Performance of WHT-STC-OFDM in Mobile Frequency Selective Channel

Luciano Leonel Mendes
Electrical Engineering
Inatel
Sta. Rita do Sapucai, MG, BR
luciano@inatel.br

Renato Baldini Filho
DECOM
UNICAMP
Campinas, SP, BR
baldini@decom.fee.unicamp.br

***Abstract*— High data rate systems usually employ Orthogonal Frequency Division Multiplexing and Space-Time Coding to improve the performance on a mobile time-variant frequency-selective channel. Nonlinearities introduced by the power amplifier can be minimized by performing a Wash-Hadamard Transform on the data symbol prior the Inverse Fast Fourier Transform (IFFT) at the transmitter. This procedure reduces the peak-to-average power ratio of the OFDM symbol. An extra advantage of the WHT combined with OFDM is the performance improvement on a frequency-selective channel. The aim of this paper is to present an analytical expression to estimate the performance of the WHT-STC-OFDM taking into account the receiver/transmitter mobility, the channel frequency response and the noise added by the channel. The theoretical results are corroborated by computer simulations.**

Keywords*— *Time-variant frequency-selective channel, OFDM, Space-Time Coding, Walsh-Hadamard Transform.

I. Introduction

Broadband wireless communication is an issue that has been the focus of attention for several years. Different standards, such as DVB (Digital Video Broadcasting) [1], ISDB (Integrated Service Digital Broadcasting) [2], Wi-FI (Wireless Fidelity) [3] and Wi-MAX (Worldwide Interoperability for Microwave Access) [4], use OFDM (Orthogonal Frequency Division Multiplexing) [5] to minimize the harmful distortions of the frequency-selective channels [6]. Space-Time coding, as proposed by Alamouti [7], can be integrated to OFDM to minimize the effects of the Doppler spread in a mobile system [4] [9].

OFDM applied to a power limited system, such as cellular communication systems, presents high PAPR (Peak-to-Average Power Ratio) [5]. High PAPR means that the signal presents high amplitude peaks that may lead the power amplifier to its saturation. An amplifier operating on a nonlinear region introduces ICI (Intracarrier Interference) [10], that reduces the overall performance of the system. The Walsh Hadamard Transform (WHT) technique can be used to reduce the PAPR of OFDM symbols [11] [12].

The WHT combined with OFDM also improves the performance on frequency-selective channels [13] due the dispersion of the information data in all OFDM subcarriers. The occurrence of a deep frequency notch in the OFDM bandwidth do not destroy all the information transmitted in the affected subcarriers.

The aim of this paper is to analyze the performance of the OFDM system combined with the WHT and STC (Space-Time Coding) [14] on a mobile frequency-selective channel. An analytical expression to estimate the performance of this system is devised. Theoretical and simulation results are compared to guarantee its validity.

This paper is organized as follows: Section II presents the principles of the OFDM combined with STC and WHT. In Section III, a theoretical expression to estimate the system performance is evaluated and theoretical and simulation curves are presented for comparison and validation. Finally, Section IV presents conclusion and final remarks.

II. Principles of WHT-STC-OFDM Systems

OFDM system transforms a high data-rate stream into N slow data-rate sub-streams transmitted by mean of N complex subcarriers. The spectral efficiency is preserved by using sub-carriers spaced by

$$\Delta f = R_{mc} = \frac{R_s}{N}, \tag{1}$$

where N is the number of subcarriers, R_{mc} is the subcarrier symbol rate and R_s is the overall symbol rate of the OFDM system. Notice that the overall occupied bandwidth of an OFDM signal is practically equals to the bandwidth of an equivalent single carrier signal [5].

The sampled OFDM signal can be stated as

$$s_m = s(mT_S) = \frac{1}{N}\sum_{i=0}^{N-1} c[i]\exp\left(j\frac{2\pi i}{N}m\right), \tag{2}$$

where $c[i]$ is the complex serial symbols to be transmitted, T_s is the time interval between adjacent samples of the OFDM symbol and m is the time index of the samples. Then, a N-point IDFT (Inverse Discrete Fourier Transform) generates the OFDM signal while a N-point DFT (Discrete Fourier Transform) recovers the desired data at the receiver.

