On the Fading Parameters Characterization of the α-μ Distribution: Measurements and Statistics

Camila Lopes Selvati, Ugo Silva Dias, Aravind Krishnan, and Michel Daoud Yacoub

***Abstract*— This paper presents the results of indoor and outdoor field trial measurements in order to obtain the probability density functions and the autocorrelation functions of the fading parameters of the α-μ distribution. The ranges of possible practical values of the α and μ fading parameters are also obtained from the empirical data. In addition, the instantaneous magnitude variations of α and μ considering the mobile receiver displacement are estimated and discussed. The results provide important information about the practical usefulness of the α-μ fading model in mobile communication systems.**

***Keywords*— α-μ distribution, channel characterization, fading parameters, field measurements.**

***Resumo*— Este artigo apresenta os resultados de medições de campo em ambientes *indoor* e *outdoor* com o objetivo de obter as funções densidade de probabilidades e as funções de autocorrelação empíricas dos parâmetros de desvanecimento da distribuição α-μ. Os intervalos de possíveis valores práticos dos parâmetros de desvanecimento α e μ também são obtidos a partir dos dados adquiridos em campo. Ainda, as variações instantâneas de magnitude de α e μ considerando o deslocamento do receptor móvel são estimadas e discutidas. Os resultados fornecem informações importantes sobre o uso prático do modelo de desvanecimento nos sistemas de comunicações móveis.**

***Palavras-Chave*— Distribuição α-μ, caracterização de canal sem fio, parâmetros de desvanecimento, medições de campo.**

I. INTRODUCTION

IN wireless communications, the multipath fading phenomenon has been characterized by several important statistics models, notably Rayleigh, Rice, Hoyt (Nakagami-*q*), Nakagami-*m*, and Weibull [1]–[3]. In [4], a general physical fading model, namely the α-μ model, has been described that considers a signal composed of clusters of multipath waves propagating in a nonhomogeneous environment. The resultant envelope in such an environment then follows the α-μ distribution, which includes as special cases Nakagami-*m* and Weibull. Its flexibility renders it adaptable to situations in which neither of these two distributions yield good fit [4], [5]. Experimental data supporting the usefulness of the Nakagami-*m* and Weibull fading models have been widely reported in the literature (e.g., [6]–[9]). Recently [10]–[12], some works presenting the performance analysis and first-order statistical modeling for the Nakagami-*m* and Rice fading parameters have been also presented. In practice, situations are easily found for which the α-μ distribution outperforms both Nakagami-m and Weibull [5], [13]. However, to the best of the authors' knowledge, works depicting practical situations in which the fading parameters of the α-μ distribution are characterized have not been reported in the literature.

In this paper, the empirical probability density functions (PDF) and the autocorrelation functions of α and μ, which are the fading parameters of the α-μ distribution, are obtained based on field measurements. In addition, the ranges of possible practical values assumed by α and μ are estimated from the empirical data and their instantaneous magnitude variations are evaluated considering the displacement of the mobile receiver in indoor and outdoor environments. Since the values of α and μ are indicators of the nonlinearities and multipath clustering of the radio channel [4], the knowledge of their PDFs, autocorrelations, and magnitude ranges, can be used in the evaluation and design of different wireless communications techniques, such as diversity combining techniques, adaptive modulation schemes, modeling and analysis of interferences, outages probabilities, design and simulation of adaptive antennas systems, among others.

The remainder of this work is structured as follows. In Section II, the α-μ fading model is revisited. In Section III, indoor and outdoor field trial measurements are conducted in order to investigate first- and high-order statistics of the α and μ fading parameters. Specially, the empirical PDFs and autocorrelations of the α and μ fading parameters are presented and discussed based on the measurement campaigns. The instantaneous variation of the α and μ magnitudes are also obtained with distance and analyzed, and the practical value ranges of α and μ parameters are estimated. Finally, in Section IV, some conclusions remarks are presented.

II. THE α-μ FADING MODEL REVISITED

The α-μ distribution is a general fading distribution that can be used to better represent the small-scale variation of the fading signal in a non line-of-sight fading condition [4]. It includes as special cases important other distributions, such as Nakagami-*m*, and Weibull. (Therefore, One-Sided Gaussian and Rayleigh are also special cases of it). As its name implies, it is written in terms of two physical parameters, namely α and μ. The power parameter $\alpha > 0$ is related to the non-linearity of the environment, whereas the parameter $\mu > 0$ is associated to the number of multipath clusters.

