Comparative Analysis of PMD and PDL on DOP in Optical Communication Systems for NRZ, RZ and CSRZ Signals

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Abstract— **This paper presents a numerical analysis of the dispersive effects of light polarization in optical fibers. This study analyzed the behavior of the degree of polarization using the Jones matrix method for modeling the transmission channel as a concatenation of 15 sections of polarization scramblers of PMD and PDL, using the NRZ, RZ and CSRZ rectangular digital signals. The numerical simulation considers the effects of first order PMD and PDL.**

Keywords- Coherence matrix, degree of polarization, PDL, PMD, Stokes parameters.

I. INTRODUCTION

The polarization of light is an important property that has been widely used in classical and quantum information systems. However, the polarization of light may suffer random variations during transmission over a communication channel. Polarization dependent loss (PDL) and polarization mode dispersion (PMD) are two properties that are found in long distance fiber optical links. These effects may also be present in optical components such as polarizers and birefringent crystals. The combined effect of PMD and PDL affects the performance of optical networks. This has motivated several studies to understand and control the properties of light polarization and the polarization changes introduced by the transmission medium and optical components used. This is done so that the full potential of systems with high transmission rates and long-distance can be used correctly [1,2]. The polarization of light is dependent on several factors, such as shape, composition of core and shell fiber, splices, mechanical stress (curvatures and pressure) and temperature. That is why a strict control of polarization must be done.

In [3] a comparative study between the models of Jones matrices with higher order PMD was done. It was shown in [4] the experimental and theoretical relationship among the degree of polarization (DOP) of signal, the PMD and the optical spectrum. The performance degradation caused by PDL in the presence and absence of PMD showed that the combined effects of both may modify the SNR (signal-to-noise ratio) of transmission systems in optical fibers [5]. The effects of PMD and PDL on the DOP for a 40 Gbps optical system, as a sequence of 127 pseudo-random NRZ signal was presented in [6]. In [7], the relationship between DOP and PMD for different types of signals is discussed in both theoretical and experimental ways. In [8] a comparison between the effects of PDL on the DOP-feedback PMD compensation in RZ and NRZ modulated systems was presented.

This paper makes a numerical analysis of the behavior of the degree of polarization as a function of PMD with and without the presence of PDL for NRZ (nonreturn-to-zero), RZ (return-to-zero) and CSRZ (carrier-suppressed RZ) rectangular pulses.

In section II a review of light polarization in optical networks (polarization, Stokes parameters, Jones matrix method, coherence matrix and degree of polarization) was done. In section III, the theory of PMD and PDL effects was presented. Section IV describes the numerical simulation used in this work and it presents the obtained results. Finally, Section V presents the conclusions.

II. REVIEW OF LIGHT POLARIZATION

A. Polarization

The polarization is the property that demonstrates the vectorial character of electromagnetic field. The phenomenon that degrades the performance of optical communication systems that use light polarization is the depolarization. The polarization of electromagnetic waves $\vec{\mathbf{E}}$ can be classified into three categories: linear, circular and elliptical. What will define the type of polarization is the relative value of the amplitudes E_{0x} and E_{0y} and the phases φ_x and φ_y in the wave equation (1) [7]:

$$\vec{\mathbf{E}} = \Re\left\{E_{0x}e^{j(\omega t-\beta z+\varphi_x)}\hat{a}_x + E_{0y}e^{j(\omega t-\beta z+\varphi_y)}\hat{a}_y\right\}. \tag{1}$$

B. Stokes Parameters and Degree of Polarization

A convenient way to represent any polarization state is given by the four Stokes parameters. These parameters are obtained from measuring the contribution of two orthogonal polarizations for the field intensity. We assume the bases H/V (horizontal/vertical), +45º/-45º (right diagonal/left diagonal), and σ+/σ– (left-hand circular /right-hand circular). The total intensity is

$$S_0 = I_{total} = I_V + I_H = I_{-45} + I_{+45} = I_{\sigma+} + I_{\sigma-}. \tag{2}$$

The difference between the polarization intensities shows S_1, S_2, S_3:

$$S_1 = I_H - I_V,\ S_2 = I_{+45} - I_{-45},\ S_3 = I_{\sigma+} - I_{\sigma-}. \tag{3}$$

The vector $\boldsymbol{S} = [S_0\ S_1\ S_2\ S_3]'$ is called Stokes vector.

