Performance of Simple and Fast Sliding Window Detectors for Spectrum Sensing in Radar Bands

Dayan A. Guimarães and Chang H. Lim

Abstract— **The recently-proposed sliding Gershgorin radii and centers ratio (SGRCR) detector makes use of multiple sensing rounds applying a short window that slides over the full sensing interval, being adequate for detecting short duration signals, like those emitted by pulse radar systems. This articles proposes the sliding Gini index detector (SGID) as a variation of the SGRCR and compares the performances of both in the context of cooperative spectrum sensing for cognitive radio applications in radar bands. These detectors have approximately the same computational complexity, but the SGID can outperform the SGRCR in most of the system parameterizations and scenarios.**

Keywords— **Cognitive radio, cooperative spectrum sensing, GID, GRCR, pulse radar, SGID, SGRCR, sliding window.**

I. INTRODUCTION

The scarcity and congestion of the radio-frequency (RF) spectrum have become formidable obstacles to the deployment of existing wireless communication systems and to the development of new ones such as the fifth generation (5G) of communication networks and the Internet of things (IoT), mainly because the unprecedented increase in the demand for new services. The problem is aggravated by the traditional static spectrum allocation policy, in which incumbent networks have the exclusive right of using a given portion of frequencies.

The cognitive radio (CR) concept [1] is being considered one of the most promising solutions to the problem of RF spectrum scarcity and congestion. Among the attributes of a CR, the spectrum sensing capability is of primary interest here. In short, the spectrum sensing allows for a secondary user (SU) to decide upon the presence or absence of the primary user (PU) signal in a given sensed band and to access the band if it is not occupied by the PU, thus leading to more efficient spectrum utilization [2].

Radar bands are potential candidates for CR networks, since they are considerably underutilized and are significant wide [3]. As a matter of fact, the Federal Communications Commission (FCC) has already regulated that wireless local area network (WLAN) devices can detect radar signals, switching to another channel to avoid interference with such signals [4]. Moreover, research efforts [5][6] have been made to allow the coexistence of radar and wireless systems for higher spectral utilization.

A pulse radar transmits RF pulses with short duration and low duty-cycle for probing the environment. In order to exploit the time sparsity of the pulse radar signal for spectrum sensing, in [7] the conventional signal detection event that takes place during a sensing interval has been converted into multiple short-time sliding window detections whose results are combined to yield the final decision upon the occupation of the sensed band.

Although there is no specific requirement for the choice of the detectors to be adopted in the sliding window approach, the Gerschgorin radii and centers ratio (GRCR) detector [8] has been chosen in [7] mainly due to its low computational complexity, which is desirable to reduce the latency of the multiple detections. Another detector recently developed, with an implementation complexity very close to the GRCR, is the Gini index detector (GID) [9]. Besides having one of the lowest computational complexities known so far, the GRCR and the GID are completely blind, exhibit the constant false alarm rate (CFAR) property, and are robust under noise and received signal powers that are not the same over all SUs and are variant over time, which is hereafter referred to as the nonuniform-dynamical noise and signal powers. The application of the GRCG to the sliding window spectrum sensing approach gave rise to the term sliding GRCR (SGRCR) in [7]. Similarly, here the application of the GID to this approach is named sliding GID (SGID). In this paper, the SGRCR and the new SGID are compared when applied to the detection of pulse radar signals under different sets of system parameters and scenarios. As a benchmark, the well-know energy detector (ED) is included in the comparisons, identified as the sliding ED (SED).

The remainder of this paper is organized as follows. Section II describes the system model, including the signal model, the GRCR, the GID and the ED test statistics, and the sliding window approach applied to these statistics. Section III is devoted to the numerical results and Section IV concludes with some opportunities for further related research.

II. SYSTEM MODEL

Following [7], here we also consider a cooperative spectrum sensing (CSS) scheme with m cognitive SUs collecting mn samples (n samples per SU) of the radar signal received during a given sensing interval.

