $\mathbf{x}^{\mathrm{u}}(n)$, and $\mathbf{s}_j$ have dimensions $K \times 1$ and represent, respectively, the complex uplink beamforming, uplink signal, and uplink steering vectors. Variable $\hat{m}_j(n)$ denotes the information signal sent to the BS from the jth user. Vector $\boldsymbol{\eta}_i(n)$ characterizes the channel noise and P_j^{u}, the uplink signal power from user j. The downlink signal scenario is shown in Fig. 2, where $\mathbf{x}^{\mathrm{d}}(n)$ and $\mathbf{w}_j^{\mathrm{d}}(n)$ are, respectively, the downlink and complex downlink vectors. Variable $m_j(n)$ denotes the signal sent to the jth user from the BS and $\eta_i(n)$, the channel noise. From Fig. 2, the received signal by user i is

$$y_i^{\mathrm{d}}(n) = \eta_i(n) + \sum_{j=1}^{M} \mathbf{s}_i^H \mathbf{w}_j^{\mathrm{d}}(n) m_j(n) = \eta_i(n) + \mathbf{s}_i^H \mathbf{x}^{\mathrm{d}}(n). \quad (3)$$

Rearranging (3) as

$$y_i^{\mathrm{d}}(n) = \mathbf{s}_i^H \mathbf{w}_i^{\mathrm{d}}(n) m_i(n) + \eta_i(n) + \sum_{j=1,\, j\neq i}^{M} \mathbf{s}_i^H \mathbf{w}_j^{\mathrm{d}}(n) m_j(n) \quad (4)$$

we observe that the received signal by user i (downlink signal) is composed of the signal of interest (first r.h.s. term), plus noise (second r.h.s. term), and interfering signals (other users, third r.h.s. term). Similarly to (4), the signal at the BS from user i (uplink signal) is expressed as

$$y_i^{\mathrm{u}}(n) = \mathbf{s}_i^H \mathbf{w}_i^{\mathrm{u}}(n) \hat{m}_i(n) + \boldsymbol{\Psi}_i^H(n) \mathbf{w}_i^{\mathrm{u}}(n) + \sum_{j=1,\, j\neq i}^{M} \mathbf{s}_j^H \mathbf{w}_i^{\mathrm{u}}(n) \hat{m}_j(n) \quad (5)$$

where $\boldsymbol{\Psi}_i(n)$ is the antenna noise vector.

Now, some assumptions on the involved signals in the system are then considered:

i) Signal $m_j(n)$ is assumed having unit power, i.e., $E[m_j^2(n)] = 1 \ \ \forall j$, where $E[\cdot]$ denotes the expected value operator.

ii) In addition, $m_j(n)$ and $m_k(n)$ are uncorrelated to each other, i.e., $E[m_j(n) m_k(n)] = 0$ for $j \neq k$.

iii) The channel noise $\eta_i(n)$ is a white Gaussian signal with variance σ_η^2, and uncorrelated with any other signal in the system.

iv) The elements $\eta_i(n)$ of the channel noise vector $\boldsymbol{\Psi}_i(n)$ are white Gaussian with variance σ_η^2, and uncorrelated with any other signal in the system.

v) For the uplink signal, one assumes that $E[\hat{m}_j^2(n)] = 1$ with $P_j^{\mathrm{u}}(n)$ representing the uplink signal power, and with $E[\hat{m}_j(n) \hat{m}_k(n)] = 0$ for $j \neq k$.

The SINR is defined as in [14] by

$$SINR = \frac{P_{\text{signal}}}{P_{\text{interference+noise}}}. \quad (6)$$

Thereby, the downlink $SINR_i^{\mathrm{d}}$, taking into account the assumptions (i) to (v) and (6), is given by

$$SINR_i^{\mathrm{d}} = \frac{\left|\mathbf{s}_i^H \mathbf{w}_i^{\mathrm{d}}\right|^2}{\sigma_\eta^2 + \sum_{j=1,\, j\neq i}^{M} \left|\mathbf{s}_i^H \mathbf{w}_j^{\mathrm{d}}\right|^2}. \quad (7)$$

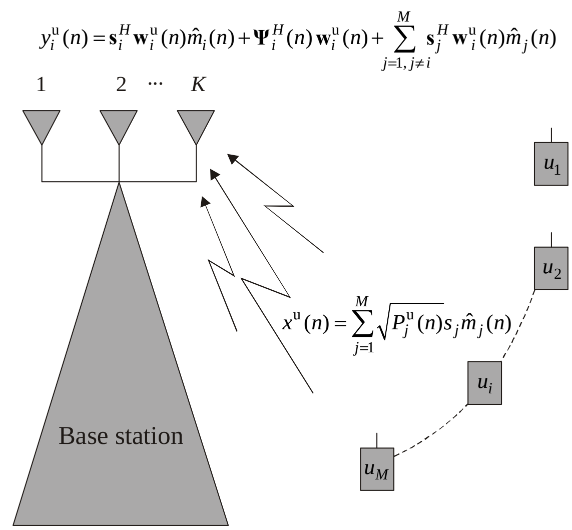


Fig. 1. Uplink signal scenario.

