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Unified Asymptotic Performance of Communication Systems in Fading Channels

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***Abstract*— The performance of wireless communications systems is affected by many components of the fading phenomenon. These components include clustering, nonlinearity, scattered waves, and line of sight. Even though several fading distributions exist which address a multitude of propagation conditions, in many cases the fading models or the associated system performance cannot be obtained in closed form. Therefore, it proves very hard to understand how each fading component impacts system performance. In this work, we introduce a unified asymptotic characterization at high signal-to-noise ratio to obtain simple, general closed-form expressions for the diversity and coding gains of essential performance metrics: symbol error rate and outage probability. Capitalizing on the fact that the asymptotic channel distribution around the origin fully determines the diversity and coding gains, our results provide new insights into how each fading component ultimately affects the performance of communication systems in fading channels.**

***Keywords*— Asymptotic characterization, error rate, fading channels, outage probability.**

I. Introduction

Wireless communications are subject to several fading conditions, including clustering, nonlinearity, scattered waves, and line of sight. Some combinations of these conditions have been incorporated into many probabilistic fading models, which range from the simple one-parameter Rayleigh distribution to the highly sophisticated seven-parameter α-η-κ-μ distribution [1]. These models can be used to assess the system performance in terms of metrics such as symbol error probability (SEP) and outage probability (OP), ultimately helping optimize system design.

For many fading scenarios, an exact performance analysis usually does not offer closed-form solutions [2]. Because of that, it is challenging to address how each fading component affects system performance. In order to overcome such difficulty, some authors have resorted to an asymptotic analysis at high (average) signal-to-noise ratio (SNR), a region of most practical interest for wireless communications [3], [4]. However, such approach has been often explored on a case-by-case basis [2], [5]–[8].

In this work we develop a unified asymptotic characterization to reveal how each of the fading components affects system performance. To this end, we provide simple, unified closed-form expressions for the high-SNR SEP and OP of communications systems in terms of two key asymptotic parameters: the diversity and coding gains. Since the high-SNR SEP and OP depend exclusively on the asymptotic channel distribution around the origin [3], our main objective is to find a comprehensive asymptotic channel distribution that embraces a broad Gaussian class of fading scenarios. As shall be demonstrated, the clustering and nonlinearity are the only fading components that affect the diversity gain, whereas all the addressed fading components affect the coding gain.

In what follows, $f_{(\cdot)}(\cdot)$ denotes probability density function (pdf); $\mathbb{E}[\cdot]$, expected value; $\mathbb{V}[\cdot]$, variance; $(\cdot)^T$, transpose; $\log[\cdot]$, the base-10 logarithm; $\Gamma(\cdot)$, the gamma function; $Q(x) \triangleq \int_x^\infty (1/\sqrt{2\pi}) \exp(-x^2/2) dx$, the Q-function; and "$\sim$", asymptotically equal to around zero, i.e., $f(x) \sim g(x) \iff \lim_{x\to 0} f(x)/g(x) = 1$.

II. Preliminaries

In order to quantify wireless system performance, two metrics commonly used are the SEP and OP, whose formulation relies on knowledge of the probability distribution that models the fading channel. For the high-SNR regime, it was observed by Wang and Giannakis [3] that the SEP and OP can be characterized by the diversity and coding gains, which afford simple parameterization in terms of the channel pdf around the origin. We now revisit how this pdf determines the diversity and coding gains of SEP and OP.

Initially, let B represent the channel power coefficient with asymptotic pdf given by

$$f_B(\beta) \sim a_{B,0} \beta^{b_{B,0}}, \tag{1}$$

where $a_{B,0}$ and $b_{B,0}$ are constants, and B is a nonnegative random variable (RV). Considering an instantaneous SEP in the form of $Q(\sqrt{\nu B \bar{\gamma}})$,[1] where ν is a positive constant that depends on the signaling scheme, and $\bar{\gamma}$ is the average SNR when $\mathbb{E}[B] = 1$, the average SEP (P_E) can be expressed at high SNR as [3]

$$P_\mathrm{E} \sim (G_\mathrm{c} \bar{\gamma})^{-G_\mathrm{d}}, \tag{2}$$

where the diversity gain G_d and the coding gain G_c are obtained in terms of $a_{B,0}$ and $b_{B,0}$ as

$$G_\mathrm{d} = b_{B,0} + 1 \tag{3a}$$

$$G_\mathrm{c} = \nu \left[\frac{2^{b_{B,0}} a_{B,0} \Gamma(b_{B,0} + 3/2)}{\sqrt{\pi}(b_{B,0} + 1)} \right]^{-\frac{1}{b_{B,0}+1}}. \tag{3b}$$