A. Signal model

The signal samples collected by the SUs in cooperation are forwarded to the fusion center (FC), where the received signal

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matrix $\mathbf{Y} \in \mathbb{C}^{m \times n}$ is formed according to

$$\mathbf{Y} = \mathbf{h}\mathbf{x}^{\mathrm{T}} + \mathbf{V}, \qquad (1)$$

where T denotes transposition, and $\mathbf{h} = [h_1, h_2, \ldots, h_m]^{\mathrm{T}}$ is the channel vector with h_i representing the complex channel gains between the radar transmitter and the i-th SU receiver, for $i = 1, \ldots, m$. These gains are assumed to be constant during the sensing interval (the sensing interval is sufficiently small compared to the channel coherence time), and independent and identically distributed between consecutive sensing rounds (the interval between two consecutive sensing events is sufficiently large compared to the channel coherence time).

In order to account for possibly different received signal powers, which characterizes the nonuniform-dynamical signal scenario, and to model a flat and slow Ricean fading channel, it follows that, in each sensing event, $\mathbf{h} = \mathbf{G}\mathbf{a}$, where $\mathbf{a} \in \mathbb{C}^{m \times 1}$ is the vector whose elements are $a_i \sim \mathbb{CN}[\sqrt{K/(2K+2)}, 1 - K/(K+1)]$, for $i = 1, \ldots, m$, which guarantees unitary second moment of the fading magnitude, with K being the Rice factor[1]. The matrix $\mathbf{G} \in \mathbb{R}^{m \times m}$ is a diagonal gain matrix given by

$$\mathbf{G} = \mathrm{diag}\left(\sqrt{\frac{\mathbf{p}}{p_{\mathrm{avg}}}}\right), \qquad (2)$$

with $\mathbf{p} = [p_1, p_2, \ldots, p_m]^{\mathrm{T}}$ being the vector containing the received signal power levels in each SU, where $p_{\mathrm{avg}} = \frac{1}{m}\sum_{i=1}^{m} p_i$ is the average received signal power over all SUs. Assuming, without loss of generality, that the overall channel power gain is unitary, the radar signal is transmitted with a constant average power p_{avg}.

The matrix $\mathbf{V} \in \mathbb{C}^{m \times n}$ in (1) is formed by independent and identically distributed zero mean complex Gaussian noise samples. Under the possibility of nonuniform-dynamical noise variances across the SUs, the elements in the i-th row of $\mathbf{V}$ have variance σ_i^2, $i = 1, \ldots, m$.

The vector $\mathbf{x} \in \mathbb{R}^{n \times 1}$ in (1) represents the radar signal samples. In practice, this signal is formed by bursts of short time pulses with very low duty-cycle [7]. From the perspective of the detector, the received signal from a rotating radar is seen as a series of bursts. If the number of pulses transmitted in a burst is N, the number of pulses eventually received by the SU depends on the aperture of the antennas and on the speed of the radar antenna rotation. Thus, here it is assumed that the number of received pulses within the sensing interval is a uniform random variable $U \sim \mathcal{U}[1, N]$ to correctly represent the asynchronous operation between the radar bursts and the spectrum sensing interval.

Denoting the average noise variance as $\sigma_{\mathrm{avg}}^2 = \frac{1}{m}\sum_{i=1}^{m} \sigma_i^2$, the approximate received signal-to-noise ratio, in dB, averaged over all SUs, is given by [7]

$$\mathrm{SNR} \approx 10\log_{10}\left[\frac{(N+1)p_{\mathrm{avg}}}{2N\sigma_{\mathrm{avg}}^2}\right]. \qquad (3)$$

[1]In a multipath fading channel, the Rice factor is the ratio between the power in the dominant multipath component and the power of the remaining ones. A larger K means a stronger line-of-sight (LoS) received signal. If $K = 0$, the Ricean fading specializes to the Rayleigh fading, which corresponds to no LoS.

This calculation is approximate due to the random number of radar pulses seen by the SUs during a sensing interval, which may be a non integer; expression (3) was derived assuming that this number is integer.