So, one way to measure how much light is polarized is using the degree of polarization (DOP) [9]:

$$DOP = \sqrt{S_1^2 + S_2^2 + S_3^2} \big/ S_0. \tag{4}$$

C. Jones Vectors

The electric field has a vectorial nature, this also induces a vectorial representation of the states of polarization. Thus, the polarization can be represented by the Jones vector. The wave electromagnetic field can be represented by $\mathbf{e} = [E_{0x}\ E_{0y}]'$, where E_{0x} and E_{0y} are the components of (1) in the x and y axes, respectively. The Jones vector can be transformed into the Stokes vector with the use of Pauli matrix [9]:

$$S_0 = \mathbf{e}^\dagger I \mathbf{e},\ S_1 = \mathbf{e}^\dagger \sigma_1 \mathbf{e},\ S_2 = \mathbf{e}^\dagger \sigma_2 \mathbf{e},\ S_3 = \mathbf{e}^\dagger \sigma_3 \mathbf{e}, \tag{5}$$

where $\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$

D. Wolf Coherence Matrix

Another representation that describes the polarization is the 2x2 matrix known as the Wolf coherence matrix [1,9]. The J_{mk} elements of the coherence matrix are defined by:

$$J_{mk} = \left\langle E_m E_k^* \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} E_m E_k^* dt \ (m,k = x,y). \tag{6}$$

The matrix $\boldsymbol{J}$ is Hermitian because $J_{xy} = J_{yx}^*$ and it is defined by:

$$\boldsymbol{J} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \vec{\mathbf{E}}^\dagger \vec{\mathbf{E}} dt. \tag{7}$$

The relation between $\boldsymbol{J}$ and the Stokes parameters is given by

$$S_0 = \left\langle E_x E_x^* \right\rangle + \left\langle E_y E_y^* \right\rangle = J_{xx} + J_{yy} \tag{8}$$

$$S_1 = \left\langle E_x E_x^* \right\rangle - \left\langle E_y E_y^* \right\rangle = J_{xx} - J_{yy} \tag{9}$$

$$S_2 = \left\langle E_x E_y^* \right\rangle + \left\langle E_y E_x^* \right\rangle = J_{xy} + J_{yx} \tag{10}$$

$$S_3 = j\left(\left\langle E_x E_y^* \right\rangle - \left\langle E_y E_x^* \right\rangle \right) = j\left(J_{xy} - J_{yx} \right). \tag{11}$$

The degree of polarization is defined as

$$DOP = \left\{ 1 - \frac{4\det(\boldsymbol{J})}{\left[Tr(\boldsymbol{J}) \right]^2} \right\}^{1/2}. \tag{12}$$

III. PMD AND PDL

Two of the physical phenomena that limit the performance of DWDM (Dense Wavelength Division Multiplexing) optical network are the PMD and PDL. The first is due to random variation of the birefringence of optical fiber over time and space, leading to light depolarization or even to the break of pulse information carrier in two. The second tends to polarize the light, since it is a partial polarizer.

Due to the ondulatory nature of light electromagnetic field, the signal energy at a given frequency is decomposed into two orthogonal polarization modes (Principal States of Polarization - PSP) with different propagation velocities due to birefringence, giving rise to polarization dispersion. The differential group delay (DGD) $\Delta\tau$ is the delay between two polarization modes at a point of the fiber in a given instant, as illustrated in Fig. 1.

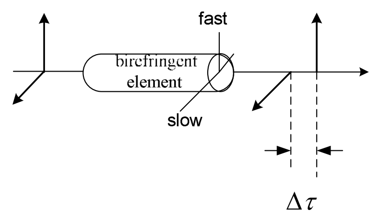

Figure 1. The signal propagates in two orthogonal polarizations with different velocities, causing a delay between the pulses (DGD).

The PMD is the time average of DGDs. The direction of PMD vector is aligned with the slow PSP and the length of PMD vector is the DGD between the fast and slow PSPs [9,10].