In a similar manner, the OP (P_out) can be expressed at high SNR as [3]

$$P_\mathrm{out} \sim (O_\mathrm{c} \bar{\gamma})^{-O_\mathrm{d}}, \tag{4}$$

[1] This condition assumes additive white Gaussian noise.

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where the diversity gain O_d and the coding gain O_c are obtained as

$$O_\mathrm{d} = b_{B,0} + 1 \tag{5a}$$

$$O_\mathrm{c} = \frac{1}{\gamma_\mathrm{th}} \left(\frac{a_{B,0}}{b_{B,0}+1} \right)^{-\frac{1}{b_{B,0}+1}}, \tag{5b}$$

and $\gamma_\mathrm{th} > 0$ is a certain outage SNR threshold. Besides the system parameters ν and γ_th, note that the diversity and coding gains of SEP and OP only depend on the parameters $a_{B,0}$ and $b_{B,0}$ of the asymptotic channel power pdf around the origin.

Once $a_{B,0}$ and $b_{B,0}$ are accordingly expressed in terms of physical fading parameters, we can outline how the fading conditions affect system performance. To this end, we introduce next a novel asymptotic analysis that leads to $a_{B,0}$ and $b_{B,0}$ in terms of all aforementioned fading components.

III. Asymptotic Analysis

In this section, we develop an asymptotic analysis considering a general fading scenario to ultimately reveal how fading parameters impact high-SNR system performance.

A. General Fading Model

Following standard practice in the literature [1], we consider a general fading model obtained from a sum of squared independent Gaussian RVs. In this way, the channel power B can be written as

$$B^{\frac{\alpha}{2}} = \sum_{i=1}^{M} X_i^2 \triangleq S, \tag{6}$$

where $\alpha > 0$ is a nonlinearity parameter, and each X_i is a Gaussian RV with mean $\mathbb{E}[X_i] = m_i$ and variance $\mathbb{V}[X_i] = \sigma_i^2$. We can arrange the components X_i into the vector form $\boldsymbol{X} \triangleq [X_1 \; X_2 \; \cdots \; X_M]^T$ so that the multivariate pdf of $\boldsymbol{X}$, $f_{\boldsymbol{X}}(\cdot)$, can be formulated in terms of its mean vector $\boldsymbol{m} \triangleq \mathbb{E}[\boldsymbol{X}]$ and covariance matrix $\boldsymbol{\Sigma} \triangleq \mathbb{E}[(\boldsymbol{X}-\boldsymbol{m})(\boldsymbol{X}-\boldsymbol{m})^T]$. Specifically, the positive-definite covariance matrix $\boldsymbol{\Sigma}$ can be expressed as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_M^2 \end{bmatrix}, \tag{7}$$

whose inverse is given by

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sigma_M^2} \end{bmatrix}. \tag{8}$$

Accordingly, the multivariate Gaussian pdf of $\boldsymbol{X}$ can be obtained as

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{\exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{m})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{x}-\boldsymbol{m})\right]}{[(2\pi)^M \det(\boldsymbol{\Sigma})]^{\frac{1}{2}}}. \tag{9}$$

Considering this general fading model, we now determine the asymptotic channel power pdf.

B. Asymptotic Channel Characterization

Our aim is to obtain the asymptotic pdf of B. We start by deriving the asymptotic pdf of $\boldsymbol{X}$, then of $\boldsymbol{X}^2 \triangleq \left[X_1^2 \; X_2^2 \; \cdots \; X_M^2\right]^T$, then of S, and finally of $B = S^{2/\alpha}$.