B. The GRCR, the GID, and the ED detectors

At the FC, the received signal sample covariance matrix (SCM) is computed from (1) as

$$\mathbf{R} = \frac{1}{n}\mathbf{Y}\mathbf{Y}^{\dagger}, \qquad (4)$$

where $\dagger$ denotes the complex conjugate and transpose.

The Gerschgorin radii and centers ratio (GRCR) test statistic defined in [8] is

$$T_{\mathrm{GRCR}} = \frac{\sum_{i=1}^{m}\sum_{j=1, j \neq i}^{m}|r_{ij}|}{\sum_{i=1}^{m} r_{ii}}, \qquad (5)$$

where r_{ij} is the element in the i-th row and j-th column of $\mathbf{R}$, for $i, j = 1, \ldots, m$.

In the case of the GID, the test statistic given in [9], apart from a constant factor that does not influence performance, is

$$T_{\mathrm{GID}} = \frac{\sum_{i=1}^{m^2}|r_i|}{\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}|r_i - r_j|}, \qquad (6)$$

where r_i is the i-th element of the vector $\mathbf{r}$ formed by stacking all columns of $\mathbf{R}$.

The energy detector test statistic is simply

$$T_{\mathrm{ED}} = \sum_{i=1}^{m}\frac{1}{\sigma_i^2}\sum_{j=1}^{n}|y_{ij}|^2, \qquad (7)$$

where y_{ij} is the element on the i-th row and j-th column of $\mathbf{Y}$. It is clear that the ED is not completely blind, since it uses the noise variance σ_i^2 at the input of the i-th SU receiver. In practice, σ_i^2 is estimated and, as a consequence, the performance of the ED significantly depends on the quality of the estimate.

As reported in [8], [9], the computational complexity of the ED is $\mathcal{O}(nm)$, which is the smallest one. The GRCR has roughly the same complexity of the GID, which is $\mathcal{O}(nm^2)$, mainly owing to the computation of the matrix $\mathbf{R}$. Hence, to the best of the authors' knowledge, the GRCR and the GID are the least complex blind detectors available in the literature, making them attractive to the sliding window approach described in the sequel in order to reduce latency.

C. The sliding window approach

The sliding window detection approach works by successively shifting a small sensing window through the whole sensing interval [7]. An intermediate decision on the presence or absence of the radar signal is made for each step of the sliding window. When the window reaches the end of the sensing interval, an operation is made among the intermediate decisions to yield the final global decision.

Working under the divide-and-conquer principle, the sliding window technique exchanges the reduced performance of individual smaller sensing windows by an increase of the

captured energy of a radar pulse that by chance falls into one or more intermediate sensing windows. This is combined with the fact that the noise energy during such small sensing windows is smaller than the one present in the whole sensing interval, potentially resulting in some performance improvement of the radar signal detection.

From Figure 1 of [7], it can be found that the number of samples collected by each SU, n, the number of radar pulses per burst, N, the radar pulse width, W_p, and the radar signal duty-cycle, D, are related via

$$n = \frac{W_p N}{D}. \tag{8}$$

Moreover, the number of steps S_n, which is the total number of sensing rounds made during the entire sensing interval, is given by [7]

$$S_n = \frac{n - S_w}{S_s} + 1, \tag{9}$$

where S_w is the sliding window size, and S_s is the step size.

In each sensing event, the SGRCR, the SGID or the SED test statistic is formed respectively from (5), (6) or (7), computed from S_n received signal matrices $\mathbf{Y}$ shorter than (1), having order $m \times S_w$ instead of $m \times n$.

The global decision is made after the OR-logic operation among all intermediate decisions. This is equivalent to say that the decision will be made in favor of the presence of the radar signal if $\max\{T(1), T(2), \ldots, T(S_n)\} > \gamma$, where γ is the decision threshold, and $T(k)$ denotes the test statistic computed at the intermediate sensing round number k, with $k = 1, \ldots, S_n$.

