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Bit Error Probability of M-QAM Signals Subject to Impulsive Noise and α-μ Fading

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***Abstract*— In this paper, a new closed-form expression is presented for determining the Bit Error Probability (BEP), P_e, of M-ary Quadrature Amplitude Modulation (M-QAM) signals subject to Double Gated Additive White Gaussian Noise (G^2AWGN) and α-μ fading, from an approach referred to as Dirac delta function approximation. In this approach, the sampling property of the Dirac delta function and an alternative representation of the tail distribution function of the standard normal distribution, known as Q-function, are used to obtain the BEP expression as a function of the Signal to Permanent Noise Ratio (SNR), Signal to Impulsive Noise Ratio (SNI) and the parameters that characterizes the channel. All BEP curves shown in this article are corroborated by simulations performed with the Monte Carlo method.**

***Keywords*— α-μ fading, bit error probability, impulsive noise.**

I. Introduction

Industrial environments are subject to numerous interferences in the 2.4 GHz frequency range, such as the presence of impulsive noise and the transmitted signal intensity variations, known as fading [1], [2], [3]. In the literature, some mathematical models are described to characterize the effects of these imperfections in these environments, such as the α-μ distribution [4], used to characterize the fading, and Double Gated Additive White Gaussian Noise (G^2AWGN) [5], used to characterize the impulsive noise.

The α-μ distribution is a generalized fading model used to characterize small-scale fading without line of sight, that includes other distributions as special cases, such as the Weibull, Rayleigh and Nakagami-m distributions, for example [4]. With regard to G^2AWGN noise, its Probability Density Function (PDF) is written as a Gaussian mixture, used to characterize the noise coming from numerous noisy sources, and encompasses, as special cases, several simpler noise models, which, depending on the application, can be used [5].

Thus, the channel model adopted in the present work, consisting of impulsive noise and α-μ fading, is considered appropriate to model channels in different environments, under different conditions, such as domestic environments, agricultural fields, shopping malls and energy substations, for example.

In this study, a new expression for the Bit Error Probability (BEP) of the M-ary Quadrature Amplitude Modulation (M-QAM) signals under G^2AWGN and α-μ fading, written in terms of elementary functions, is obtained using the method described in [6]. In this method, an alternative representation for the Q-function and the sampling property of the Dirac delta function are used to calculate the BEP for M-QAM signals subject to fading and noise. This approach is called the delta approximation and is relatively simple when compared to other methods for performance analysis of wireless communication channels subject to different types of fading [6], [7], [8], [9].

In addition to this introductory section, this article is divided as follows: in Section II, the mathematical model of impulsive noise adopted in this work is presented. In Section III, an expression for the BEP of the M-QAM in a channel with impulsive noise and α-μ fading, using the delta approximation, is shown. BEP curves, corroborated by simulations performed with Monte Carlo method, as a function of Signal to Permanent Noise Ratio (SNR), are plotted under different parameters that characterize mathematically the channel in Section IV. Finally, the conclusions obtained from this study are presented in Section V.

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II. Impulsive Noise Model

The G^2AWGN noise, represented by $\eta(t)$, can be written as [5, Equation 1]

$$\eta(t) = C_0(t)\eta_g(t) + C_1(t)C_2(t)\eta_i(t), \quad (1)$$

in which $\eta_i(t)$ represents a white Gaussian random process with zero mean and variance σ_i^2 and the signals $C_0(t)$, $C_1(t)$ and $C_2(t)$ represent continuous-time Bernoulli random processes that take discrete values in the set $\{0, 1\}$. From the value that the signal $C_0(t)$ assumes, other simpler noise models, encompassed by the general model, can be obtained. The signals $C_1(t)$ and $C_2(t)$ are used to characterize mathematically the random occurrences of noisy bursts and pulses, respectively; and the term $\eta_g(t)$ represents the permanent noise, defined as the background Gaussian noise that always appears in the system, characterized by a white Gaussian process with zero mean and variance σ_g^2.