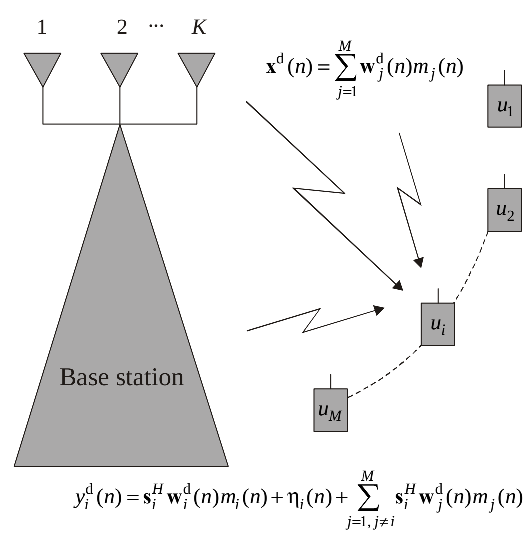


Fig. 2. Downlink signal scenario.

Similarly to the downlink $SINR_i^{\mathrm{d}}$, from (i) to (v) and (6), we get

$$SINR_i^{\mathrm{u}} = \frac{\left|\mathbf{s}_i^H \mathbf{w}_i^{\mathrm{u}}\right|^2 P_i^{\mathrm{u}}}{\left|\mathbf{w}_i^{\mathrm{u}}\right|^2 \sigma_\eta^2 + \sum_{j=1,\, j\neq i}^{M} \left|\mathbf{s}_j^H \mathbf{w}_i^{\mathrm{u}}\right|^2 P_j^{\mathrm{u}}}. \tag{8}$$

Note that in (8) the radiated power from user *i* to the BS is a parameter to be controlled by the adaptive algorithm [1], [4]-[6].

III. Problem Statement and Proposed Algorithm

In this section, we start with the problem given in [4] and next introduce the proposed solution. An LMS-based algorithm for performing an uplink beamforming is devised, imposing restrictions on the user delivered power and minimum SINR values. For such, general expressions of the downlink and uplink problem are derived, respectively, by using (1) and (2) [6], [7]. The block diagram considering the reference signal is shown in Fig. 3.

A. Proposed Algorithm

To derive the DCLMS algorithm, we express (1) in terms of the instantaneous quadratic error as

$$\min_{\mathbf{w}_i^{\mathrm{u}}(n)} [\varepsilon_i^2(n)] \tag{9}$$

$$\text{subject to } SINR_i^{\mathrm{u}}(n) \geq \delta_i \tag{10}$$

with

$$\varepsilon_i(n) = d_i(n) - [\mathbf{x}^{\mathrm{u}}(n)]^H \mathbf{w}_i^{\mathrm{u}}(n) \tag{11}$$

where $\varepsilon_i(n)$ is the error signal and $d_i(n)$, the reference signal for user *i*. Then, using the gradient rule, $\mathbf{w}_i^{u}(n)$ is expressed as

$$\mathbf{w}_i^{\mathrm{u}}(n+1) = \mathbf{w}_i^{\mathrm{u}}(n) - \frac{\mu}{2}\nabla\boldsymbol{\varepsilon}_i^2(n). \tag{12}$$

Now, substituting (11) into (12), we get

$$\mathbf{w}_i^{\mathrm{u}}(n+1) = \mathbf{w}_i^{\mathrm{u}}(n) + \mu\varepsilon_i(n)\mathbf{x}^{\mathrm{u}}(n) \tag{13}$$

where μ is the step-size parameter.

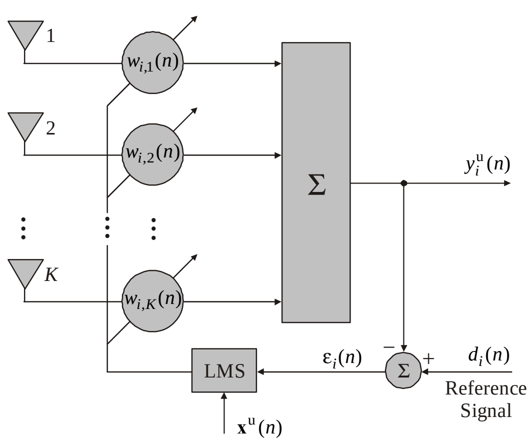


Fig. 3. Block diagram of an antenna array using a reference signal.