The PDL concerns with the loss of energy that is preferred for a given polarization state, that is, a polarization mode suffers more loss than the other. This differential loss changes the output polarization state. The PDL can be represented by a matrix exponential operator given by [1].

$$P = e^{-\alpha/2} \exp\left(\frac{\vec{\alpha} \cdot \vec{\sigma}}{2} \right), \tag{13}$$

where $\vec{\sigma} = [\sigma_1, \sigma_2, \sigma_3]'$, α the loss coefficient $\vec{\alpha} = \alpha\hat{\alpha}$ is the PDL vector and $\hat{\alpha}$ is the unitary vector in the Stokes space that points to the direction of maximum transmission. The PDL in dB is defined by the expression [1,10]:

$$PDL_{dB} = 10\ln\left(\frac{1+\alpha}{1-\alpha} \right). \tag{14}$$

The optical systems often have components such as amplifiers, optical couplers and isolators that may have PDL. These components are inserted between birefringent optical devices on a network, creating an interaction between PMD and PDL that will change the polarization state.

The combination of PMD and PDL creates effects which are highly complex and can affect the communication systems. An example is the power fading caused by the variation of polarization in the input element with PDL [3] due to the existence of at least one PMD element before the element with PDL.

IV. NUMERICAL SIMULATION AND RESULTS

In order to analyze the behavior of light polarization degree in the OOK (on-off keying) modulation, for NRZ, RZ and CSRZ rectangular pulses, it was adopted the channel model used in [6]. The transmission channel is modeled as a concatenation of 15 sections [11] polarization scramblers (*PS*), with PMD and PDL elements in each section as shown in Fig. 2.

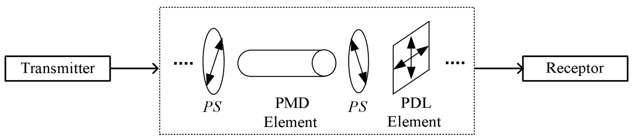

Figure 2. Model of an optical communication system simulated with concatenated birefringent elements.

The Jones transfer matrix of the channel can be modeled in the frequency domain as

$$T(\omega) = e^{-(\alpha_f + j\bar{\beta})L} U(\omega), \tag{15}$$

where α_f, $\bar{\beta}$ and L are the optical fiber constant attenuation, the average propagation constant and the fiber length, respectively, and $U(\omega)$ is a 2x2 matrix that describes the effects of PMD and PDL.

The Jones matrix representing the DGD units of first order $J_\beta(\tau_m)$ and of PDL $J_\alpha(\alpha_m)$ are shown below:

$$J_\beta(\tau_m) = \begin{bmatrix} e^{i\tau_m\omega/2} & 0 \\ 0 & e^{-i\tau_m\omega/2} \end{bmatrix}, \tag{16}$$

$$J_\alpha(\alpha_m) = \begin{bmatrix} e^{\alpha_m/2} & 0 \\ 0 & e^{-\alpha_m/2} \end{bmatrix}. \tag{17}$$

α_m is the loss coefficient and τ_m the DGD of the mth section of the channel. If the total average DGD introduced by a channel of M sections is $\langle \mathrm{DGD} \rangle$, then τ_m is given by [3,10]

$$\tau_m = \sqrt{\frac{3\pi}{8M}} \langle DGD \rangle \cdot \tag{18}$$

Each *PS* is represented by two Jones matrix, as shown below [6]:

$$J_\theta(\theta_{mj}) = \begin{bmatrix} \cos\theta_{mj} & -\sin\theta_{mj} \\ \sin\theta_{mj} & \cos\theta_{mj} \end{bmatrix}, \tag{19}$$

$$J_\phi(\phi_{mj}) = \begin{bmatrix} \cos\left(\frac{\phi_{mj}}{2}\right) & isen\left(\frac{\phi_{mj}}{2}\right) \\ isen\left(\frac{\phi_{mj}}{2}\right) & \cos\left(\frac{\phi_{mj}}{2}\right) \end{bmatrix}, \tag{20}$$

where $m = 1 \sim M$ and $j = 1 \sim 2$. $J_\theta(\theta_{mj})$ denotes a polarization rotator, which rotates the azimuthal angle of θ_{mj}. $J_\phi(\phi_{mj})$ indicates a phase shift of ϕ_{mj}. θ_{mj} e ϕ_{mj} are random between 0 and 2π to simulate the statistical nature of the fiber.