In (1), the product $C_1(t)C_2(t)$ characterizes the gated noise $\eta_i(t)$. As the noise gating of $\eta_i(t)$ is double, the noise obtained so far is called G^2AWGN. The signal $C_1(t)$ in (1) is given by [5, Equation 5]

$$C_1(t) = \sum_{k=-\infty}^{\infty} m_k P_{R_1}(t - kT_1), \quad (2)$$

in which m_k is the k-th bit of the alphabet {0,1} with probability distribution $p(m_k = 1) = p_1$ and $p(m_k = 0) = 1 - p_1$. The pulse $P_{R_1}(t)$ of duration T_1 assumes unit amplitude in $0 \leq t \leq \Delta_2 T_1$, with Δ_2 assuming values between zero and one. The signal $C_2(t)$ assumes the values zero and one randomly and is given by [5, Equation 2]

$$C_2(t) = \sum_{l=-\infty}^{\infty} m_l P_{R_2}(t - lT_2), \tag{3}$$

in which m_l is the l-th bit of the alphabet {0,1} with probability distribution $p(m_l = 1) = p_2$ and $p(m_l = 0) = 1 - p_2$. The pulse $P_{R_2}(t)$ of duration T_2 assumes unit amplitude in $0 \leq t \leq \Delta_1 T_2$, with Δ_1 assuming values between zero and one.

III. BEP OF M-QAM SIGNALS IN A CHANNEL SUBJECT TO IMPULSIVE NOISE AND α-μ FADING

It was shown in [10, Equation 26] that the BEP of M-QAM signals, with the channel subject to G^{2}AWGN noise and fading, conditioned to the intensity of the fading envelope, z, denoted by $P(e|z)$, can be written as

$$\begin{aligned} P(e|z) = &\frac{2}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} w(i,k,M) \\ &\times \Bigg\{ \Delta_1\Delta_2 p_1 p_2 \mathrm{Q}\left(\sqrt{a(i,M) z^2 \frac{\delta_g \delta_i}{\delta_g + \delta_i}}\right) \\ &+ (1 - \Delta_1\Delta_2 p_1 p_2)\mathrm{Q}\left(\sqrt{a(i,M) z^2 \delta_g}\right)\Bigg\}, \end{aligned} \tag{4}$$

in which

$$w(i,k,M) = (-1)^{\left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \cdot \left(2^{k-1} - \left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right), \tag{5}$$

$$a(i,M) = \frac{3(2i+1)^2}{(M-1)} \log_2 M, \tag{6}$$

M is the order of the constellation, δ_g is the SNR, defined as the ratio of the signal power to the power of the background Gaussian noise that is always present in the system, and δ_i is the Signal to Impulsive Noise Ratio (SNI), defined as the ratio of the power of the signal to the power of the impulsive noise that acts in the system.

For the model of received signal [8],

$$Y(t) \;=\; z(t)X(t) + \eta(t), \tag{7}$$

in which $X(t)$ is the transmitted signal, $Y(t)$ is the received signal, $z(t)$ is the fading and $\eta(t)$ is the G^{2}AWGN noise, the corresponding BEP, P_e, can be obtained weighting (4) by the PDF of the fading factor z, that is, [8, Equation 8.102]

$$P_e = \int_0^{\infty} P(e|z) f_Z(z) dz. \tag{8}$$

In (7), the fading is considered slow and non-selective in frequency, implying that the multiplicative parameter z can be considered constant during a signaling interval.

Considering that the PDF of α-μ distribution is given by [4, Equation 1]

$$f_Z(z) = \frac{\alpha\mu^{\mu} z^{\alpha\mu-1}}{\hat{z}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{z^{\alpha}}{\hat{z}^{\alpha}}\right) \mathrm{u}(z), \tag{9}$$

in which $\mathrm{u}(\cdot)$ represents the unit step function, $\Gamma(\cdot)$ represents the Gamma function, the parameter α is related to the non-linearity of the function that characterizes the fading envelope, μ is associated with the number of multipath groupings,

$$\hat{z} = \sqrt[\alpha]{E\left(z^{\alpha}\right)} \tag{10}$$

and

$$\mu = \frac{E^2\left(z^{\alpha}\right)}{V\left(z^{\alpha}\right)}, \tag{11}$$

in which $E(\cdot)$ and $V(\cdot)$ denote the operators expectance and variance, respectively, it follows that the BEP of M-QAM signals, under G^{2}AWGN noise and α-μ fading, is given by

