Blind Joint Detection and Channel Estimation in Space-Frequency Diversity Systems Using Time-Varying Linear Constellation Precoding

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***Resumo*— Este artigo formula um novo receptor para a decodificação cega de códigos espaço-frequência em sistemas multiantena. A proposta para o projeto do sinal transmitido combina espalhamento em frequência do tipo Vandermonde com uma precodificação linear de constelação variante no tempo. Esta última consiste em estender a rotação de constelação sobre múltiplos períodos de símbolo. Explorando a estrutura algébrica resultante deste processamento combinado na transmissão, o sinal recebido é formulado como um modelo em fatores paralelos aninhado (do inglês, "nested parallel factor") com uma rica esrtutura multilinear. Um processo de estimação conjunta do canal e de decodificação de símbolos é derivado lançando mão de um algoritmo de mínimos quadrados alternados e aninhados. As condições de identificabilidade para o receptor proposto são também discutidas, trazendo à luz os compromissos intrínsecos envolvendo as diversidades de espaço, tempo e frequência exploradas pelo receptor proposto. Resultados numéricos são fornecidos para a avaliação da taxa de erro de bits em algumas configurações de sistema.**

***Palavras-Chave*— receptor cego, precodificação de constelação, diversidade espaço-frequência, modelagem PARAFAC.**

***Abstract*— This work formulates a new multiantenna receiver for the blind decoding of space-frequency codes in multiantenna systems. The proposed transmit signal design combines frequency-domain Vandermonde spreading with a time-varying linear constellation precoding. The latter consists in extending constellation rotation across multiple symbols periods. Exploiting the algebraic structure of this combined transmit processing, the received signal is formulated as a three-way array following a nested parallel factor model with a rich multilinear structure. A joint channel estimation and symbol decoding process is derived out by resorting to a nested alternating least squares algorithm. We also discuss identifiability conditions for the proposed receiver, shedding light on the intrinsic tradeoffs involving space, time and frequency diversities exploited by the proposed blind receiver. Numerical results are provided for bit error rate (BER) performance evaluation for some system configurations.**

***Keywords*— blind receiver, constellation precoding, space-frequency diversity, PARAFAC modeling.**

I. Introduction

The combination of multiple input multiple output (MIMO) systems and orthogonal frequency division multiplexing (OFDM) has been focus of a large number of works [1]. In MIMO-OFDM, the transmit antennas can be employed to achieve high data rates via spatial multiplexing as well as to improve link reliability in frequency-selective channels through space-time/space-frequency or space-time-frequency coding [2]–[5]. In a number of recent works, space-time codes with blind detection have been proposed using tensor decompositions [6], [7], [8], [9], [10], [11]. The approach of [6] proposes a blind-decodable space-time codes based on three-way array factor model known as parallel factors (PARAFAC) [14], [15]. The work [7] is based on the same idea as that of [6] but considers frequency selective channels. In [8], joint space-time multiplexing and linear precoding is considered by resorting to a constrained three-way factor model. In [10], a generalization of the model of [8] therein called constrained factors (CONFAC), is proposed to derive a wide class of space-time multiantenna schemes enjoying blind detection. A space-time coding model relying on the PARATUCK2 tensor model was proposed recently in [11].

In this work, we consider the problem of blind decoding of space-frequency block codes in MIMO-OFDM systems. A new receiver combining space-frequency and time domain processing is proposed that allows an iterative joint blind channel estimation and symbol decoding. The proposed transmit signal design combines frequency-domain Vandermonde spreading with a time-varying linear constellation precoding. The first operation consists in spreading the information symbols across a set of neighboring frequency bins (subcarriers) over which the channel is considered to be invariant. Such a spreading operation is performed by a semi-unitary matrix having a Vandermonde structure. The second operation consists in a time-varying linear constellation precoding (TV-LCP), which consists in extending the LCP operation across a small number of successive OFDM symbols.

Exploiting the algebraic structure of this combined transmit processing, the received signal is formulated as a three-way array following a nested parallel factor model. A joint channel estimation and symbol decoding process based on nested alternating least squares is derived. The identifiability conditions for the proposed receiver are also discussed, shedding light on the tradeoffs involving space, time and frequency diversities exploited by the proposed blind receiver.