Using the Maclaurin series expansion of the exponential function in (9) and taking its first term, the asymptotic pdf of $\boldsymbol{X}$ can be written as

$$f_{\boldsymbol{X}}(\boldsymbol{x}) \sim \frac{\exp\left[-\frac{1}{2}\boldsymbol{m}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{m}\right]}{[(2\pi)^M \det(\boldsymbol{\Sigma})]^{\frac{1}{2}}}. \tag{10}$$

For convenience, we let $k_i \triangleq m_i^2/\sigma_i^2$, $\forall i$, such that (10) reduces to

$$f_{\boldsymbol{X}}(\boldsymbol{x}) \sim (2\pi)^{-\frac{M}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{M} k_i\right] \prod_{i=1}^{M} \frac{1}{\sigma_i}. \tag{11}$$

Furthermore, via a simple transformation of variables, we can obtain from (11) the asymptotic pdf of $\boldsymbol{X}^2$:

$$f_{\boldsymbol{X}^2}\left(\boldsymbol{x}^2\right) \sim (2\pi)^{-\frac{M}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{M} k_i\right] \prod_{i=1}^{M} \frac{1}{\sigma_i x_i}. \tag{12}$$

In order to obtain from (12) the asymptotic pdf of S (defined in (6)), we capitalize on a key finding for correlated RVs [4]: under mild conditions, positive correlated RVs behave asymptotically around zero as an equivalent set of mutually independent RVs. Eq. (12) meets those conditions [4, eq. (5)]. Let $\{\check{X}_i^2\}_{i=1}^M$ be this set of independent RVs asymptotically equivalent to $\{X_i^2\}_{i=1}^M$. Also, let the Maclaurin series expansion of the pdf of each $\check{X}_i^2$ be given by

$$f_{\check{X}_i^2}\left(\check{x}_i^2\right) = \sum_{n=0}^{\infty} a_{i,n} \left(\check{x}_i^2\right)^{b_{i,n}} \sim a_{i,0} \left(\check{x}_i^2\right)^{b_{i,0}}, \tag{13}$$

and the Maclaurin series expansion of the pdf of S be expressed by

$$f_S(s) = \sum_{n=0}^{\infty} a_n s^{b_n} \sim a_0 s^{b_0}, \tag{14}$$

where $a_{i,n}, a_n, b_{i,n}$, and b_n are constants, $\forall i, n$. Note that $a_{i,0}\left(\check{x}_i^2\right)^{b_{i,0}}$ and $a_0 s^{b_0}$ denote the asymptote of $f_{\check{X}_i^2}(\cdot)$ and $f_S(\cdot)$, respectively [4]. Comparing (12) with [4, eq. (5)], we obtain $a_{i,0}$ and $b_{i,0}$ as

$$a_{i,0} = (2\pi)^{-\frac{1}{2}} \exp\left[-\frac{1}{2M}\sum_{i=1}^{M} k_i\right] \prod_{j=1}^{M} \sigma_j^{-\frac{1}{M}} \tag{15a}$$

$$b_{i,0} = -\frac{1}{2}, \tag{15b}$$

and substituting (15) into [4, eq. (4)], we attain

$$a_0 = \frac{\exp\left[-\frac{1}{2}\sum_{i=1}^{M} k_i\right]}{2^{\frac{M}{2}}\Gamma\left(\frac{M}{2}\right)} \prod_{i=1}^{M} \frac{1}{\sigma_i} \tag{16a}$$

$$b_0 = \frac{M}{2} - 1. \tag{16b}$$

As a final step, we obtain the asymptotic pdf of $B = S^{2/\alpha}$ by performing once again a transformation of variables, which leads to

$$f_B(\beta) \sim a_{B,0}\beta^{b_{B,0}} = \frac{\alpha a_0}{2} \beta^{\frac{\alpha(b_0+1)-2}{2}}. \tag{17}$$

Using (16) and (17), $a_{B,0}$ and $b_{B,0}$ are then expressed as

$$a_{B,0} = \alpha \frac{\exp\left[-\frac{1}{2}\sum_{i=1}^{M} k_i\right]}{2^{\frac{M+2}{2}}\Gamma\left(\frac{M}{2}\right)} \prod_{i=1}^{M} \frac{1}{\sigma_i} \tag{18a}$$

$$b_{B,0} = \frac{\alpha M}{4} - 1. \tag{18b}$$

From (18), we can obtain a comprehensive characterization of the asymptotic system performance in terms of the various fading parameters, as provided next.