$$\begin{aligned} P_e = &\frac{2}{\sqrt{M}\log_2\sqrt{M}} \frac{\alpha\mu^{\mu}}{\hat{z}^{\alpha\mu}\Gamma(\mu)} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} w(i,k,M) \\ &\times \Bigg\{ \Delta_1\Delta_2 p_1 p_2 \int_0^{\infty} \mathrm{Q}\left(z\sqrt{a(i,M)\frac{\delta_g\delta_i}{\delta_g+\delta_i}}\right) \\ &\times z^{\alpha\mu-1} \exp\left(-\frac{\mu z^{\alpha}}{\hat{z}^{\alpha}}\right) dz \\ &+ (1 - \Delta_1\Delta_2 p_1 p_2) \int_0^{\infty} \mathrm{Q}\left(z\sqrt{a(i,M)\delta_g}\right) \\ &\times z^{\alpha\mu-1} \exp\left(-\frac{\mu z^{\alpha}}{\hat{z}^{\alpha}}\right) dz \Bigg\}. \end{aligned} \tag{12}$$

Defining

$$\begin{aligned} I_1 = &\int_0^{\infty} \mathrm{Q}\left(x\sqrt{a(i,M)\frac{\delta_g\delta_i}{\delta_g+\delta_i}}\right) x^{\alpha\mu-1} \\ &\times \exp\left(-\mu\left(\frac{x}{\hat{z}}\right)^{\alpha}\right) dx \end{aligned} \tag{13}$$

and considering that [6, Equation 4]

$$\mathrm{Q}(x) \approx \frac{1}{\sqrt{2\pi}} x^{-1} e^{-\frac{x^2}{2}} - \frac{1}{\sqrt{2\pi}} (x+1)^{-1} e^{-\frac{(x+1)^2}{2}}, \tag{14}$$

one can write (13) as

$$\begin{aligned} I_1 = &\frac{1}{\sqrt{2\pi}} \left(a(i,M)\frac{\delta_g\delta_i}{\delta_g+\delta_i}\right)^{-\frac{1}{2}} \\ &\times \int_0^{\infty} x^{-1} \exp\left(-\frac{a(i,M)}{2}\frac{\delta_g\delta_i}{\delta_g+\delta_i}x^2\right) \\ &\times x^{\alpha\mu-1} \exp\left(-\mu\left(\frac{x}{\hat{z}}\right)^{\alpha}\right) dx \\ &- \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \left(x\sqrt{a(i,M)\frac{\delta_g\delta_i}{\delta_g+\delta_i}} + 1\right)^{-1} \\ &\times \exp\left(-\frac{1}{2}\left(x\sqrt{a(i,M)\frac{\delta_g\delta_i}{\delta_g+\delta_i}} + 1\right)^2\right) \\ &\times x^{\alpha\mu-1} \exp\left(-\mu\left(\frac{x}{\hat{z}}\right)^{\alpha}\right) dx. \end{aligned} \tag{15}$$

If

$$v(x) = x^2, \tag{16}$$

such that

$$dv = 2x dx \tag{17}$$

and

$$dx = \frac{1}{2} v^{-\frac{1}{2}} dv, \tag{18}$$

it follows that

$$\begin{aligned} I_1 = &\frac{1}{2\sqrt{2\pi}} \left(a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i} \right)^{-\frac{1}{2}} \\ &\times \int_0^\infty \exp\left(-\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} v \right) v^{\frac{1}{2}(\alpha\mu - 1) - 1} \\ &\times \exp\left(-\mu \left(\frac{\sqrt{v}}{\hat{z}} \right)^\alpha \right) dv \\ &- \frac{1}{2\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} v \right) \\ &\times v^{\frac{1}{2}\alpha\mu - 1} \left(\sqrt{v a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i}} + 1 \right)^{-1} \\ &\times \exp\left(-\frac{1}{2} \left(1 + 2\sqrt{v a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i}} \right) \right) \\ &\times \exp\left(-\mu \left(\frac{\sqrt{v}}{\hat{z}} \right)^\alpha \right) dv. \end{aligned} \tag{19}$$