III. NUMERICAL RESULTS

The metrics often used to assess the spectrum sensing performance are the probability of detection and the probability of false alarm, defined as $P_d = \Pr(\text{decision} = \mathcal{H}_1|\mathcal{H}_1)$ and $P_{fa} = \Pr(\text{decision} = \mathcal{H}_1|\mathcal{H}_0)$, respectively, where $\mathcal{H}_1$ and $\mathcal{H}_0$ are the hypotheses of the presence (i.e. $\mathbf{Y} = \mathbf{h}\mathbf{x}^T + \mathbf{V}$) and absence (i.e. $\mathbf{Y} = \mathbf{V}$) of the radar signal, respectively, and $\Pr(\cdot)$ is the probability of the underlying event.

A typical graphical tool for analyzing P_d and P_{fa} simultaneously is the receiver operating characteristic (ROC) curve, which trades P_{fa} versus P_d by varying the decision threshold γ.

A condensed metric also often used is the area under the ROC curve (AUC). The worst and useless performance, which corresponds to a ROC curve with $P_d = P_{fa}$, gives AUC = 0.5. The best performance corresponds to a ROC curve attaining $P_d = 1$ and $P_{fa} = 0$, yielding AUC = 1.

In the following we present results of the AUC as a function of variations in all system parameters that are relevant to the spectrum sensing performance. These results were produced by computer simulations using the MATLAB version R2018a, from 20000 Monte Carlo events in which the received signal matrix $\mathbf{Y}$ was generated under the hypothesis $\mathcal{H}_1$ (to estimate P_d) and $\mathcal{H}_0$ (to estimate P_{fa}). The AUCs were computed using the built-in MATLAB function $-\texttt{trapz}(P_{fa}, P_d)$. When a given parameter is not the one that is varied, its value was set to: $m = 5$ SUs, radar signal with duty-cycle $D = 5\%$, maximum of $N = 4$ pulses per burst during the sensing

interval under the $\mathcal{H}_1$ hypothesis, $n = 1200$ samples collected by each SU, $W_p = 15$ samples per radar pulse, sensing window size S_w equal to W_p and equal to the step size of the sliding window, S_s, average SNR $= -18$ dB, and sensing channel Rice factor $K = 6$.

When noise and signal powers are uniform (the same across the SUs and constant over time), $\sigma_i^2 = \sigma_{avg}^2 = 1$, and $p_i = p_{avg}$ according to the desired SNR; see (3). In the more realistic scenario in which noise and signal powers are nonuniform-dynamical (different across the SUs and time-varying), $\sigma_i^2 \sim \mathcal{U}[0.05\sigma_{avg}^2, 1.95\sigma_{avg}^2]$ and $p_i \sim \mathcal{U}[0.05p_{avg}, 1.95p_{avg}]$ in each spectrum sensing event.

Since the radar burst and the sensing interval are not synchronous to each other, the beginning of the first radar pulse over the sensing interval is random. By restricting its beginning to be uniformly distributed over $[0, n - W_p - 1]$, it is guaranteed that, under $\mathcal{H}_1$, at least a single entire pulse is present during the whole sensing interval.

Figure 1 gives the AUC as a function of the sliding window size S_w, assuming uniform noise and received signal powers (left), and nonuniform-dynamical noise and received signal powers (right). In this case, $n = 1200$ samples, and $S_n = 80$, 40, 20, 10, 5, 2, and 1 sensing rounds per sensing interval, respectively for $S_w = S_s = 15, 30, 60, 120, 240, 600$, and 1200 samples.

A first conclusion obtained from Figure 1 is that the detection of a radar signal benefits from the sliding window technique, since the spectrum sensing performance may be improved especially for small window sizes. It is worth highlighting that, as we noted in the previous paper [7], the computational complexity for obtaining S_n SCMs in terms of complex multiplications is $S_n S_w m^2$, which turns out to be nm^2 when $S_w = S_s$; see (9). Thus, when $S_w = S_s$ the computational complexity does not vary with the sliding window size S_w.

Another interesting conclusion that can be drawn from Figure 1 is that only the SGRCR shows almost the same performances for the two situations of uniform and nonuniform-dynamical signal and noise powers, meaning that its robustness to variations of noise and signal levels over time was inherited from the robustness of the GRCR detector demonstrated in [8]. Unexpectedly, the SGID inherited the robustness of the GID demonstrated in [9] only for small window sizes, as will be confirmed by all subsequent results. However, the justification for this behavior is still unknown by the authors.