Assuming that the channel frequency response is $H[n]$, then the signal delivered to the decision device is given by

$$c'[i] = H[i]c[i] + W[i], \tag{3}$$

where $W[i]$ is a sample of the complex gaussian noise in the frequency domain for the i^{th} subcarrier. The detector normalizes the received signal by $H[i]$ in order to equalize the channel frequency response. This process increases the noise interference at the subcarriers when $|H[i]| < 1$ and reduces it when $|H[i]| > 1$. The signal-to-noise ratio at each subcarrier is weighted by the channel frequency response at that specific frequency.

The STC uses two transmitting and L_r receiving antennas to obtain a $2L_r$-order diversity gain [7]. The STC is associated with the OFDM to improve the efficiency of high data rate transmission in mobile frequency-selective channels.

Figure 1 presents the block diagram of an STC-OFDM transmitter and Figure 2 presents the block diagram of an STC-OFDM receiver with a single antenna. The transmitted data are arranged in a matrix given by Table I.

TABLE I
STC-OFDM TRANSMISSION MATRIX.

	Antenna 0	Antenna 1
i^{th} subcarrier, n^{th} symbol	$c[n]$	$-c[n+1]^*$
i^{th} subcarrier, $(n+1)^{th}$ symbol	$c[n+1]$	$c[n]^*$

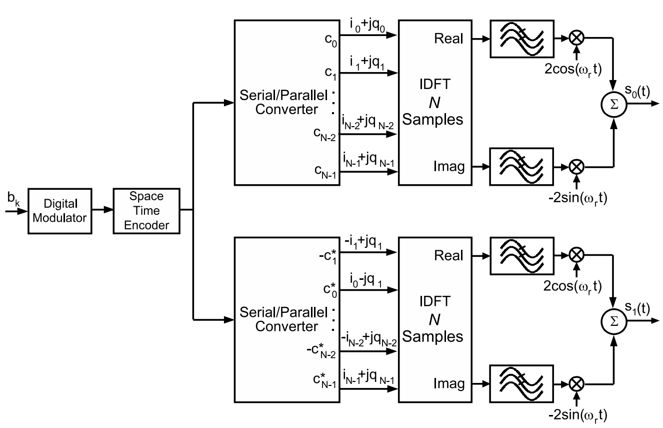

Fig. 1. Block diagram of an STC-OFDM transmitter.

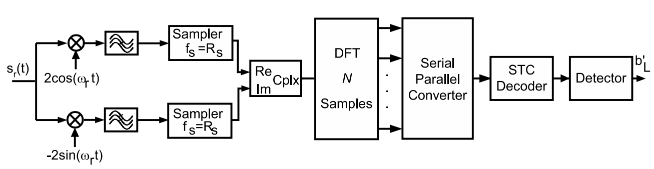

Fig. 2. Block diagram of an STC-OFDM receiver.

The received signal at the input of the DFT block is given by

$$\overrightarrow{s_r} = \overrightarrow{s_0} \star h_0 + \overrightarrow{s_1} \star h_1 + \overrightarrow{w} \quad (4)$$

where $\overrightarrow{s_r}$ is the received complex vector with N samples, $\overrightarrow{s_0}$ is the signal transmitted by the first antenna, $\overrightarrow{s_1}$ is the signal transmitted by the second antenna, h_0 is the channel impulse response between the first transmitting antenna and the receiving antenna, h_1 is the channel impulse response between the second transmitting antenna and the receiving antenna, $\overrightarrow{w}$ is the complex noise vector and $(\star)$ denotes linear convolution operation.

Applying the DFT in (4) leads to the received signal in each subcarrier. The received signals in the i^{th} subcarrier of the n^{th} and $(n+1)^{th}$ OFDM symbols are, respectively, given by

$$\begin{aligned} S_r[n,i] &= c[n]H_0[n,i] - c^*[n+1]H_1[n,i] + W[n,i] \\ S_r[n+1,i] &= c[n+1]H_0[n+1,i] + c^*[n]H_1[n+1,i] + \\ &+ W[n+1,i] \end{aligned} \quad (5)$$

where $c[n]$ is the original transmitted data symbol, $H_0[n,i]$ and $H_1[n,i]$ are the frequency response of the channels at n^{th} time instant and i^{th} subcarrier and $W[n,i]$ is the amplitude spectrum of the noise at the n^{th} time instant and i^{th} subcarrier. Because the i^{th} subcarrier of two adjacent OFDM symbols is employed to construct the codeword, it is not necessary to consider the frequency index i.The channel frequency response is assumed time invariant during the transmission of two adjacent OFDM symbols, which means that $H_x[n,i] = H_x[n+1,i] = H_x[n]$ [15].