C. Diversity and Coding Gains

Based on the above analysis, we can use (18) to obtain the gains for the SEP and OP, thereby providing an insightful system performance characterization. Accordingly, substituting (18) into (3), we obtain the gains for SEP:

$$G_{\mathrm{d}} = \frac{\alpha M}{4} \tag{19a}$$

$$G_{\mathrm{c}} = \nu \left\{ M \pi^{\frac{1}{2}} 2^{\frac{M}{2}\left(1-\frac{\alpha}{2}\right)} \Gamma\left(\frac{M}{2}\right) \left[\Gamma\left(\frac{\alpha M}{4} + \frac{1}{2}\right)\right]^{-1} \times \exp\left[\frac{1}{2}\sum_{i=1}^{M} k_i\right] \prod_{i=1}^{M} \sigma_i \right\}^{\frac{4}{\alpha M}}. \tag{19b}$$

Also, substituting (18) into (5), we obtain the gains for OP:

$$O_{\mathrm{d}} = G_{\mathrm{d}} \tag{20a}$$

$$O_{\mathrm{c}} = \frac{1}{\gamma_{\mathrm{th}}} \left\{ M 2^{\frac{M}{2}-1} \Gamma\left(\frac{M}{2}\right) \exp\left[\frac{1}{2}\sum_{i=1}^{M} k_i\right] \prod_{i=1}^{M} \sigma_i \right\}^{\frac{4}{\alpha M}}. \tag{20b}$$

Note the implications of (19) and (20). For any values of ν and γ_{th}, the diversity and coding gains of SEP and OP are given in terms of the various elements of the fading model in (6): the number of multipath clusters (M), the nonlinearity of the transmission medium and transceiver electronics (α), the line of sight (m_i), and the mean power of the scattered waves (σ_i^2). In other words, we have a simple and thorough understanding about how each physical aspect of fading impacts system performance at high-SNR conditions, usually required in practice.

IV. Particular Cases

We can reduce the general fading model considered in the previous section to a variety of scenarios and existing fading distributions. Initially, we address the particular case where the multipath clusters X_i are exchangeable RVs [9], which we call commutative scenario.

A. Commutative Scenario

Consider the commutative scenario as the one where the order of the RVs X_i in (6) is irrelevant. Under such constraint, we can eliminate the indices to let $m_i = m$, $\sigma_i = \sigma$, and $k_i = k$, $\forall i$. Using this into (19) and (20), the diversity and coding gains reduce to

$$G_{\mathrm{d}} = O_{\mathrm{d}} = \frac{\alpha M}{4} \tag{21a}$$

$$G_{\mathrm{c}} = \nu \left\{ \sigma^M M \pi^{\frac{1}{2}} 2^{\frac{M}{2}\left(1-\frac{\alpha}{2}\right)} \Gamma\left(\frac{M}{2}\right) \left[\Gamma\left(\frac{\alpha M}{4} + \frac{1}{2}\right)\right]^{-1} \times \exp\left[\frac{kM}{2}\right] \right\}^{\frac{4}{\alpha M}} \tag{21b}$$

$$O_{\mathrm{c}} = \frac{1}{\gamma_{\mathrm{th}}} \left\{ \sigma^M M 2^{\frac{M}{2}-1} \Gamma\left(\frac{M}{2}\right) \exp\left[\frac{kM}{2}\right] \right\}^{\frac{4}{\alpha M}}. \tag{21c}$$

From these expressions, we provide in Section V some insights into the system performance in terms of each fading parameter. Previously, we reduce (19) and (20) to many existing fading models, as detailed next.