This way, the $U(\omega)$ matrix in (15) is modeled by [6]

$$U(\omega) = \prod_{m=1}^{M} \left\{ \left[J_\theta(\theta_{m1}) J_\phi(\phi_{m1}) J_\beta(\tau_m) J_\phi(-\phi_{m1}) J_\theta(-\theta_{m1}) \right] \cdot \left[J_\theta(\theta_{m2}) J_\phi(\phi_{m2}) J_\alpha(\alpha_m) J_\phi(-\phi_{m2}) J_\theta(\theta_{m2}) \right] \right\} \tag{21}$$

The relationship between the input and output fields is given by:

$$\vec{\mathbf{E}}_{out}(\omega) = T(\omega)\vec{\mathbf{E}}_{in}(\omega), \tag{22}$$

$\vec{\mathbf{E}}_{in}(\omega)$ and $\vec{\mathbf{E}}_{out}(\omega)$ are, respectively, the Fourier transforms of input and output vectors of the electric field of the link. If we consider that the input field is a stationary stochastic process, then the frequency domain, equation (7) can be rewritten as

$$\boldsymbol{J} = \lim_{T\to\infty} \frac{1}{2\pi T} \int_{-\infty}^{\infty} E\left\{ \vec{\mathbf{E}}_{out}^{\dagger}(\omega) \vec{\mathbf{E}}_{out}(\omega) \right\} d\omega, \tag{23}$$

where $E\{\cdot\}$ is the expected value or mean. Since the power spectral density (PSD) of the input signal is given by

$$S(\omega) = \lim_{T\to\infty} \frac{1}{T} E\left\{ \left| \vec{\mathbf{E}}_{in}(\omega) \right|^2 \right\}, \tag{24}$$

one can relate the degree of polarization in (12) with (23).

A. Numerical Simulation of the Degree of Polarization with PMD and PDL

From (12), the behavior of the degree of polarization in the presence of PMD and PDL was numerically calculated. It was analyzed the evolution of the DOP for a 20 Gb/s for rectangular NRZ, RZ (50% and 33%) and CSRZ pulses. The axis of PSP is parallel to the S_1 axis in the Stokes space.

It was simulated numerically the relationship between DOP and PMD with and without the effect of PDL in an optical fiber divided into 15 sections, based on 1000 statistically independent samples for α_m 0, 10, 20 dB. It was used a standard deviation of 20% in the average DGD per section. This standard deviation is known to provide sufficient randomness of the DGD values per section so as to avoid undesired periodicities in the frequency autocorrelation function (which do not occur in real fibers) [12,13].

The power spectral densities for the NRZ, RZ and CSRZ 1/2 rectangular pulses are, respectively, [10]

$$S_{NRZ}(\omega) = \frac{P_0 T_b}{4} \mathrm{sinc}^2(\omega T_b/2) + \frac{\pi}{4} P_0, \tag{25}$$

$$S_{RZ}(\omega) = \frac{P_0 T_b d_c}{4} \mathrm{sinc}^2(\omega T_b d_c/2) \left[1 + \frac{2\pi}{T_b} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi n}{T_b}\right) \right] \tag{26}$$

$$S_{CSRZ}(\omega) = \frac{P_0 T_b}{8} \mathrm{sinc}^2(\omega T_b/4) \left[1 + \sum_{n=-\infty}^{+\infty} (-1)^n e^{jn\omega T_b} \right] \tag{27}$$

P_0, T_b and d_c are the initial power, the bit duration and duty cycle, respectively.

Fig. 3 shows the behavior of the DOP for NRZ rectangular pulses. It is noted a monotonically decreasing of DOP as the DGD increases. One can also see a slight improvement of the DOP with increasing PDL.

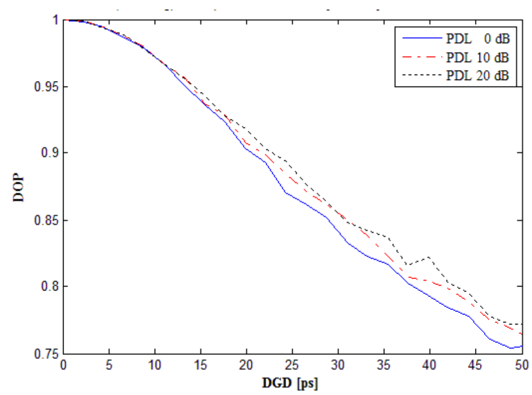

Figure 3. DOP x DGD for NRZ rectangular pulse for α_m 0, 10 e 20 dB.

In Fig. 4, the DOP as a function of DGD is shown for the RZ 50% rectangular pulse. The behavior of DOP is the same for NRZ pulses. The decrease rate of DOP is greater to RZ 50% pulse than NRZ pulse. We can notice a greater sensitivity of DOP in relation to the variation of PDL.