B. Determining the Uplink and Downlink Powers

Once the uplink beamforming vector is computed, the following step is to update the uplink power. Then, from (10), we have

$$P_i^{\mathrm{u}}(n+1) = P_i^{\mathrm{u}}(n)\frac{\delta_i}{SINR_i^{\mathrm{u}}} \tag{14}$$

for which convergence of (14) occurs when the $SINR_i^{\mathrm{u}}$ is equal to the constraint δ_i. The $SINR_i^{\mathrm{u}}$ can be obtained from the expected value of the quadratic error by determining $\min\{E[\varepsilon_i^2(n)]\}$. Thus, comparing the expression obtained from the Wiener filter [15] (after substituting it into the quadratic error expression) with the SINR expression, given by

$$SINR_i^{\mathrm{u}} = \frac{P_i^{\mathrm{u}}\left|\mathbf{s}_i^H \mathbf{w}_i^{\mathrm{u}}\right|^2}{\sum_{j=1,\, j\neq i}^{M} P_j^{\mathrm{u}}\left|\mathbf{s}_j^H \mathbf{w}_i^{\mathrm{u}}\right|^2 + \sigma_\eta^2\left\|\mathbf{w}_i^{\mathrm{u}}\right\|^2} \tag{15}$$

one obtains the following expression in terms of $\min\{E[\varepsilon_i^2(n)]\}$ [1]. Thus,

$$SINR_i^{\mathrm{u}} = \frac{1-\min\{E[\varepsilon_i^2(n)]\}}{\min\{E[\varepsilon_i^2(n)]\}}. \tag{16}$$

Thereby, (14) is rewritten as

$$P_i^{\mathrm{u}}(n+1) = P_i^{\mathrm{u}}(n)\,\delta_i\,\frac{\min\{E[\varepsilon_i^2(n)]\}}{1-\min\{E[\varepsilon_i^2(n)]\}}. \tag{17}$$

Now, using the duality approach, the weight vector for the downlink channel is given by

$$\mathbf{w}_i^{\mathrm{d}}(n) = \sqrt{P_i^{\mathrm{d}}(n)}\mathbf{w}_i^{\mathrm{u}}(n). \tag{18}$$

Then, from (18), the downlink power update $P_i^{\mathrm{d}}(n+1)$ is obtained by [2]

$$P_i^{\mathrm{d}}(n+1) = \frac{\left\|\mathbf{w}_i^{\mathrm{d}}(n+1)\right\|^2}{\left\|\mathbf{w}_i^{\mathrm{u}}(n+1)\right\|^2}. \tag{19}$$

Note that $\left\|\mathbf{w}_i^{\mathrm{d}}(n+1)\right\|^2$ is not known at the instant of computing $P_i^{\mathrm{d}}(n+1)$. However, by using the Duality Theory, uplink and downlink cost functions given, respectively, by

$$f^{\mathrm{u}}(n) = \sum_{i=1}^{M} P_i^{\mathrm{u}}(n) \tag{20}$$

and

$$f^{\mathrm{d}}(n) = \sum_{i=1}^{M}\left\|\mathbf{w}_i^{\mathrm{d}}(n)\right\|^2 = \sum_{i=1}^{M} P_i^{\mathrm{d}}(n) \tag{21}$$

are equal $f^{\mathrm{u}}(n) = f^{\mathrm{d}}(n)$ providing the algorithm convergence is achieved. In this case, the downlink power $P_i^{\mathrm{d}}(n+1)$ is estimated from the uplink power by

$$P_i^{\mathrm{d}}(n+1) = \frac{P_i^{\mathrm{u}}(n+1)}{\left\|\mathbf{w}_i^{\mathrm{u}}(n+1)\right\|^2}. \tag{22}$$

Finally, (13), (17), (18), and (22) represent the operations of the DCLMS algorithm. Note that the knowledge of the DoA of the signals is not necessary.

IV. Simulation Results

Aiming to verify the behavior of the proposed algorithm, we consider a scenario formed by a linear array with K isotropic elements uniformly spaced by half a wavelength calculated from the center frequency of the system operating band (in all simulations reported here, we have used the center frequency normalized to 1 Hz); additionally, one considers multipath-free signals. The involved signals are corrupted by white Gaussian noise. The step-size value μ is determined experimentally assuring the algorithm stability [16].