The STC decoder combines the received signal at the same subcarrier of two adjacent OFDM symbols in order to obtain diversity gain. Thus, the signal delivered to the detector corresponding to the i^{th} subcarrier of the n^{th} OFDM symbol is given by [15]

$$\begin{aligned} d[n] =& H_0^*[n] \cdot S_r[n] + H_1[n] \cdot S_r^*[n+1] \\ =& \left(|H_0[n]|^2 + |H_1[n]|^2\right) c[n] + H_0^*[n]W[n] + \\ &+ H_1[n]W^*[n+1] \end{aligned} \quad (6)$$

while the signal corresponding to the i^{th} subcarrier of the $(n+1)^{th}$ OFDM symbol is given by

$$\begin{aligned} d[n+1] =& H_0^*[n] \cdot S_r[n+1] - H_1[n] \cdot S_r^*[n] \\ =& \left(|H_0[n]|^2 + |H_1[n]|^2\right) c[n+1] + \\ &+ H_0[n]W[n+1] - H_1[n]W^*[n] \end{aligned} \quad (7)$$

Notice that the receiver knows both channel frequency responses in order to combine the signals received at i^{th} subcarrier of the n^{th} and $(n+1)^{\text{th}}$ STC-OFDM symbols. Pilot subcarriers can be used to estimate the channel frequency response of both channels. The combined symbols, $d[n]$ and $d[n+1]$ are normalized by $\left(|H_0[n]|^2 + |H_1[n]|^2\right)$ to obtain the decision variables that will be used to estimate the transmitted bits.

Observing (6) and (7) it is possible to conclude that a diversity gain of order 2 has been obtained. Notice also that there are two noise samples added at each received symbol. Thus, a 3 dB penalty is expected for the STC scheme when compared with the performance of a Maximum Ratio Combiner Receiver (MRCR), for same overall transmitter mean power in both cases [7].

As already stated, WHT can be used with STC-OFDM to reduce the PAPR of the signal, minimizing the probability of nonlinear distortions in the amplifier [8]. The WHT multiplies the symbol data vector of length N, $c[n]$, by a $N \times N$ matrix given by

$$\Omega_N = \begin{bmatrix} \Omega_{N/2} & \Omega_{N/2} \\ \Omega_{N/2} & -\Omega_{N/2} \end{bmatrix}, \tag{8}$$

where N is a base-2 number and $\Omega_1 = [1]$. The resulting transmitted sequence is given by

$$c_\Omega[n] = c[n] \times \Omega_N. \tag{9}$$

Figure 3(a) presents the block diagram of the WHT-STC-OFDM transmitter while Figure 3(b) presents the block diagram of the WHT-STC-OFDM receiver.

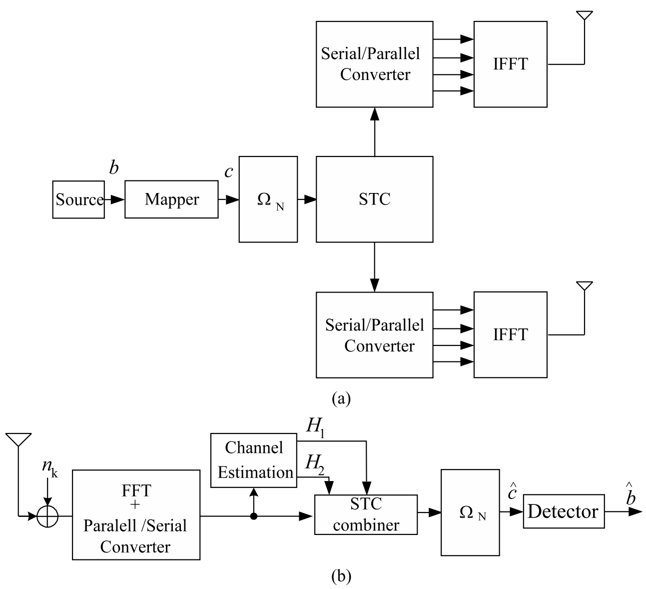

Fig. 3. Block diagram of an WHT-STC-OFDM system. (a) Transmitter. (b) Receiver.