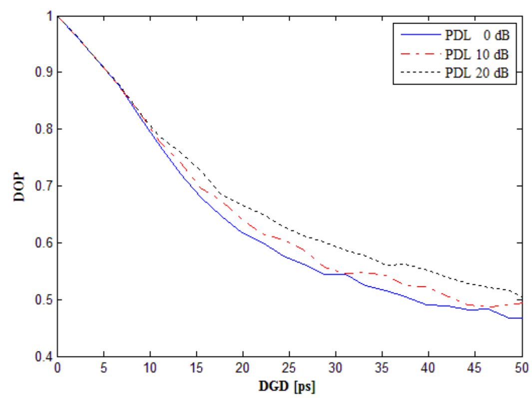

Figure 4. DOP x DGD for RZ 50% rectangular pulse for α_m 0, 10, 20 dB.

The behavior of DOP for RZ 33% rectangular pulses is shown in Fig. 5. We can notice again, that the values of DOP become better with the increase of PDL, but has an inferior performance compared with RZ 50%.

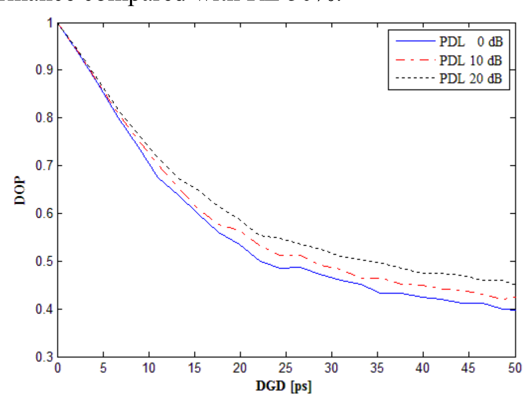

Figure 5. DOP x DGD for RZ 33% rectangular pulse for α_m 0, 10, 20 dB.

Fig. 6 shows the DOP versus DGD for the CSRZ rectangular pulse. The DOP also gets worse with the increase of DGD and improves with the increase of PDL.

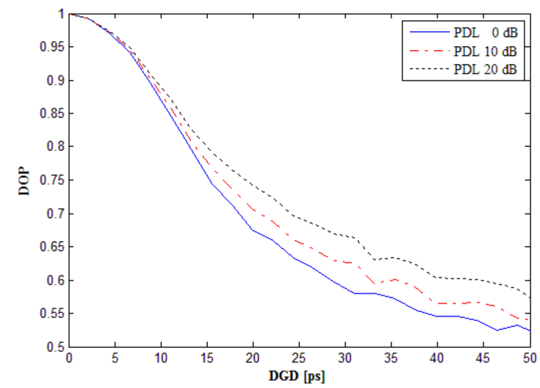

Figure 6. Degree of polarization versus DGD for the CSRZ rectangular pulse for α_m 0, 10 and 20 dB.

Fig. 7 shows the DOP versus DGD without the PDL effect for NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses. We can observe that the NRZ format has the lowest degradation of DOP, followed by CRSZ, RZ 50% and RZ 33%.

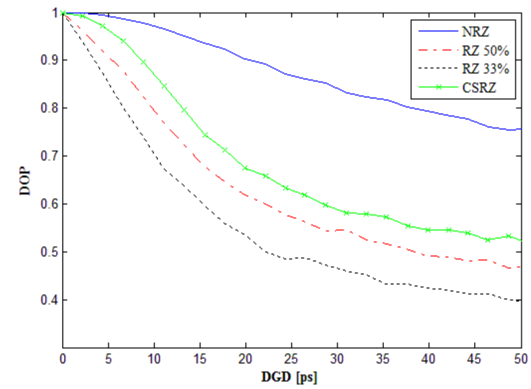

Figure 7. Degree of polarization versus DGD for the NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses for α_m 0 dB.

In Figs. 8 and 9 the behavior of the DOP as a function of DGD for NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses is shown for a PDL of 10 dB and 20 dB, respectively.