If

$$v(y) = y^N, \tag{20}$$

such that

$$dv = N y^{N-1} dy, \tag{21}$$

it follows that

$$\begin{aligned} I_1 = &\frac{1}{2\sqrt{2\pi}} \left(a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i} \right)^{-\frac{1}{2}} \\ &\times \int_0^\infty \exp\left(-\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} y^N \right) N y^{\frac{1}{2}(\alpha\mu - 1)N - 1} \\ &\times \exp\left(-\mu \left(\frac{\sqrt{y^N}}{\hat{z}} \right)^\alpha \right) dy \\ &- \frac{1}{2\sqrt{2\pi}} \int_0^\infty \exp\left(-\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} y^N \right) N y^{\frac{1}{2}\alpha\mu N - 1} \\ &\times \left(\sqrt{y^N a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i}} + 1 \right)^{-1} \\ &\times \exp\left(-\frac{1}{2} \left(1 + 2\sqrt{y^N a(i,M) \frac{\delta_g \delta_i}{\delta_g + \delta_i}} \right) \right) \\ &\times \exp\left(-\mu \left(\frac{\sqrt{y^N}}{\hat{z}} \right)^\alpha \right) dy. \end{aligned} \tag{22}$$

According to Jang [6, Equation 8], if

$$g(y^N) = \exp(-a y^N) N y^{cN-1}, \tag{23}$$

then

$$\int_0^\infty g(y^N) dy = \frac{\Gamma(c)}{a^c}. \tag{24}$$

Hence,

$$\lim_{N \to \infty} g(y^N) = \frac{\Gamma(c)}{a^c} \delta\left(y^N - \frac{c}{a} \right) \tag{25}$$

and

$$\begin{aligned} I_1 = &\frac{2^{-\frac{1}{2}}}{2\sqrt{2\pi}} \Gamma\left(\frac{1}{2}(\alpha\mu - 1) \right) \left(\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} \right)^{-\frac{\alpha\mu}{2}} \\ &\times \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{(\alpha\mu - 1)}{a(i,M)} \left(\frac{1}{\delta_g} + \frac{1}{\delta_i} \right)} \right)^\alpha \right) \\ &- \frac{1}{2\sqrt{2\pi}} \Gamma\left(\frac{\alpha\mu}{2} \right) (\sqrt{\alpha\mu} + 1)^{-1} \exp\left(-\frac{1}{2} (1 + 2\sqrt{\alpha\mu}) \right) \\ &\times \left(\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} \right)^{-\frac{\alpha\mu}{2}} \\ &\times \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{\alpha\mu}{a(i,M)} \left(\frac{1}{\delta_g} + \frac{1}{\delta_i} \right)} \right)^\alpha \right). \end{aligned} \tag{26}$$

Defining

$$\beta_1 = \frac{1}{4\sqrt{\pi}} \Gamma\left(\frac{1}{2}(\alpha\mu - 1) \right) \tag{27}$$

and

$$\begin{aligned} \beta_2 = &\frac{1}{2\sqrt{2\pi}} \Gamma\left(\frac{\alpha\mu}{2} \right) (\sqrt{\alpha\mu} + 1)^{-1} \\ &\times \exp\left(-\frac{1}{2} (1 + 2\sqrt{\alpha\mu}) \right), \end{aligned} \tag{28}$$

one can write I_1 as

$$\begin{aligned} I_1 = &\left(\frac{a(i,M)}{2} \frac{\delta_g \delta_i}{\delta_g + \delta_i} \right)^{-\frac{\alpha\mu}{2}} \\ &\times \left(\beta_1 \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{(\alpha\mu - 1)}{a(i,M)} \left(\frac{1}{\delta_g} + \frac{1}{\delta_i} \right)} \right)^\alpha \right) \right. \\ &\left. - \beta_2 \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{\alpha\mu}{a(i,M)} \left(\frac{1}{\delta_g} + \frac{1}{\delta_i} \right)} \right)^\alpha \right) \right). \end{aligned} \tag{29}$$

Defining

$$I_2 = \int_0^\infty \mathrm{Q}\left(x\sqrt{a(i,M)\delta_g} \right) x^{\alpha\mu - 1} \exp\left(-\mu \left(\frac{x}{\hat{z}} \right)^\alpha \right) dx \tag{30}$$

and considering that $\mathrm{Q}(\cdot)$ is defined by (14), it is possible to write (30) as

$$\begin{aligned} I_2 = &\left(\frac{a(i,M)}{2} \delta_g \right)^{-\frac{\alpha\mu}{2}} \left(\beta_1 \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{(\alpha\mu - 1)}{a(i,M)\delta_g}} \right)^\alpha \right) \right. \\ &\left. - \beta_2 \exp\left(-\frac{\mu}{\hat{z}^\alpha} \left(\sqrt{\frac{\alpha\mu}{a(i,M)\delta_g}} \right)^\alpha \right) \right), \end{aligned} \tag{31}$$

in which β_1 and β_2 are given respectively by (27) and (28).