Still from Figure 1, it can be inferred that the SGID can beat the SGRCR, and even the SED when noise and received signal powers across the SUs are uniform. Here, the SED is assumed to have the perfect knowledge of noise powers, which needs to be taken into account for performance comparisons.

The AUC versus the sensing channel Rice factor K is presented in Figure 2. It confirms that the superiority of the GID with respect to the GRCR is maintained in the SGID and SGRCR variants when some line-of-sight component exists in the received radar signal. The GID exploits the degree of variation of the elements of an SCM for deciding on the presence of a PU signal, and may benefit from higher values of K that may decrease the denominator of its test statistic (6).
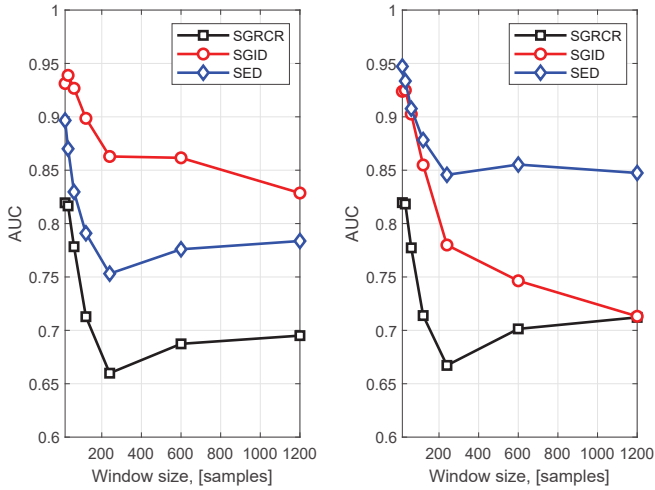
Fig. 1. AUC versus window size (S_{w}): uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right). The smallest window size is $S_{\mathrm{w}} = W_{\mathrm{p}} = 15$ samples; the largest is $S_{\mathrm{w}} = n = 1200$.

In the results presented hereafter, it has been adopted $K = 6$ to represent a mild line-of-sight situation.

Still referring to Figure 2, again it can be verified that the SGID can beat even the SED in the case of uniform noise and received signal powers. Moreover, as anticipated in the comments regarding Figure 1, it can be seen in Figure 2 that the SGID is quite robust against nonuniform-dynamical noise and received signal powers for small sensing window sizes ($S_{\mathrm{w}} = 15$ in this case).
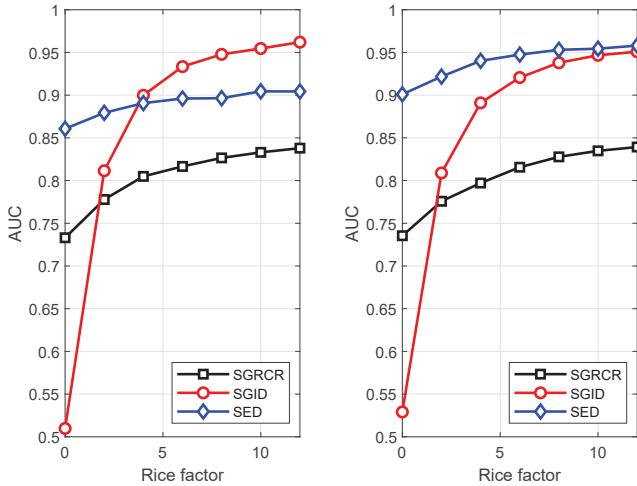

Fig. 2. AUC versus Rice factor (K): uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right).