This paper is organized as follows. In Section II, the system model is described. Section III formulates the received signal as a three-way array following a nested PARAFAC model. This section also presents the proposed blind receiver and discusses identifiability conditions. Numerical results are given in Section IV, and the paper is concluded in Section V.

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Notations: Scalars are denoted by lower-case letters $(a, b, \ldots)$, vectors are written as boldface lower-case letters $(\mathbf{a}, \mathbf{b}, \ldots)$, matrices as boldface capitals $(\mathbf{A}, \mathbf{B}, \ldots)$, and tensors as calligraphic letters $(\mathcal{A}, \mathcal{B}, \ldots)$. $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ stand for transpose and pseudo-inverse of $\mathbf{A}$, respectively. $\mathbf{A}_{i.} \in \mathbb{C}^{1\times R}$ is a row vector denoting the i-th row of $\mathbf{A} \in \mathbb{C}^{I\times R}$. The operator *diag*$(\mathbf{a})$ forms a diagonal matrix from its vector argument. The operator vec$(\mathbf{A})$ yields an RI-dimensional vector that stacks the R columns of $\mathbf{A} \in \mathbb{C}^{I\times R}$ on top of each other. The operator vecdiag$(\mathbf{D})$ forms a vector $\mathbf{d}$ from the diagonal of matrix $\mathbf{D} \in \mathbb{C}^{R\times R}$, while $D_i(\mathbf{A})$ is a diagonal matrix constructed out of the i-th row of $\mathbf{A}$. The Khatri-Rao product between two matrices $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_R] \in \mathbb{C}^{I\times R}$ and $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_R] \in \mathbb{C}^{J\times R}$, denoted by $\diamond$, is defined as

$$\mathbf{A} \diamond \mathbf{B} \doteq \begin{bmatrix} \mathbf{B}D_1(\mathbf{A}) \\ \vdots \\ \mathbf{B}D_I(\mathbf{A}) \end{bmatrix} = [\mathbf{a}_1 \otimes \mathbf{b}_1, \ldots, \mathbf{a}_R \otimes \mathbf{b}_R], \quad (1)$$

where $\otimes$ denotes the Kronecker product. We make use of the following property of the Khatri-Rao product:

$$\text{vec}(\mathbf{A}\text{diag}(\mathbf{x})\mathbf{B}^T) = (\mathbf{B} \diamond \mathbf{A})\mathbf{x}, \quad (2)$$

where $\mathbf{x}$ is an R-dimensional vector.

II. System Model

We consider a MIMO-OFDM wireless communication system employing M transmit antennas and K receive antennas. In the frequency domain, information transmission is partitioned into groups of F neighboring subcarriers, and space-frequency coding is applied across these F subcarriers. The transmission time-frame is composed of a collection of N short time-slots of P symbol periods each. Assuming that the channel is constant over a time-slot, the discrete-time baseband equivalent model for the received signal is given by

$$\mathbf{Y}_{n,p} = \sqrt{\frac{\rho}{M}}\mathbf{H}_p\mathbf{C}_{n,p} + \mathbf{V}_{n,p} \in \mathbb{C}^{K\times F}, \quad (3)$$

where $\mathbf{C}_{n,p} \in \mathbb{C}^{M\times F}$ denotes complex space-frequency code matrix transmitted during the p-th symbol period of the n-th time-slot, with $E[\text{trace}(\mathbf{C}_{n,p}\mathbf{C}_{n,p}^H)] = FM$, and $n = 1, \ldots, N$, $p = 1, \ldots, P$, $\mathbf{V}_{n,p} \in \mathbb{C}^{K\times F}$ denotes zero-mean i.i.d. in space and frequency circular Gaussian (*CN*(0,1)) noise, and $\mathbf{Y}_{n,p} \in \mathbb{C}^{K\times F}$ denotes the complex received signal matrix during the p-th symbol period of the n-th time-slot. The channel matrix $\mathbf{H}_p \in \mathbb{C}^{K\times M}$ has i.i.d *CN*(0,1) entries, with $E[\text{trace}(\mathbf{H}_p\mathbf{H}_p^H)] = MK$, and ρ denotes the signal-to-noise (SNR) ratio at each receive antenna.