Thus, the BEP of M-QAM with the channel subject to G^2AWGN noise and α-μ fading is given by

$$\begin{aligned} P_e = &\frac{2}{\sqrt{M}\log_2\sqrt{M}} \frac{\alpha\mu^\mu}{\hat{z}^{\alpha\mu}\Gamma(\mu)} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} w(i,k,M) \\ &\times \{ \Delta_1 \Delta_2 p_1 p_2 I_1 + (1 - \Delta_1 \Delta_2 p_1 p_2) I_2 \}, \end{aligned} \tag{32}$$

in which I_1 and I_2 are given respectively by (29) and (31).

IV. Results

In this section, theoretical and approximate BEP curves are presented for different parameters that characterize mathematically the channel. The values considered for the G^{2}AWGN noise are based on the simulation conditions described in [5] for impulsive noise. In the simulations, the Monte Carlo method was applied. Additionally, for comparison purposes, the BEP curve for a channel subject to α-μ fading and only AWGN noise was included in all figures. This curve is a lower bound to the system performance.

In Fig. 1, curves of BEP, P_e, of 64-QAM modulation scheme under impulsive noise and α-μ fading, as a function of the SNR, δ_g, are presented, considering four values of SNI, $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$, $\mu = 2.5$, $\alpha = 1.5$ and $\hat{z} = 1.0$. The BEP curves, corroborated by computational simulations, are obtained by using delta approximation as well as by the expression of P_e, given by (12). It is observed that the curves obtained using the delta approximation approach practically overlap the one obtained with (12). It is also observed that the BEP decreases with the increase of the SNR, for fixed values of the SNI. For $\delta_i = 5$ dB as well as for $\delta_i = 10$ dB, the BEP is not less than 10^{-3} for $\delta_g < 40$ dB. Notice that, for $\delta_g < 10$ dB, the four values of δ_i under consideration take practically the same values of P_e.

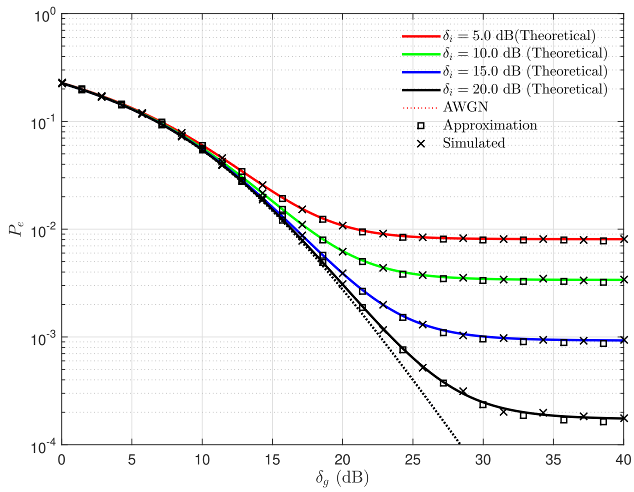

Fig. 1. BEP of 64-QAM scheme under G^{2}AWGN noise and α-μ fading, for different values of signal to impulsive noise ratio, considering $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$, $\mu = 2.5$, $\alpha = 1.5$ and $\hat{z} = 1.0$.

It should be mentioned that the BEP curves obtained in this article, by using delta approximation, have a good adherence to theoretical curves, both for low and for high values of SNR. In [6], a good adherence for the BEP is also presented, with the channel subject to AWGN noise and Nakagami-m, Nakagami-q or Nakagami-n fading.

In Fig. 2, BEP curves for G^{2}AWGN noise and α-μ fading for four different values of M, considering $\delta_i = 20$ dB, $\mu = 1.5$, $\alpha = 2.0$, $\hat{z} = 1.0$ and $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$ are shown. Since the parameter α in this simulation is equal to 2.0, one has, as a special case, the Nakagami-m distribution. In this distribution, the parameter m is equivalent to the parameter μ in the distribution α-μ. In Fig. 2, it is noted that the less dense the constellation is, the smaller the BEP obtained, since the symbols are more spaced and therefore less susceptible to the effects of noise. It is observed that a BEP equal to 10^{-3} is obtained with $\delta_g \approx 20.9$ dB for $M = 16$ while it is obtained with $\delta_g \approx 35.4$ dB for $M = 256$.