Figures 3 and 4 give the AUC as a function of the average SNR across the SUs, and as a function of the number m of SUs, respectively. As expected, larger values of SNRs or m yield better spectrum sensing performances, but in a diminishing-return fashion. The SGID is shown to be superior to the SGRCR for most of the SNRs. A very small advantage of the SGRCR over the SGID is observed in the scenario of

nonuniform-dynamical noise and received signal powers, for SNRs around -12.5 dB and above. The SGID can outperform the benchmark SED under uniform noise and received signal powers, for SNRs around -15 dB and below.
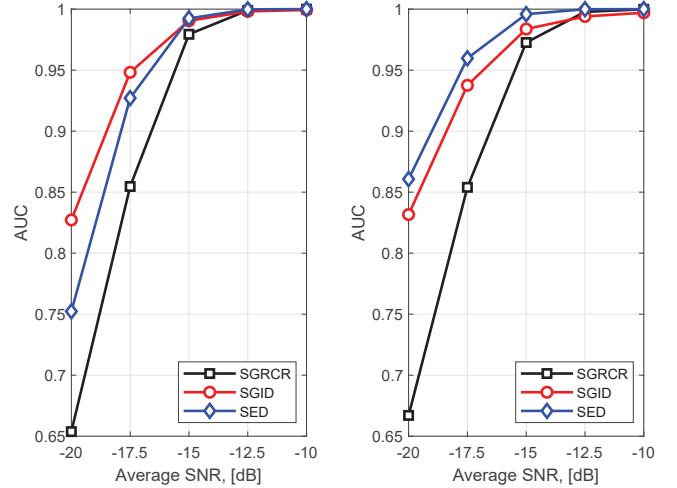

Fig. 3. AUC versus average SNR over all SUs: uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right).

Regarding the number of SUs (see Figure 4), the SGID outperformed the SGRCR for all analyzed values of m. The SGID can even beat the SED for $m \geq 4$, in the uniform noise and received signal powers setting. The robustness of the SGRCR and the SGID is once again confirmed in Figures 3 and 4, with a slight advantage of the SGRCR.
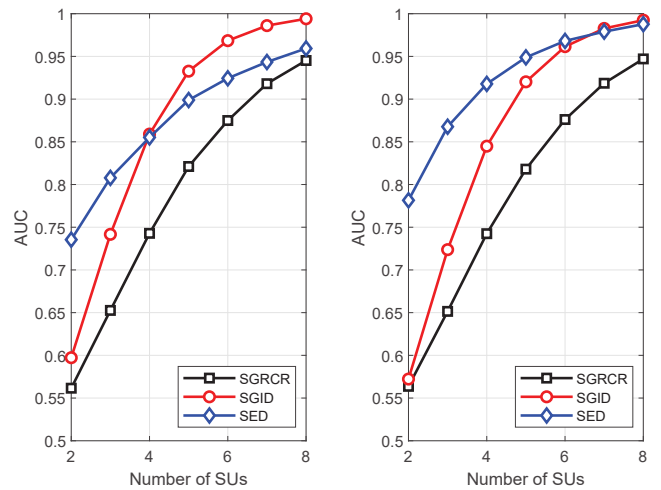

Fig. 4. AUC versus the number of SUs (m): uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right).

Finally, the AUC versus the radar signal duty-cycle D, and versus the radar pulse width W_{p} are shown in Figures 5 and 6, respectively. In the case of Figure 5, it follows that $n = 6000$, 3000, 1200, 600, and 300 samples, and $S_{\mathrm{n}} = 400$, 200, 80, 40, and 20 sensing rounds, respectively for $D = 0.01$, 0.02, 0.05, 0.1, and 0.2. In the case of Figure 6, it follows that

$n = 160$, 240, 400, 800, 1600 and 2400 samples, respectively for $W_\mathrm{p} = S_\mathrm{s} = S_\mathrm{w} = 2$, 3, 5, 10, 20 and 30 samples, with a fixed $S_\mathrm{n} = 80$ sensing rounds.