For the encoding of the information symbols, we consider a combination of a linear block-coding in the frequency-domain with linear constellation precoding (LCP) that acts over space and time domains. The information symbols stream is first parsed into symbol vectors $\mathbf{s}_n \in \mathbb{C}^{M\times 1}$, with $E[\text{trace}(\mathbf{s}_n^H\mathbf{s}_n)] = M$. Each symbol vector $\mathbf{s}_n$ is linearly precoded across P time-slots by means of a set of unitary space-time modulation matrices $\{\mathbf{G}_1, \ldots, \mathbf{G}_P\}$ of dimension $M \times M$. For the p-th symbol period, $\mathbf{G}_p$ rotates the components of the symbol vector $\mathbf{s}_n$ and loads a combination of these components into the M transmit antennas. Let $\mathbf{x}_{n,p} = \mathbf{G}_p\mathbf{s}_n \in \mathbb{C}^{M\times 1}$ be the signal vector resulting from this linearly precoding operation. Finally, the space-frequency code matrix is generated by "diagonally" spreading the precoded symbol vector $\mathbf{x}_{n,p}$ across F subcarriers using a linear block-coding matrix $\mathbf{W} \in \mathbb{C}^{F\times M}$. This operation can be translated into the following equation:

$$\mathbf{C}_{n,p} = \text{diag}\big(\mathbf{G}_p\mathbf{s}_n\big)\mathbf{W}^T. \quad (4)$$

If the channel is constant over a time-frame, the received signal can be modeled as

$$\mathbf{Y}_{n,p} = \sqrt{\frac{\rho}{M}}\mathbf{H}\text{diag}\big(\mathbf{G}_p\mathbf{s}_n\big)\mathbf{W}^T + \mathbf{V}_{n,p} \in \mathbb{C}^{K\times F}. \quad (5)$$

A. Choice of $\mathbf{G}_p$

We propose the following structure for this LCP matrix:

$$\mathbf{G}_p \doteq \mathbf{\Theta}\mathbf{D}^p, \quad (6)$$

where $\mathbf{\Theta}$ is a discrete Fourier transform (DFT) matrix and $\mathbf{D}_p = \text{diag}([1, \alpha_p, \ldots, \alpha_p^{(M-1)}])$, with $\alpha = \exp(j\phi)$, is a phase rotation vector, ϕ being an elementary rotation that can be optimized for a given M and modulation type [12], [13]. We call attention that the proposed blind receiver does not need to know the rotation matrix $\mathbf{D}$ in (6), which can alternatively contain phase sweeping vectors randomly varied at the transmitter and unknown at the receiver.

It worth noting that the time-varying LCP design (6) follows the basic structure of [12], [13]. The difference is on the introduction of the time dependency on the diagonal rotation matrix $\mathbf{D}^p \in \mathbb{C}^{M\times M}$. This feature provides time-domain modulation diversity by exploiting P symbol periods over which the channel is reused. As will be seen later, the introduction of time-varying rotations can be efficiently exploited for blind detection purposes. The price to pay for this added feature is a reduction on the rate of the resulting code by a factor of P. Note also that for $P = 1$ the proposed design reduces to traditional LCP.

B. Choice of $\mathbf{W}$

Recall that the linear block-coding matrix $\mathbf{W} \in \mathbb{C}^{F\times M}$ encodes the linearly precoded symbols in the frequency domain across F subcarriers. Our code construction condition requires that $\mathbf{W}$ have full rank. Herein, we focus in the case $F \le M$ and choose $\mathbf{W}$ as a Vandermonde matrix with (f, m)-th entry given by:

$$[\mathbf{W}]_{f,m} \doteq e^{j\frac{2\pi}{M}(f-1)(m-1)}. \quad (7)$$

As pointed out in [6], the Vandermonde design provides flexibility in a sense that we can trade off diversity for transmission rate by truncating the rows of $\mathbf{W}$ to any intermediate value $1 \le F \le M$, while ensuring the maximum possible diversity gain for each choice of F.