It should be mentioned that the curves shown in Fig. 2 are the same as those presented in [11, Fig. 3]. In [11], an expression for the BEP is presented considering the channel subject to G^{2}AWGN noise and Nakagami-m fading, written is terms of Gauss hypergeometric function. In [11], the BEP expression is obtained by a method in which the multiplicative fading is transformed in an additive noise R obtained by dividing the received signal by the estimated fading envelope. This method can be applied to reduce the computational complexity of signal detection in the presence of fading, when compared to the conventional method that consists of multiplying the transmitted signal by the fading added to noise.

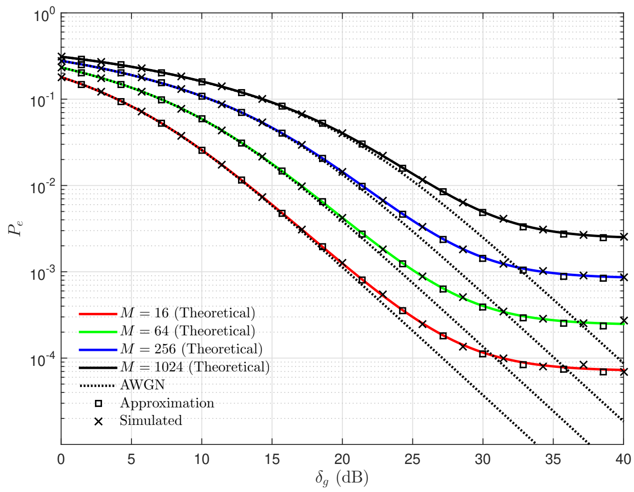

Fig. 2. BEP of M-QAM scheme under G^{2}AWGN noise and α-μ fading, for different values of the order of the constellation M, considering $\delta_i = 20$ dB, $\mu = 1.5$, $\alpha = 2.0$, $\hat{z} = 1.0$ and $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$.

Fig. 3 shows the BEP curves of the 64-QAM modulation scheme under G^{2}AWGN noise and α-μ fading. The curves are plotted against the SNR, for different values of the parameter α, with $\delta_i = 20$ dB, $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$, $\mu = 1.0$ and $\hat{z} = 1.0$. As the parameter α is related to the non-linearity of the function that characterizes the fading envelope, it follows that as α increases, lower values of BEP are obtained for fixed values of δ_g. It is also observed that BEP = 10^{-3} is obtained with $\delta_g \approx 23.3$ dB for $\alpha = 4.0$ whereas it is obtained with $\delta_g \approx 34.6$ dB for $\alpha = 3.0$. For the configuration of parameters $\alpha = 2.0$ and $\mu = 1.0$, as presented in one of the BEP curves in Fig. 3, we have, as a particular model of the α-μ distribution, the Rayleigh distribution, used to characterize mathematically cases in which the fading in the channel acts more severely. For $\delta_g = 40$ dB, a BEP difference of one order of magnitude is observed in the curves corresponding to $\alpha = 1.0$ and $\alpha = 3.0$.

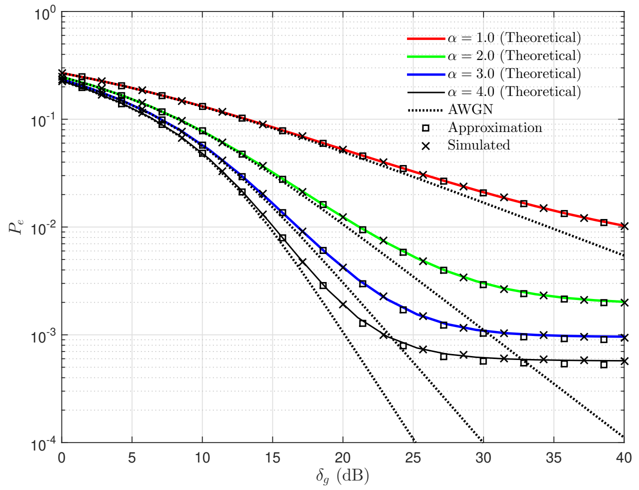

Fig. 3. BEP of 64-QAM scheme under G^2AWGN noise and α-μ fading, for different values of α, with δ_i = 20 dB, $\Delta_1 = \Delta_2 = p_1 = p_2 = 0.5$, $\mu = 1.0$ and $\hat{z} = 1.0$.