B. Existing Fading Models

There are many distributions available in the literature that model the fading channel. In order to reduce our analysis to each specific case by following a common notation used in the literature [1], let

$$K \triangleq \frac{\sum_{i=1}^{M} m_i^2}{\sum_{i=1}^{M} \sigma_i^2}, \quad P \triangleq \frac{M_{\mathrm{x}}}{M_{\mathrm{y}}}, \quad \text{and} \quad Q \triangleq \frac{1}{P}\frac{\sum_{i=1}^{M_{\mathrm{x}}} k_i}{\sum_{i=M_{\mathrm{x}}+1}^{M_{\mathrm{x}}+M_{\mathrm{y}}} k_i},$$

where $M_{\mathrm{x}} > 0$ and $M_{\mathrm{y}} > 0$ are the number of multipath clusters of in-phase and quadrature components, respectively, such that $M = M_{\mathrm{x}} + M_{\mathrm{y}}$.

Table I presents the general gains for many fading models, from the simple Rayleigh distribution to the highly sophisticated α-η-κ-μ distribution. Assume that σ_{x}^2 and σ_{y}^2 denote the variances of the in-phase and quadrature components of the corresponding fading model, respectively.

V. Numerical Results

In this section, we evaluate how the diversity and coding gains vary with the fading parameters M, α, σ, and k. In particular, we illustrate the analysis under the commutative scenario, with $\nu = \gamma_{\mathrm{th}} = 1$. For convenience, in each figure we fix the values of some parameters, where $\sigma = 1$ represents scattered waves of unit powers; $k = 0$, absence of line of sight; $\alpha = 2$ and $\alpha = 10$, linear and nonlinear media, respectively.

A. Diversity Gain

The analysis for the diversity gains is quite simple: note from (21a) that G_{d} and O_{d} are directly proportional to the medium nonlinearity (α) and the clustering (M). This indicates that as α and M increase, so does the magnitude of the slope in the SEP and OP curves, as expected from (2) and (4). Therefore, the fading parameters α and M increase the diversity gain, as illustrated in Figs. 1, 2, 3, and 4, and ultimately dominate the system performance at high SNR.

B. Coding Gain

The analysis for the coding gains is less straightforward. Figs. 1, 2, 3, and 4 depict the coding gains in terms of M, σ, k, and α, respectively. As M increases, Fig. 1 shows that O_{c} increases and, more interestingly, G_{c} decreases. However, this

TABLE I: Diversity and coding gains for existing fading models.