The received signal vector at the output of the STC combiner is applied to the Inverse WHT, that is similar to the matrix product performed at the WHT, since

$$c[n] = c_\Omega[n] \times \Omega_N. \tag{10}$$

III. Performance Analysis

Two different channels are considered in the performance analysis realized in this paper: i) time-invariant frequency-selective channel and; ii) time-variant frequency-selective channel. For each channel, the performance of conventional OFDM and WTH-OFDM are evaluated. The performance of STC-OFDM and WHT-STC-OFDM are also evaluated for the mobile channel.

A. Performance of OFDM and WHT-OFDM on frequency selective channel.

The performance of an M-QAM-OFDM system is equivalent to the performance of a M-QAM single carrier system for large number of subcarriers ($N \geq 64$) [5]. Thus, the symbol error rate evaluated for single carrier M-QAM can be used to estimate the performance of an M-QAM-OFDM system. Therefore, the symbol error rate of an M-QAM-OFDM system is estimated by [5]

$$p_e \approx \frac{4(L-1)}{L} \mathrm{Q}\left(\sqrt{\frac{3\bar{E}}{(L^2-1)\,N_0}}\right), \tag{11}$$

where $L = \sqrt{M}$ is the number of projections of the M possible symbols of the constellation in the x or y axis, $\bar{E}$ is the average symbol energy of the constellation and N_0 is the noise power spectral density. The performance of the M-QAM-OFDM system on a frequency-selective channel is disturbed by the channel frequency response, $H[i]$, that defines the amplitude gain and phase rotation of the i^{th} subcarrier. The symbol error probability of the M-QAM-OFDM system on a frequency-selective channel can be stated as the symbol error probability averaged by the channel frequency response. Thus,

$$p_{e_s} \approx \frac{4(L-1)}{NL} \sum_{i=0}^{N-1} \mathrm{Q}\left(\sqrt{|H_i|^2 \frac{3\bar{E}}{(L^2-1)\,N_0}}\right). \tag{12}$$

For high values of the argument, the Q$-function$ can be tightly approximated by

$$Q(x) \approx \frac{x}{\sqrt{2\pi}\,(1+x^2)} e^{-\frac{x^2}{2}}. \tag{13}$$

Applying (13) in (12) leads to

$$p_{e_s} \approx \frac{4(L-1)}{\sqrt{2\pi}\,NL} \sum_{i=0}^{N-1} \frac{\gamma_i}{1+\gamma_i^2} \times e^{-\frac{\gamma_i^2}{2}}, \tag{14}$$

where

$$\gamma_i = \sqrt{|H_i|^2 \frac{3\bar{E}}{(L^2-1)\,N_0}} \tag{15}$$

Considering the use of the WHT in the M-QAM-OFDM system, each subcarrier transmits a linear combination of N data symbols, given by (9). Then, a deep notch in the channel frequency response will mainly affect the symbols transmitted in the subcarriers positioned at this notch. However, the information conveyed at those subcarriers may be recovered using the data from other subcarriers.

Let the symbol transmitted at the i^{th} subcarrier be:

$$c_\Omega[i] = \sum_{k=0}^{N-1} a_{k,i}\, c[k], \tag{16}$$

where $a_{k,i}$ is the element of the k^{th} row and i^{th} column of (8). Thus, the data received at the output of the frequency domain equalizer in the receiver side are given by

$$c'_\Omega[i] = c_\Omega[i] + \frac{W[i]}{H[i]}. \tag{17}$$

Applying the Inverse WHT in (17) leads to

$$c'[i] = Nc[i] + \sum_{k=0}^{N-1} a_{k,i} \frac{W[i]}{H[i]}, \tag{18}$$

Since $W[i]$ is a gaussian random variable, the value of $a_{k,i}$ does not affect this analysis. It is possible to notice from (18) that if the channel is flat with unitary gain, then the performance of the WHT-OFDM is the same of the conventional OFDM. However, if the channel if frequency-selective, then the effect of the channel on the signal-to-noise ratio of each received symbol will be given by

$$H_{mod} = \left(\sum_{i=0}^{N-1} \frac{1}{N \cdot |H[i]|^2} \right)^{-\frac{1}{2}}, \tag{19}$$

which is constant for all subcarriers.