III. Blind Receiver

A. Nested PARAFAC decomposition

We introduce a fourth-order PARAFAC based decomposition which consists in expressing a four-way array in the form of two nested three-way PARAFAC decompositions of ranks R and S, respectively. For a four-way array $\mathcal{Y} \in \mathbb{C}^{I \times L \times J \times K}$, let us consider the following decomposition:

$$y_{i,l,j,k} = \sum_{r=1}^{R} \underbrace{\left(\sum_{q=1}^{Q} a_{i,q}^{(1)} a_{l,q}^{(2)} a_{r,q}^{(3)} \right)}_{a_{i,l,r}} b_{j,r} c_{k,r}, \quad (8)$$

where $i = 1, \ldots, I$, $l = 1, \ldots, L$, $j = 1, \ldots, J$, $k = 1, \ldots, K$. The scalars $a_{i,q}^{(1)}$, $a_{l,q}^{(2)}$, $a_{r,q}^{(3)}$, $b_{j,r}$, $c_{k,r}$ are typical elements of matrices $\mathbf{A}^{(1)} \in \mathbb{C}^{I \times Q}$, $\mathbf{A}^{(2)} \in \mathbb{C}^{L \times Q}$, $\mathbf{A}^{(3)} \in \mathbb{C}^{R \times Q}$, $\mathbf{B} \in \mathbb{C}^{J \times R}$, $\mathbf{C} \in \mathbb{C}^{K \times R}$, respectively. The term within parenthesis corresponds to a PARAFAC decomposition [14], [15] of the three-way array $\mathcal{A} \in \mathbb{C}^{I \times L \times R}$ with typical entry $a_{i,l,r}$. It can be shown that the i-th matrix slice $\mathbf{A}_{i\cdot\cdot} \in \mathbb{C}^{L \times R}$, obtained by fixing the first dimension of $\mathcal{A} \in \mathbb{C}^{I \times L \times R}$ to index i, admits the following factorization [6]:

$$\mathbf{A}_{i\cdot\cdot} = \mathbf{A}^{(2)} D_i(\mathbf{A}^{(1)}) \mathbf{A}^{(3)T} \quad (9)$$

Define

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{1\cdot\cdot} \\ \vdots \\ \mathbf{A}_{I\cdot\cdot} \end{bmatrix} \in \mathbb{C}^{IL \times R} \quad (10)$$

as un "unfolded" representation of the three-way array $\mathcal{A} \in \mathbb{C}^{I \times L \times R}$. Using property (1), we can write $\mathbf{A}$ as:

$$\mathbf{A} = (\mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)}) \mathbf{A}^{(3)T}. \quad (11)$$

Using this definition, we can recast (8) as a PARAFAC decomposition of the three-way array $\overline{\mathcal{Y}} \in \mathbb{C}^{IL \times J \times K}$ in terms of matrix factors $\mathbf{A} \in \mathbb{C}^{IL \times R}$, $\mathbf{B} \in \mathbb{C}^{J \times R}$ and $\mathbf{C} \in \mathbb{C}^{K \times R}$, as follows

$$\overline{y}_{t,j,k} = \sum_{r=1}^{R} a_{t,r} b_{j,r} c_{k,r}, \quad (12)$$

where $t = 1, \ldots, IL$. The t-th first-mode matrix slice $\overline{\mathbf{Y}}_{t\cdot\cdot} \in \mathbb{C}^{J \times K}$ of $\overline{\mathcal{Y}} \in \mathbb{C}^{IL \times J \times K}$ admits the following factorization:

$$\overline{\mathbf{Y}}_{t\cdot\cdot} = \mathbf{B} D_t(\mathbf{A}) \mathbf{C}^T \quad (13)$$

Using (11), we can rewrite (13) as

$$\overline{\mathbf{Y}}_{t\cdot\cdot} = \mathbf{B} D_t\Big((\mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)}) \mathbf{A}^{(3)T}\Big) \mathbf{C}^T \quad (14)$$

Defining

$$\overline{\mathbf{Y}} = \begin{bmatrix} \overline{\mathbf{Y}}_{1\cdot\cdot} \\ \vdots \\ \overline{\mathbf{Y}}_{IL\cdot\cdot} \end{bmatrix}, \quad (15)$$

and applying property (1), yields

$$\overline{\mathbf{Y}} = \underbrace{\Big(\underbrace{\big[(\mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)}) \mathbf{A}^{(3)T}\big]}_{\text{inner PARAFAC}} \diamond \mathbf{B} \Big) \mathbf{C}^T}_{\text{outer PARAFAC}}. \quad (16)$$

This nested factorization expresses the (outer) PARAFAC decomposition of tensor $\overline{\mathcal{Y}} \in \mathbb{C}^{IL \times J \times K}$ in terms of the (inner) PARAFAC decomposition of tensor $\mathcal{A} \in \mathbb{C}^{I \times L \times R}$.