The most important conclusion obtained from Figure 5 and Figure 6 does not come from the absolute performance variations, but from the relative ones. This is because the variation of D and W_p are accompanied by the variation of other performance-relevant parameter, n, according to (8) and (9). It can be observed that the SGID can outperform the SGRCR and even the SED for a wide range of D; the performance differences become smaller as D becomes small.
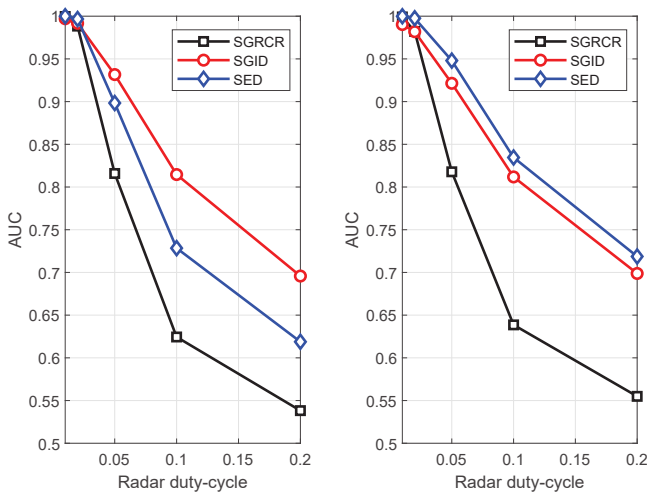

Fig. 5. AUC versus radar signal duty-cycle (D): uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right).
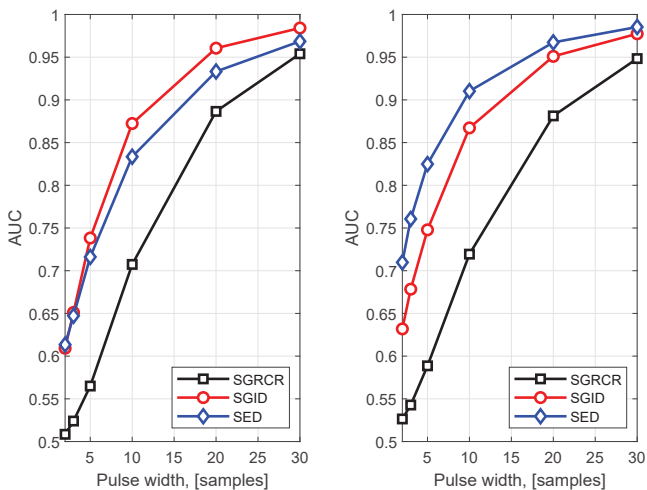

Fig. 6. AUC versus the the radar pulse width (W_p): uniform signal and noise powers (left), nonuniform-dynamical signal and noise powers (right).

IV. CONCLUSIONS

This paper applied the sliding window approach, originally proposed with the GRCR detector, to the GID and the ED, and presented extensive comparisons between the performances of

the resultant detectors under variations of all performance-relevant system parameters.

In spite of winning in some situations, the SED has shown to be sensitive to the nonuniform and dynamical noise and received signal powers in all situations investigated throughout the paper. Nevertheless, one must recall that the SED has been included in the comparisons just as a benchmark, since it is not fully blind: it needs the knowledge of the noise variances across the SUs, while the SGID and the SGRCR do not.

It has been shown that the SGRCR maintains the robustness of the GRCR with respect to received signal and noise power variations in any circumstance, while the SGID is robust only for the most practical appealing situation of small sensing window sizes. It has also been demonstrated that the SGID can outperform the SGRCR in most of the situations, if the sensing channel has some dominant multipath component (recall that the GID was developed to work in this scenario). Hence, considering that the SGRCR and the SGID have approximately the same computational complexity and are completely blind, the SGRCR is preferred in the non line-of-sight sensing channel, whereas the SGID is more attractive for a Rician fading channel with a dominant path component, as happens during a line-of-sight propagation condition.

Since the GRCR and the GID test statistics are derived from the sample covariance matrix, they might have some complementary property. Thus, as an opportunity for further developments, one could think of combining these detectors into a hybrid one, aiming at improved performance, mainly under small Rice factors. As other possibilities targeting the same objective, one could switch between the GRCR and the GID in successive intermediate sensing rounds, or run both detectors in parallel throughout these rounds, combining the decisions of both at the end.

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