The result presented in (14) can be used to estimate the performance of the WHT-OFDM system, but the value of γ_i must be modified considering the result presented in (19). Since for a WHT-OFDM system, γ_i is constant and independent of the frequency index, i, (14) can be rewritten for the WHT-OFDM system as

$$p_{e_{\Omega s}} \approx \frac{4(L-1)}{\sqrt{2\pi}\, L} \times \frac{\gamma_\Omega}{1+\gamma_\Omega^2} \times e^{-\frac{\gamma_\Omega^2}{2}}, \tag{20}$$

where

$$\gamma_\Omega = \sqrt{|H_{mod}|^2 \frac{3\bar{E}}{(L^2-1)\, N_0}} \tag{21}$$

Figure 4 presents the performance of OFDM and WHT-OFDM systems with parameters stated in Table II on a channel with impulse response given by

$$h(t) = 0.6\delta(t) + 0.2\delta\,(t - 1.375\mu s) + 0.1\delta\,(t - 2.125\mu s) \tag{22}$$

TABLE II
SYSTEM PARAMETERS.

Parameters	**Value**
Mapping	16-QAM
Total number of subcarriers	2048
OFDM symbol duration (T)	256 μs
Time Guard Interval	$T/32 = 8$ μs
Sampling rate	8 MHz

Notice from Figure 4 that the theoretical equations evaluated in (14) and (20) can be used to estimate the performance of conventional OFDM and WHT-OFDM, respectively, on a frequency selective channel. The use of the WHT combined with OFDM also results in a better performance when compared with a conventional OFDM system.

B. Performance of OFDM, WHT-OFDM and WHT-STC-OFDM on time-variant frequency-selective channel.

The performance of a conventional M-QAM-OFDM system on a time-variant flat-fading channel with Rayleigh

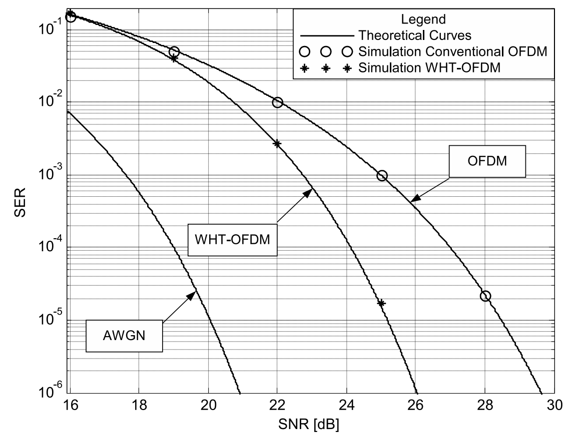

Fig. 4. Performance of conventional OFDM and WHT-OFDM on a frequency-selective channel.

distribution is given by [6]

$$p_{e_m} \approx \frac{2(L-1)}{L} \left(1 - \sqrt{\frac{\eta}{1+\eta}} \right), \tag{23}$$

where

$$\eta = \frac{3\sigma_r^2}{L^2-1} \times \frac{\bar{E}}{N_0} \tag{24}$$

and σ_r^2 is the variance of the orthogonal gaussian random variables used to model the Rayleigh random variable [6].