The PARAFAC decomposition of $\overline{\mathcal{Y}} \in \mathbb{C}^{IL \times J \times K}$ in (12) enjoys the essential uniqueness property, which means that each matrix factor can be determined up to column scaling and permutation. More specifically, this means that any alternative solution $\tilde{\mathbf{A}} \in \mathbb{C}^{IL \times R}$, $\tilde{\mathbf{B}} \in \mathbb{C}^{J \times R}$ and $\tilde{\mathbf{C}} \in \mathbb{C}^{K \times R}$ satisfying the model is linked to the true model parameters by $\tilde{\mathbf{A}} = \mathbf{A}\mathbf{\Pi}\mathbf{\Delta}_a$, $\tilde{\mathbf{B}} = \mathbf{B}\mathbf{\Pi}\mathbf{\Delta}_b$, $\tilde{\mathbf{C}} = \mathbf{C}\mathbf{\Pi}\mathbf{\Delta}_c$, where $\mathbf{\Pi}$ is a column permutation matrix and $\mathbf{\Delta}_a \mathbf{\Delta}_b \mathbf{\Delta}_c = \mathbf{I}_R$. A sufficient condition for such an uniqueness was firstly derived in [16]. It says that the inner decomposition is essentially unique if $k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2R + 2$, where $k_{\mathbf{A}}$ denotes the Kruskal-rank [16] (also called k-rank), of $\mathbf{A}$. The k-rank of $\mathbf{A}$ corresponds to the greatest integer $k_{\mathbf{A}}$ such as *any* set of $k_{\mathbf{A}}$ columns of $\mathbf{A}$ is linearly independent. Assuming that neither $\mathbf{A}$, nor $\mathbf{B}$, nor $\mathbf{C}$ has a pair of proportional columns, then $k_{\mathbf{A}} = \text{rank}(\mathbf{A})$ and uniqueness of the outer decomposition (12) is guaranteed if [6]:

$$\min(IL, R) + \min(J, R) + \min(K, R) \geq 2R + 2. \quad (17)$$

In an analogous manner, provided that $\mathbf{A}$ is unique, the inner decomposition (8) is also unique if:

$$\min(I, Q) + \min(L, Q) + \min(R, Q) \geq 2Q + 2. \quad (18)$$

B. Receiver formulation

Note that with definition (6), the received signal (5) can be expressed as:

$$\mathbf{Y}_{n,p} = \sqrt{\frac{\rho}{M}} \mathbf{H} D_p\big(\mathbf{A} D_n(\mathbf{S}) \mathbf{\Theta}^T\big) \mathbf{W}^T + \mathbf{V}_{n,p} \in \mathbb{C}^{K \times F}, \quad (19)$$

where

$$\mathbf{A} = \begin{bmatrix} \text{vecdiag}(\mathbf{D}^1)^T \\ \vdots \\ \text{vecdiag}(\mathbf{D}^P)^T \end{bmatrix} = \begin{bmatrix} 1 & \alpha_1 & \cdots & \alpha_1^{(M-1)} \\ 1 & \alpha_2 & \cdots & \alpha_2^{(M-1)} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \alpha_P & \cdots & \alpha_P^{(M-1)} \end{bmatrix} \in \mathbb{C}^{P \times M} \quad (20)$$

collects phase rotation and hopping factors acting over the P symbol periods, and $\mathbf{S} \in \mathbb{C}^{N \times M}$ is a symbol matrix whose rows are the symbol vectors $\mathbf{s}_n^T$, $n = 1, \ldots, N$. Defining $\mathbf{Y}_n = [\mathbf{Y}_{n,1}^T, \ldots, \mathbf{Y}_{n,P}^T]^T \in \mathbb{C}^{PK \times F}$ as the data matrix collecting the received samples during the P symbol periods of the n-th time-slot. From property (1), we have:

$$\mathbf{Y}_n = \sqrt{\frac{\rho}{M}} \big[(\mathbf{A} D_n(\mathbf{S}) \mathbf{\Theta}^T) \diamond \mathbf{H}\big] \mathbf{W}^T + \mathbf{V}_n \in \mathbb{C}^{PK \times F}, \quad (21)$$

where the noise term $\mathbf{V}_n$ is defined in the same way as $\mathbf{Y}_n$. Likewise, defining a bigger matrix $\mathbf{Y} = [\mathbf{Y}_n^T, \ldots, \mathbf{Y}_n^T]^T \in \mathbb{C}^{NPK \times F}$ collecting the received data for all N time-slots, yields:

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}} \Big(\big[(\mathbf{S} \diamond \mathbf{A}) \mathbf{\Theta}^T\big] \diamond \mathbf{H} \Big) \mathbf{W}^T + \mathbf{V} \in \mathbb{C}^{NPK \times F}. \quad (22)$$

The signal part of (22) corresponds to the nested PARAFAC decomposition (16). Note that none of the system design matrices $\mathbf{A}$, $\mathbf{\Theta}$ and $\mathbf{W}$ has proportional columns. Moreover, under the assumption that neither $\mathbf{S}$ nor $\mathbf{H}$ contains proportional columns, we can apply the uniqueness conditions (17)-(18) to model (22) to yield:

$$\begin{aligned} &\min(NP, M) + \min(K, M) + \min(F, M) \geq 2M + 2, \\ &\min(N, M) + \min(P, M) \geq M + 2. \end{aligned} \tag{23}$$

In this work, we are interested in two system configurations.

1) Assuming $P < M \leq N$, condition (23) simplifies to:

$$\min(\min(K, M) + \min(F, M) - M, P) \geq 2; \tag{24}$$

2) Assuming $N < M \leq P$, condition (23) simplifies to:

$$\min(\min(K, M) + \min(F, M) - M, N) \geq 2. \tag{25}$$

These two conditions ensure blind channel and symbol recovery from the nested PARAFAC model (22). They show that N and P play symmetrical roles, establishing a tradeoff between the minimum number of time-slots N and the number of symbol periods P necessary for obtaining unique blind estimates of the model parameters. Note also that in both (24) and (25), condition $\min(K, M) + \min(F, M) \geq M + 2$ establishes a tradeoff between the number of receive antennas K and the number of subcarriers F for ensuring uniqueness.

C. Alternating least squares algorithm

We assume that channel state information is not available at the receiver. The receiver consists in fitting the nested PARAFAC model to the received data using an alternating least squares (ALS) algorithm [6].

In the formulation of the receiver algorithm, we assume that the frequency block-coding matrix $\mathbf{W}$ and the fixed constellation rotation matrix $\mathbf{\Theta}$ are perfectly known at the receiver (i.e. they do not need to be estimated). Let us define $\bar{\mathbf{X}} = \sqrt{\frac{P}{M}}(\mathbf{S} \diamond \mathbf{A})\mathbf{\Theta}^T \in \mathbb{C}^{NP \times M}$ as a matrix representing the (time-domain) linearly constellation precoded signal. The algorithm consists in two ALS based estimation stages. The first one provides estimates of $\bar{\mathbf{X}}$ and $\mathbf{H}$ by fitting a PARAFAC model to the received signal tensor $\mathcal{Y}$, as follows

$$J_1 = \sum_t \sum_k \sum_f \left| y_{t,k,f} - \sum_m \bar{x}_{t,m} h_{k,m} w_{f,m} \right|^2, \tag{26}$$

where $\bar{x}_{t,m}$ is an element of matrix $\bar{\mathbf{X}}$. In an analogous manner, the second stage of the algorithm provides estimates of $\mathbf{S}$ by fitting a PARAFAC model to tensor $\mathcal{X}$, i.e.:

$$J_2 = \sum_n \sum_p \sum_m \left| x_{n,p,m} - \sum_r s_{n,r} a_{p,r} \theta_{m,r} \right|^2, \tag{27}$$