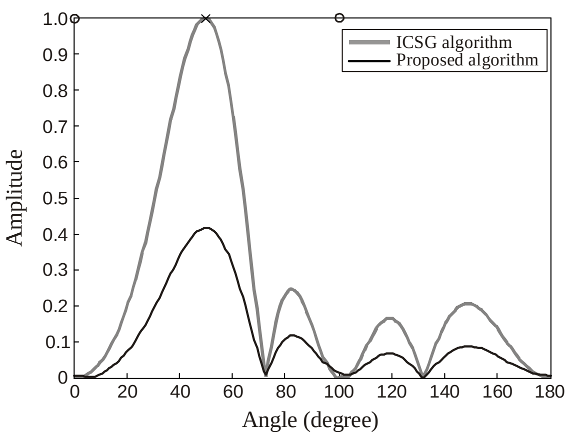
A. Example 1

This example compares the beamforming of the ICSG algorithm [12] with the proposed one. In this example, the figures illustrate the radiation patterns obtained by both the beamforming algorithms. The antenna array is composed of $K = 6$ elements, the channel noise variance is $\sigma_\eta^2 = 0.1$, and the step size used is $\mu = 0.001$. The desired user has AoA of 50° (marked with $\times$). Two pairs of AoAs are considered for the two interfering users, i.e., $(0^\circ, 100^\circ)$ and $(40^\circ, 60^\circ)$ (marked with $\circ$), which are illustrated, respectively, by the radiation patterns shown in Figs. 4(a) and (b).

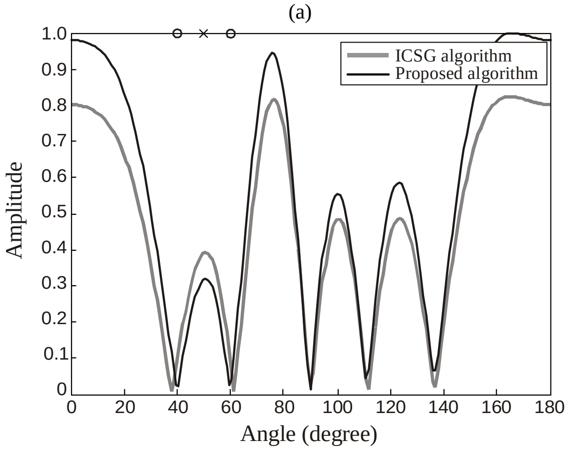
From Fig. 4(a), we observe that when the sources are apart from each other both methods provide almost identical radiation pattern shapes (except for the amplitude differences), placing nulls at the AoA of the interferers. When the AoAs are getting closer, for the case $(40^\circ, 60^\circ)$ [see Fig. 4(b)], the analysis of the obtained results is the same as in Fig. 4(a). In addition, note from Fig. 4 that the proposed approach requires a lower value of power for illuminating the desired user than the ICSG algorithm given in [12].

B. Example 2

In this example, the proposed algorithm behavior is illustrated for the case in which one desired user changes its positions from 0° to 45°, while two other interferers remain at positions 50° and 100°. Fig. 5 shows the radiation patterns and SINR evolution curves, before and after the change at iteration # 30. The constraint value used is $\delta = \delta_i = 1$. For this example, the parameters used are $K = 6$, channel noise variance $\sigma_\eta^2 = 0.1$, and step size $\mu = 0.001$. From the SINR curves, one verifies that the SINR constraints $(SINR_i^{\mathrm{d}} = SINR_i^{\mathrm{u}} = 1)$ are fulfilled for all users. After the AoA change, there exists a transient period in which the SINR value tends to 1 as the algorithm converges [see Fig. 6(a)]. From Figs. 6(b) and (c), we notice that the algorithm adjusts the involved powers aiming to obtain the SINR constraint values equal to 1 in this case.

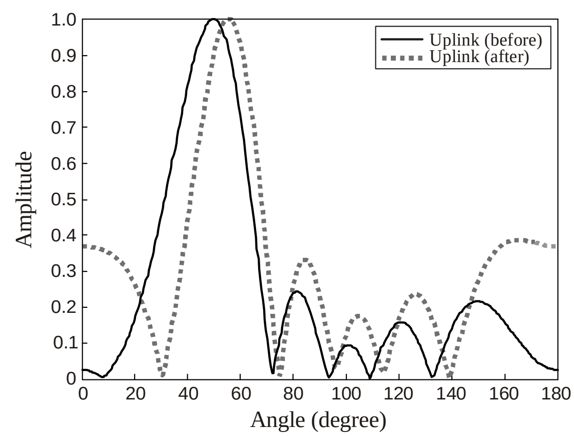


(a)