V. Conclusion

In this paper, a new closed-form expression is presented for the computation of the Bit Error Probability (BEP), P_e, of M-ary Quadrature Amplitude Modulation (M-QAM) signals subject to impulsive noise and α-μ fading. The novel expression was obtained by an approach referred to as delta approximation, which consists in using an approximation for the Q-function and the sampling property of the Dirac delta function. Exact BEP curves and approximated BEP curves using this approach, as a function of the Signal to Permanent Noise Ratio (SNR), were obtained and plotted for different parameters that characterize mathematically the channel. All BEP curves shown in this article are corroborated by simulations performed with the Monte Carlo method.

As future works, the authors intend to determine new expressions for calculating the BEP of the modulation scheme M-QAM, with the channel subject to impulsive noise and α-η-κ-μ [12] fading.

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Referências

[1] P. Angskog, C. Karlsson, J. F. Coll, P. Chilo and J. Stenumgaard. *Sources of Disturbances on Wireless Communication in Industrial and Factory Environments*. In Proceedings of the Asia-Pacific Symposium on Electromagnetic Compatibility, pp. 281-284, 2010.

[2] L. Tang, K. C. Wang, Y. Huang and F. Gu. *Channel Characterization and Link Quality Assessment of IEEE 802.15.4-Compliant Radio for Factory Environments*. IEEE Transactions on Industrial Informatics, vol. 3, no. 2, pp. 99-110, May 2007.

[3] E. Tanghe, W. Joseph, L. Verloock, L. Martens, H. Capoen, K. V. Herwegen and W. Vantomme. *The Industrial Indoor Channel: Large-Scale and Temporal Fading at 900, 2400, and 5200 MHz*. IEEE Transactions on Wireless Communications, vol. 7, no. 7, pp. 2740-2751, July 2008.

[4] M. D. Yacoub. *The α-μ Distribution: A Physical Fading Model for the Stacy Distribution*. IEEE Transactions on Vehicular Technology, vol. 56, no. 1, pp. 27-34, January 2007.

[5] E. R. Araújo, W. J. L. Queiroz, F. Madeiro, W. T. A. Lopes and M. S. Alencar. *On Gated Gaussian Impulsive Noise in M-QAM with Optimum Receivers*. Journal of Communication and Information Systems, vol. 30, no. 1, pp. 1-10, March 2015.

[6] W. M. Jang. *Quantifying Performance in Fading Channels Using the Sampling Property of a Delta Function*. IEEE Communications Letters, vol. 15, no. 3, pp. 266-268, March 2011.

[7] Z. Wang and G. B. Giannakis. *A Simple and General Parameterization Quantifying Performance in Fading Channels*. IEEE Transactions Communications, vol. 51, no. 8, pp. 1389-1398, August 2003.

[8] M. K. Simon and M. S. Alouini. *Digital Communications over Fading Channels: A Unified Approach to Performance Analysis*. 2nd edition, New York: John Wiley and Sons, 2000. 565 pp.

[9] M. Chiani, D. Dardari, and M. K. Simon. *New Exponential Bounds and Approximations for the Computation of Error Probability in Fading Channels*. IEEE Transactions Wireless Communications, vol. 2, no. 4, pp. 840-845, July 2003.

[10] W. J. L. Queiroz, W. T. A. Lopes, F. Madeiro and M. S. Alencar. *On the Performance of M-QAM for Nakagami Channels Subject to Gated Noise*. Telecommunication Systems, vol 58, no. 1, pp. 1-10, May 2018.

[11] H. S. Silva, M. S. Alencar, W. J. L. Queiroz, D. B. T. Almeida and F. Madeiro. *Closed-form Expression for the Bit Error Probability of the M-QAM for a Channel Subjected to Impulsive Noise and Nakagami Fading*. Wireless Communications and Mobile Computing, vol. 2018, pp. 1-9, October 2018.

[12] M. D. Yacoub. *The α-η-κ-μ Fading Model*. IEEE Transactions Antennas Propagation, vol. 64, no. 8, pp. 3597-3610, August 2016.