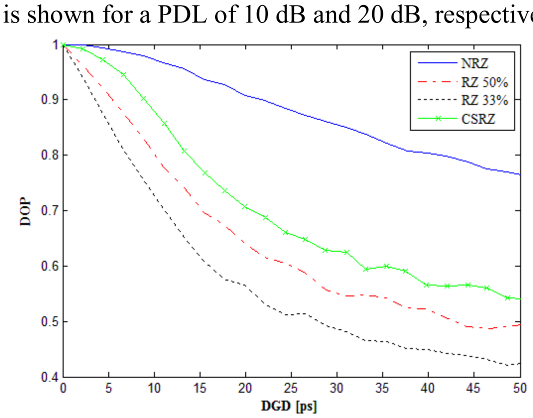

Figure 8. Degree of polarization versus DGD for the NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses for α_m 10 dB.

In Fig. 9, the RZ 50% and CSRZ pulses have DOP values very close for DGD values greater than 20 ps. Again, the

NRZ has a better performance compared with other forms of pulses.

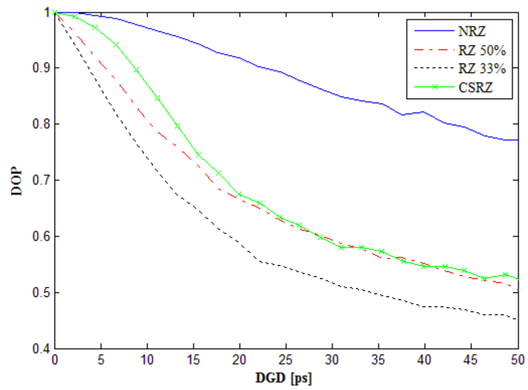

Figure 9. Degree of polarization versus DGD for the NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses for α_m 20 dB.

In Fig. 10, we analyzed the behavior of DOP as a function of PDL for an average DGD of 11ps. It is noticed that the NRZ rectangular pulse keeps the DOP values practically constant for the range of considered PDL values and also has high values of DOP for the same values of PDL than the other pulse shapes analyzed (RZ 50%, RZ 33%, CSRZ). The NRZ format has a lower degradation of DOP for all values of PDL, providing a better system performance under the terms of degree of polarization.

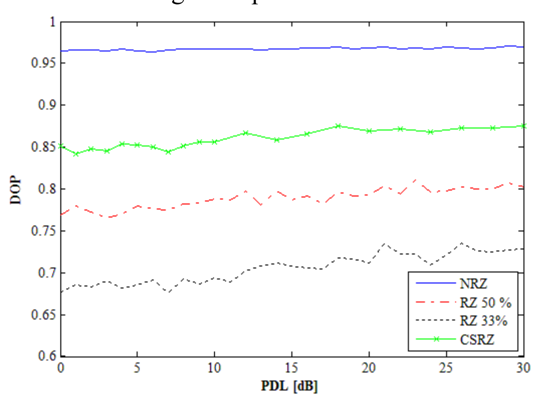

Figure 10. Degree of polarization versus PDL for NRZ, RZ 50%, RZ 33% and CSRZ rectangular pulses for $\langle DGD \rangle = 11$ ps.

V. CONCLUSIONS

This work initially introduced the concept of light polarization and analyzed the depolarization in optical fibers during the transmission process. The DOP, in the output of an optical link, was characterized using mathematical models that allowed the numerical analysis of the behavior of the DOP of optical pulses in a channel with PMD and PDL. Thus, a model was presented and analyzed, being evidenced that the results were correct and consistent when faced with the existing theory.

The DOP has improved with the increase of PDL in all cases presented, but for a fixed DGD of 11 ps the DOP has remained fairly stable for NRZ rectangular pulses, having an average value of DOP of 0.9673 and a standard deviation of 1.5×10^{-3} for a variation of PDL from 0 to 30 dB. While for RZ 50% rectangular pulses, the average DOP was 0.7891 with a standard deviation of 1.21×10^{-2}. The average DOP for the pulse RZ 33% was 0.7054 with a standard deviation of 1.79×10^{-2}. Moreover, the CSRZ rectangular pulse had a mean value of 0.8594 of DOP with a deviation of 1.12×10^{-2}.

It was observed that in RZ signals, when the duty cycle decreases, the greater the degradation of DOP on the increase of DGD.

In general, among the pulse shapes analyzed, the one that shows a better response, considering the DOP, in the presence of dispersive effects of PMD and PDL is the rectangular NRZ. In spite of this advantage, the use of NRZ format becomes questionable at bit rates higher than 10 Gb/s, but it could be a good option in quantum key distribution with weak pulses using polarization encoded qubit.

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