Fading Model	Original Parameterization	Our Parameterization	$G_{\mathrm{d}} = O_{\mathrm{d}}$	G_{c}	O_{c}
Rayleigh	(Ω)	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\Omega}{2}\right]^{\frac{1}{2}}$	1	$2\nu\Omega$	$\frac{\Omega}{\gamma_{\mathrm{th}}}$
Hoyt	(b, Ω)	$\sigma_{\mathrm{x}} = \left[\frac{\Omega(1+b)}{2}\right]^{\frac{1}{2}}$ $\sigma_{\mathrm{y}} = \sigma_{\mathrm{x}}\left[\frac{1-b}{1+b}\right]^{\frac{1}{2}}$	1	$2\nu\Omega\left[(1+b)(1-b)\right]^{\frac{1}{2}}$	$\frac{\Omega}{\gamma_{\mathrm{th}}}\left[(1+b)(1-b)\right]^{\frac{1}{2}}$
Rice	(k, Ω)	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\Omega}{2(k+1)}\right]^{\frac{1}{2}}$ $K = k$	1	$2\nu\Omega\frac{\exp[k]}{k+1}$	$\frac{\Omega}{\gamma_{\mathrm{th}}}\frac{\exp[k]}{k+1}$
Nakagami-m	(m, Ω)	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\Omega}{2m}\right]^{\frac{1}{2}}$ $M = 2m$	m	$\frac{\nu\Omega}{2}\left[\frac{2\sqrt{\pi}\Gamma(m)}{m^{m-1}\Gamma\left(m+\frac{1}{2}\right)}\right]^{\frac{1}{m}}$	$\frac{\Omega}{\gamma_{\mathrm{th}}}\left[\frac{\Gamma(m)}{m^{m-1}}\right]^{\frac{1}{m}}$
Weibull	(α, Ω)	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\Omega}{2}\right]^{\frac{1}{2}}$	$\frac{\alpha}{2}$	$\frac{\nu}{2}\left[\frac{2\sqrt{\pi}\Omega}{\Gamma\left(\frac{\alpha}{2}+\frac{1}{2}\right)}\right]^{\frac{2}{\alpha}}$	$\frac{\Omega^{\frac{2}{\alpha}}}{\gamma_{\mathrm{th}}}$
α-μ	$(\alpha, \mu, \hat{r})$	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\hat{r}^{\alpha}}{2\mu}\right]^{\frac{1}{2}}$ $M = 2\mu$	$\frac{\alpha\mu}{2}$	$\frac{\nu\hat{r}^2}{2}\left[\frac{2\sqrt{\pi}\Gamma(\mu)}{\mu^{\mu-1}\Gamma\left(\frac{\alpha\mu}{2}+\frac{1}{2}\right)}\right]^{\frac{2}{\alpha\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left[\frac{\Gamma(\mu)}{\mu^{\mu-1}}\right]^{\frac{2}{\alpha\mu}}$
η-μ	$(\eta, \mu, \hat{r})$	$\sigma_{\mathrm{x}} = \left[\frac{\eta\hat{r}^2}{2\mu(\eta+1)}\right]^{\frac{1}{2}}$ $\sigma_{\mathrm{y}} = \frac{\sigma_{\mathrm{x}}}{\sqrt{\eta}}$ $M = 4\mu$	2μ	$\frac{\nu\hat{r}^2}{2}\left[\frac{4\sqrt{\pi}\eta^{\mu}\Gamma(2\mu)}{\mu^{2\mu-1}(\eta+1)^{2\mu}\Gamma\left(2\mu+\frac{1}{2}\right)}\right]^{\frac{1}{2\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left[\frac{2\eta^{\mu}\Gamma(2\mu)}{\mu^{2\mu-1}(\eta+1)^{2\mu}}\right]^{\frac{1}{2\mu}}$
κ-μ	$(\kappa, \mu, \hat{r})$	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\hat{r}^2}{2\mu(\kappa+1)}\right]^{\frac{1}{2}}$ $M = 2\mu$ $K = \kappa$	μ	$\frac{\nu\hat{r}^2}{2}\left[\frac{2\sqrt{\pi}\Gamma(\mu)\exp[\kappa\mu]}{\mu^{\mu-1}(\kappa+1)^{\mu}\Gamma\left(\mu+\frac{1}{2}\right)}\right]^{\frac{1}{\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left[\frac{\Gamma(\mu)\exp[\kappa\mu]}{\mu^{\mu-1}(\kappa+1)^{\mu}}\right]^{\frac{1}{\mu}}$