Define $\mathbf{Y}' \in \mathbb{C}^{KF \times NP}$, $\mathbf{Y}'' \in \mathbb{C}^{FNP \times K}$, $\mathbf{X}' \in \mathbb{C}^{PM \times N}$ and $\mathbf{X}'' \in \mathbb{C}^{MN \times P}$ as follows

$$[\mathbf{Y}']_{(k-1)F+f,t} \doteq y_{t,k,f} \tag{28}$$

$$[\mathbf{Y}'']_{(f-1)NP+t,k} \doteq y_{t,k,f} \tag{29}$$

$$[\mathbf{X}']_{(p-1)M+m,n} \doteq x_{n,p,m} \tag{30}$$

$$[\mathbf{X}'']_{(m-1)N+n,p} \doteq x_{n,p,m}. \tag{31}$$

The multilinearity of the PARAFAC decomposition allows to represent these four matrices as

$$\mathbf{Y}' = (\mathbf{H} \diamond \mathbf{W})\bar{\mathbf{X}}^T + \mathbf{V}', \tag{32}$$

$$\mathbf{Y}'' = (\mathbf{W} \diamond \bar{\mathbf{X}})\mathbf{H}^T + \mathbf{V}'', \tag{33}$$

$$\mathbf{X}' = (\mathbf{A} \diamond \mathbf{\Theta})\mathbf{S}^T, \tag{34}$$

$$\mathbf{X}'' = (\mathbf{\Theta} \diamond \mathbf{S})\mathbf{A}^T. \tag{35}$$

Note that $\bar{\mathbf{X}}$ is linked to $\mathbf{X}'$ and $\mathbf{X}''$ by "reshaping" operations, more specifically:

$$[\bar{\mathbf{X}}]_{(n-1)P+p,m} = [\mathbf{X}']_{(p-1)M+m,n} = [\mathbf{X}'']_{(m-1)N+n,p}. \tag{36}$$

The four factorizations (32)-(35) lead to least squares (LS) updating equations of $\bar{\mathbf{X}}$, $\mathbf{H}$, $\mathbf{S}$ and $\mathbf{A}$, respectively. The algorithm is summarized as follows:

RECEIVER ALGORITHM

First stage

Initialization: Set $i = 0$; Randomly initialize $\widehat{\mathbf{H}}$;

(1.1) $i = i + 1$;

(1.2) Compute the LS estimate of $\bar{\mathbf{X}}$:

$$\widehat{\bar{\mathbf{X}}}^T(i) = (\widehat{\mathbf{H}}(i-1) \diamond \mathbf{W})^\dagger \mathbf{Y}';$$

(1.3) Compute the LS estimate of $\mathbf{H}$:

$$\widehat{\mathbf{H}}^T(i) = (\mathbf{W} \diamond \widehat{\bar{\mathbf{X}}}(i))^\dagger \mathbf{Y}'';$$

(1.4) Repeat steps (1.1)-(1.3) until convergence.

Second stage

Initialization: Construct $\mathbf{X}'$ and $\mathbf{X}''$ from $\widehat{\bar{\mathbf{X}}}(i)$ using (36);
Set $j = 0$; Randomly initialize $\widehat{\mathbf{A}}$;

(2.1) $j = j + 1$;

(2.2) Compute the LS estimate of $\widehat{\mathbf{S}}$:

$$\widehat{\mathbf{S}}^T(j) = (\widehat{\mathbf{A}}(j-1) \diamond \mathbf{\Theta})^\dagger \mathbf{X}';$$

(2.3) Compute the LS estimate of $\mathbf{A}$:

$$\widehat{\mathbf{A}}^T(j) = (\mathbf{\Theta} \diamond \widehat{\mathbf{S}}(j))^\dagger \mathbf{X}''$$

(2.4) Repeat steps (2.1)-(2.4) until convergence.

It is worth mentioning that due the knowledge of matrices $\mathbf{W}$ and $\mathbf{\Theta}$, permutation ambiguity does not exist. Note that scaling ambiguity can be removed from the estimated channel matrix $\widehat{\mathbf{H}}$ in the first stage, by exploiting the fact that the first row of $\mathbf{W}$ and $\mathbf{X}$ are known by definition. Likewise, scaling ambiguity can be removed from the estimated symbol matrix $\widehat{\mathbf{S}}$ in the second stage, by exploiting the fact that the first row of $\mathbf{A}$ and $\mathbf{\Theta}$ are also known. The convergence of both ALS stages is usually achieved within less than 100 iterations.