For a frequency-selective channel, the SNR of each subcarrier is also averaged by the channel frequency response. Thus, the performance of the M-QAM-OFDM system on a frequency-selective mobile channel is given by

$$p_{e_{sm}} \approx \frac{2(L-1)}{NL} \sum_{i=0}^{N-1} \left(1 - \sqrt{\frac{\eta \times |H[i]|^2}{1+\eta \times |H[i]|^2}} \right), \tag{25}$$

If a WHT-OFDM is employed, the modified channel frequency response is obtained by (19), then the symbol error rate is stated as

$$p_{e_{sm\Omega}} \approx \frac{2(L-1)}{L} \left(1 - \sqrt{\frac{\eta \times |H_{mod}|^2}{1+\eta \times |H_{mod}|^2}} \right). \tag{26}$$

Since the channel frequency response of the mobile channel time variant, its performance is actually an average, where the notch position is located in different subcarriers for different OFDM symbols. Therefore, the WHT-OFDM in such a mobile channel does not result in any performance gain when compared to a conventional OFDM system, as can be seen in Figure 5.

The performance of a STC-OFDM system with a single receiving antenna on a mobile flat channel is given by [15]

$$p_{e_{stc}} = \frac{L-1}{L} \times \left(2 - 3\varsigma + \varsigma^3 \right) \tag{27}$$

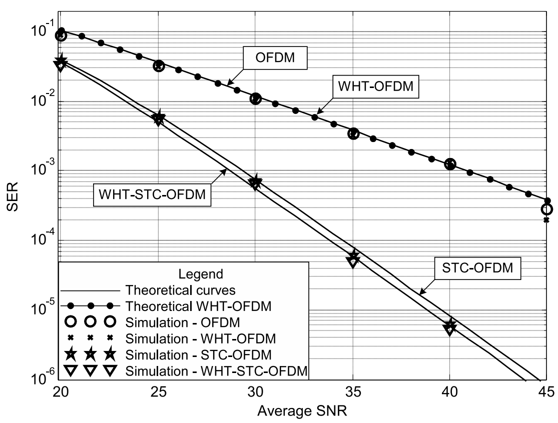

Fig. 5. Performance of conventional OFDM and WHT-OFDM on a frequency-selective mobile channel. Rayleigh distribution with $\sigma_r = 1$.

where

$$\varsigma = \sqrt{\frac{0.25\eta}{1+0.25\eta}} \tag{28}$$

For a frequency-selective mobile channel, the performance of the STC-OFDM is given by

$$p_{es_{stc}} = \frac{L-1}{NL}\sum_{i=0}^{N-1}\left(2-3\varsigma_s+\varsigma_s^3\right) \tag{29}$$

where

$$\varsigma_s = \sqrt{\frac{0.25\eta|H_e[i]|^2}{1+0.25\eta|H_e[i]|^2}} \tag{30}$$

and

$$H_e[i] = \sqrt{|H_0[i]|^2+|H_1[i]|^2} \tag{31}$$

is the equivalent channel frequency response at the input of the slicer in the receiver end.

Finally, if the STC-OFDM is combined with the WHT, the performance of the WHT-STC-OFDM system is evaluated by

$$p_{es_{stc}} = \frac{L-1}{L}\left(2-3\varsigma_\Omega+\varsigma_\Omega^3\right) \tag{32}$$

where

$$\varsigma_\Omega = \sqrt{\frac{0.25\eta|H_{e_{mod}}|^2}{1+0.25\eta|H_{e_{mod}}|^2}} \tag{33}$$

and $H_{e_{mod}}$ is obtained by applying (31) in (19). Figure 5 also presents the theoretical and simulation results obtained for STC-OFDM and WHT-STC-OFDM on a time-variant frequency-selective channel. The use of the WHT slightly increases the overall system performance.

IV. Conclusion

The use of the WHT combined with an OFDM system results in a PAPR reduction. This paper presents analytical approximations that can be used to estimate the symbol error probability of OFDM, WHT-OFDM, STC-OFDM and WHT-STC-OFDM on time-variant frequency-selective channel. All theoretical expressions have been compared with computational simulation, showing that analytical curves fit the simulated ones.

From the results presented in Section III, it is possible to conclude that the use of WHT combined with OFDM results in a large gain when the channel frequency response is not flat. However, in a time-variant frequency-selective channel, the WHT does not result in any extra gain, because the channel frequency response does not present a deep notch in any specific frequency for a large period of time. For the WHT-STC-OFDM, a small gain is observed when compared with STC-OFDM, mainly because the WHT can slightly reduce the chance of deep notches simultaneously occur in a specific frequency on both channels.

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