η-κ (Beckmann)	$(\eta, \kappa, \hat{r})$	$\sigma_{\mathrm{x}} = \left[\frac{\eta\hat{r}^2}{(\eta+1)(\kappa+1)}\right]^{\frac{1}{2}}$ $\sigma_{\mathrm{y}} = \frac{\sigma_{\mathrm{x}}}{\sqrt{\eta}}$ $K = \kappa$	1	$\nu\hat{r}^2\frac{4\sqrt{\eta}\exp\left[\frac{\kappa(\eta+1)(q+1)}{2(\eta q+1)}\right]}{(\eta+1)(\kappa+1)}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\frac{2\sqrt{\eta}\exp\left[\frac{\kappa(\eta+1)(q+1)}{2(\eta q+1)}\right]}{(\eta+1)(\kappa+1)}$
α-η-μ	$(\alpha, \eta, \mu, \hat{r})$	$\sigma_{\mathrm{x}} = \left[\frac{\eta\hat{r}^{\alpha}}{2\mu(\eta+1)}\right]^{\frac{1}{2}}$ $\sigma_{\mathrm{y}} = \frac{\sigma_{\mathrm{x}}}{\sqrt{\eta}}$ $M = 4\mu$	$\alpha\mu$	$\frac{\nu\hat{r}^2}{2}\left[\frac{4\sqrt{\pi}\mu^{1-2\mu}\eta^{\mu}\Gamma(2\mu)}{(\eta+1)^{2\mu}\Gamma\left(\alpha\mu+\frac{1}{2}\right)}\right]^{\frac{1}{\alpha\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left[\frac{2\eta^{\mu}\Gamma(2\mu)}{\mu^{2\mu-1}(\eta+1)^{2\mu}}\right]^{\frac{1}{\alpha\mu}}$
α-κ-μ	$(\alpha, \kappa, \mu, \hat{r})$	$\sigma_{\mathrm{x}} = \sigma_{\mathrm{y}} = \left[\frac{\hat{r}^{\alpha}}{2\mu(\kappa+1)}\right]^{\frac{1}{2}}$ $M = 2\mu$ $K = \kappa$	$\frac{\alpha\mu}{2}$	$\frac{\nu\hat{r}^2}{2}\left[\frac{2\sqrt{\pi}\mu^{1-\mu}\Gamma(\mu)\exp[\kappa\mu]}{(\kappa+1)^{\mu}\Gamma\left(\frac{\alpha\mu}{2}+\frac{1}{2}\right)}\right]^{\frac{2}{\alpha\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left[\frac{\Gamma(\mu)\exp[\kappa\mu]}{\mu^{\mu-1}(\kappa+1)^{\mu}}\right]^{\frac{2}{\alpha\mu}}$
α-η-κ-μ	$(\alpha, \eta, \kappa, \mu, p, q, \hat{r})$	$\sigma_{\mathrm{x}} = \left[\frac{\eta\hat{r}^{\alpha}\frac{(p+1)}{(\eta+1)}}{2\mu p(\kappa+1)}\right]^{\frac{1}{2}}$ $\sigma_{\mathrm{y}} = \sigma_{\mathrm{x}}\left[\frac{p}{\eta}\right]^{\frac{1}{2}}, M = 2\mu$ $K = \kappa, P = p, Q = q$	$\frac{\alpha\mu}{2}$	$\frac{\nu\hat{r}^2}{2}\left\{\frac{2\sqrt{\pi}(p+1)^{\mu}\left(\frac{\eta}{p}\right)^{\frac{p\mu}{p+1}}}{\mu^{\mu-1}(\eta+1)^{\mu}(\kappa+1)^{\mu}} \times\frac{\Gamma(\mu)\exp\left[\frac{\kappa\mu(\eta+1)(pq+1)}{(p+1)(\eta q+1)}\right]}{\Gamma\left(\frac{\alpha\mu}{2}+\frac{1}{2}\right)}\right\}^{\frac{2}{\alpha\mu}}$	$\frac{\hat{r}^2}{\gamma_{\mathrm{th}}}\left\{\frac{(p+1)^{\mu}\left(\frac{\eta}{p}\right)^{\frac{p\mu}{p+1}}\Gamma(\mu)}{\mu^{\mu-1}(\eta+1)^{\mu}(\kappa+1)^{\mu}} \times\exp\left[\frac{\kappa\mu(\eta+1)(pq+1)}{(p+1)(\eta q+1)}\right]\right\}^{\frac{2}{\alpha\mu}}$