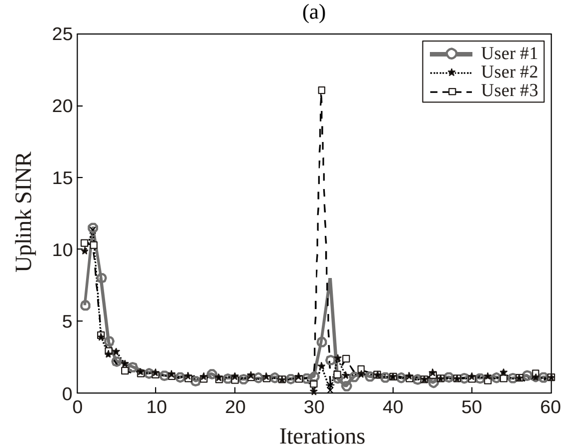


(b)

Fig. 4. Example 1. (a)-(b) Radiation patterns.



(a)



(b)

Fig. 5. Example 2. (a) Uplink channel radiation patterns. (b) Uplink SINR curves.

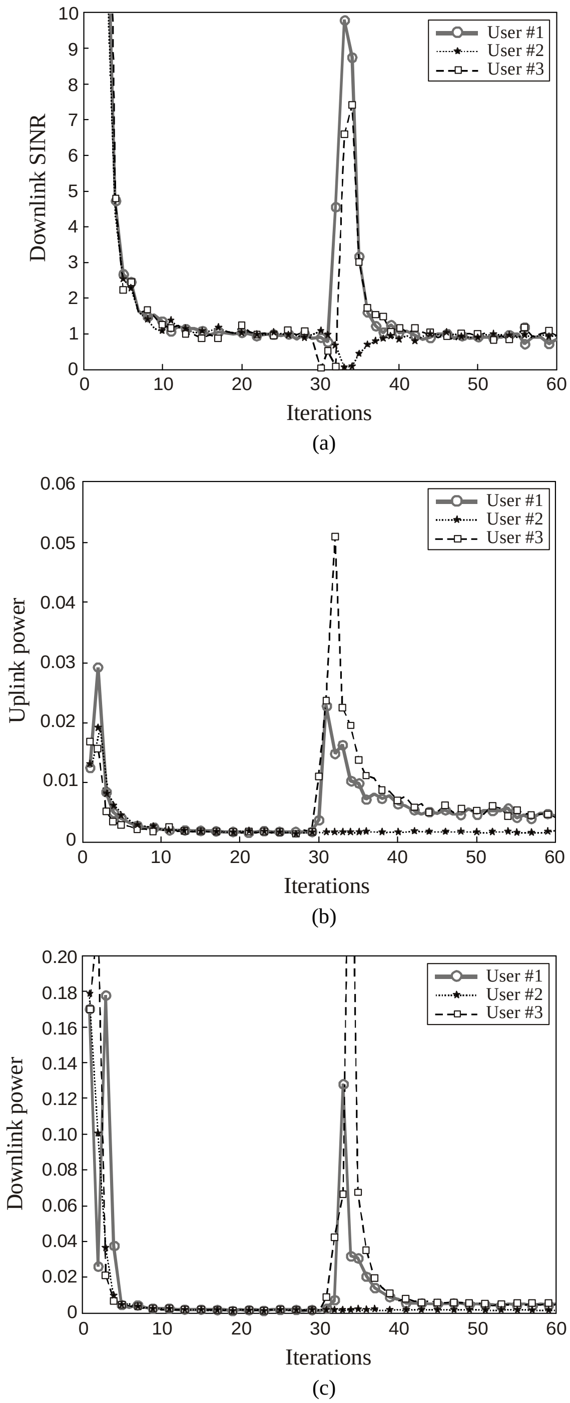


Fig. 6. Example 2. (a) Downlink SINR curve. (b) Uplink and (c) Downlink power curves.

V. Concluding Remarks

This paper presented an adaptive beamforming algorithm for antenna arrays in cellular systems. The proposed algorithm is based on a duality between uplink and downlink channels along with a constrained LMS algorithm. To circumvent the use of DoA algorithms, a reference signal is considered as is made in CDMA systems. By using the proposed scheme, we have achieved better convergence with minimum delivered power. Thereby, the generated self interference in the system is minimized. Numerical simulation results confirmed very good performance of the proposed algorithm.

Acknowledgment

The authors are thankful to the Committee for Postgraduate Courses in Higher Education (CAPES) and the National Council for Scientific and Technological Development (CNPq) by the financial supporting of this research.

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