IV. NUMERICAL RESULTS

We evaluate the bit-error-rate (BER) performance of the proposed blind receiver. Each BER curve is an average of 10000 Monte Carlo runs, each one representing a channel realization, the coefficients of which are drawn from an i.i.d. complex-valued Gaussian generator. At each run, the transmitted symbols of all the data streams are drawn from

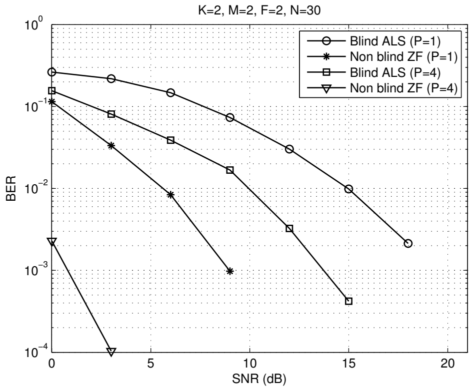

Fig. 1. ALS (blind) vs. ZF (perfect channel knowledge)

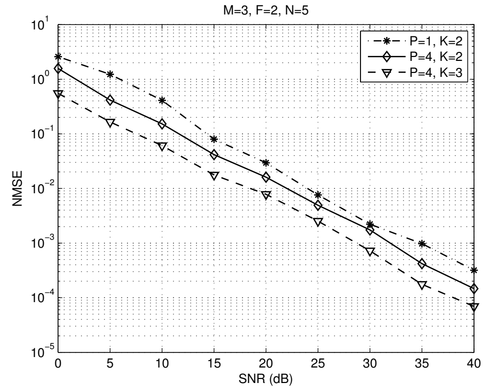

Fig. 2. NMSE of the estimated channel.

a QPSK sequence and the additive noise power is generated according to the signal-to-noise ratio (SNR) value given by SNR $= 10\log_{10}(\|\mathbf{Y}\|_F^2/\|\mathbf{V}\|_F^2)$. In order to provide a performance reference of the proposed receiver, we have plotted the performance of the nonblind zero forcing (ZF) receiver. Contrarily to the proposed receiver, the nonblind ZF one assumes perfect knowledge of the channel matrix. Using our notation, the ZF receiver consists in a single-step estimation of the symbol matrix given by:

$$\widehat{\mathbf{s}}_n^{(\text{ZF})} = \begin{bmatrix} (\mathbf{W} \diamond \mathbf{H})(\boldsymbol{\Theta} \diamond \mathbf{A}_{1\,.}) \\ \vdots \\ (\mathbf{W} \diamond \mathbf{H})(\boldsymbol{\Theta} \diamond \mathbf{A}_{P\,.}) \end{bmatrix}^{\dagger} \begin{bmatrix} \text{vec}(\mathbf{Y}_{n,1}) \\ \vdots \\ \text{vec}(\mathbf{Y}_{n,P}) \end{bmatrix}, \quad (37)$$

where $\mathbf{H}$ is perfectly known. In this comparison, we consider $K = 2$, $M = 2$, $F = 2$ and $N = 30$. It can be seen from Figure 1 that the gap between ALS and ZF increases as P increases. We can observe that both ZF and ALS present the same BER improvement for higher SNRs.

We now evaluate the accuracy of the blind channel estimation from the normalized mean square error (NMSE), averaged over 500 Monte Carlo runs and defined as follows:

$$\text{NMSE}(\mathbf{H}) = \sum_{l=1}^{500} \frac{\left\|\widehat{\mathbf{H}}(l) - \mathbf{H}\right\|_F^2}{\|\mathbf{H}\|_F^2}, \quad (38)$$

where $\widehat{\mathbf{H}}(l)$ is the channel matrix estimated at convergence of the l-th run. In this experiment, we consider a challenging configuration with a short number of time-slots equal to $N = 5$. The other system parameters are $M = 3$ and $F = 2$. The values of P and K are varied. Figure 2 shows a linear decrease in the NMSE as a function of the SNR for all configurations. We can also observe that increasing P and/or K provides an improved channel estimation accuracy, as expected.

V. Conclusion

The proposed blind receiver relies on the formulation of the received signal as a nested three-way PARAFAC model that arises by combining space-frequency spreading with time-varying constellation rotation. Sufficient conditions for unique blind channel and symbol recovery have been derived, which put in evidence some tradeoffs involving space, time and frequency diversities. Numerical results have illustrated the performance of the receiver.

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