[a] See [1, Sec. VI] for further details on the original parameterization of the existing fading models.

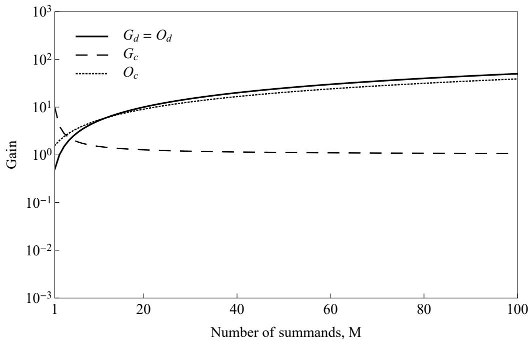

Fig. 1: Diversity and coding gains as function of the number of summands for $\alpha = 2$, $\sigma = 1$, and $k = 0$.

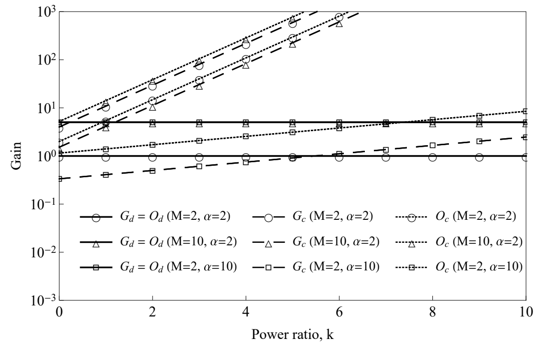

Fig. 3: Diversity and coding gains as function of the power ratio for $\sigma = 1$ and some combinations of M and α.

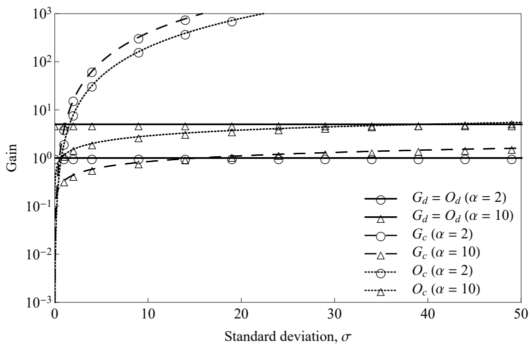

Fig. 2: Diversity and coding gains as function of the standard deviation for varying α, $M = 2$, and $k = 0$.

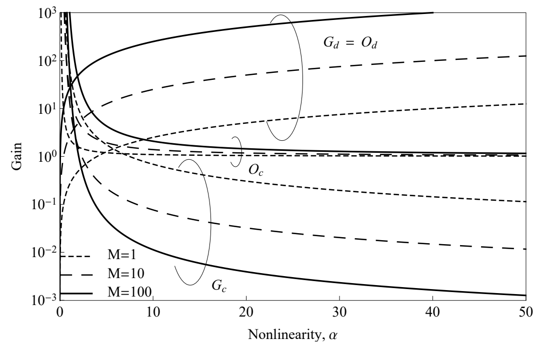

Fig. 4: Diversity and coding gains as function of the nonlinearity for varying M, $\sigma = 1$, and $k = 0$.

does not necessarily mean that the SEP increases, since the diversity gain $G_{\text{d}} = \frac{\alpha M}{4}$ increases with M, and this latter aspect dominates the system performance at high SNR, as already mentioned. In Fig. 2, σ increases the coding gains by $(40/\alpha)\log[\sigma]$ dB, and in Fig. 3, k increases them by $20(k/\alpha)\log[e]$ dB, where e is the Euler's number. These three figures reveal that a growth in the number of multipath clusters (M), in the power of scaretted waves (σ^2), or in the power of specular components (m^2) improves the system performance. Indeed, such growth corresponds to more signal replicas (M) or signal power (σ^2 and m^2) reaching the receiver, thereby decreasing metrics such as SER and OP. In Fig. 4, the coding gains approach infinity as α approaches zero, a region that corresponds to a severe fading condition [1]. Conversely, as α increases, O_{c} approaches unity (zero decibel) and G_{c} approaches zero. In this case, even though G_{c} tends to deteriorate the channel as α approaches infinity, note that $G_{\text{d}} = \frac{\alpha M}{4}$ is also function of α, which dominates the SEP at high SNR. Hence, the system performance improves when the parameters M, σ, k, and α increase, as expected [10], [11].

VI. Conclusions

In this work we have proposed a simple and unified asymptotic performance analysis for communications systems operating over fading channels. The proposed analysis offers insights on how fading parameters affect wireless system performance at high SNR. This can be readily used to evaluate many communications schemes operating in a broad family of propagation scenarios, thus helping optimize system design.

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