For a α-μ fading signal with envelope R, an arbitrary parameter $\alpha > 0$, and a α-root mean value $\hat{r} = \sqrt[\alpha]{E(R^\alpha)}$, in

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C. L. Selvati and U. S. Dias are with the Microwave and Wireless Systems Laboratory (MWSL), Department of Electrical Engineering, University of Brasilia, 70910-900, Brasília, DF, Brazil (e-mail: cselvati@redes.unb.br, udias@unb.br).

A. Krishnan and M. D. Yacoub is with the Wireless Technology Laboratory (WissTek), Department of Communications, School of Electrical and Computer Engineering, State University of Campinas, 13083-852, Campinas, SP, Brazil (e-mail: [aravind,michel]@wisstek.org).

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which $E(\cdot)$ signifies the expectation operator, the α-μ envelope PDF, $f_{\mathrm{R}}(r)$, is written as

$$f_R(r) = \frac{\alpha \mu^{\mu} r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu \frac{r^{\alpha}}{\hat{r}^{\alpha}}\right), \qquad (1)$$

for which $\mu > 0$ is the inverse of the normalized variance of R^{α}, i.e.,

$$\mu = \frac{E^2(R^{\alpha})}{V(R^{\alpha})}, \qquad (2)$$

$\Gamma(\cdot)$ is the Gamma function [14, Eq. 6.1.1], and $V(\cdot)$ denotes the variance operator.

III. FIELD TRIALS AND STATISTICAL ANALYSIS

A series of field trials was conducted at the University of Campinas (Unicamp), Brazil, in order to (i) estimate the value ranges of the α and μ parameters, (ii) characterize the amplitude variations with distance of the α and μ fading parameters, (iii) obtain the empirical PDFs of the α and μ parameters, and (iv) obtain the empirical autocorrelation functions of such fading parameters. To this end, the transmitter was placed on the rooftop of one of the buildings and the receiver travelled through the campus as well as within the buildings. The mobile reception equipment was especially assembled for this purpose. Basically, the setup consisted of a vertically polarized omnidirectional receiving antenna, a low noise amplifier, a spectrum analyzer, data acquisition apparatus, a notebook computer, and a distance transducer for carrying out the signal sampling. The transmission consisted of a continuous wave tone at 1.8 GHz. The spectrum analyzer was set to zero span and centered at the desired frequency, and its video output used as the input of the data acquisition equipment with a sampling interval of $\lambda/14$ [15]–[17]. The local mean was estimated by the moving average method, with optimum windows length from 45λ to 60λ [10], equivalent to 630 and 840 measured envelope samples, respectively. From the collected data, the long term fading was filtered out, then the α and μ fading parameters could be estimated using the following moment-based estimator.

A. The Moment-based α-μ Estimator

The moments of the α-μ envelope are given as [4] $E(R^k) = \frac{\hat{r}^k\Gamma(\mu+k/\alpha)}{\mu^{k/\alpha}\Gamma(\mu)}$. From this, an equality is defined that is useful for the estimation of the parameters of this distribution. In essence [4],

$$\frac{E^2(R^{\beta})}{E(R^{2\beta}) - E^2(R^{\beta})} = \frac{\Gamma^2(\mu+\beta/\alpha)}{\Gamma(\mu)\Gamma(\mu+2\beta/\alpha) - \Gamma^2(\mu+\beta/\alpha)} \qquad (3)$$

in which $\beta > 0$ is chosen arbitrarily. For two distinct and arbitrary values of β, two equations are set up so that the physical parameters α and μ are encountered. For a particular case in which $\beta = 1$ and $\beta = 2$, (3) yields an estimator in terms of the first and second moments. Of course, from (3), other moment-based estimators can be used.

B. The Empirical Autocorrelation

The normalized empirical autocorrelation was computed according to

$$\widehat{A}_R(\Delta) = \frac{\sum_{i=1}^{N-\Delta} r_i r_{i+\Delta}}{\sum_{i=1}^{N-\Delta} r_i^2} \qquad (4)$$

where r_i is the i-th sample of the amplitude sequence, N is the total number of samples, Δ is the α or μ discrete relative distance difference, and $\widehat{A}_R(.)$ denotes an empirical estimate of $A_R(.)$.

C. Numerical Results and Discussion

Figures 1, 2, and 3 show sample plots of the magnitude variations of the α and μ fading parameters with distance in indoor and outdoor environments. Note that when α increases μ decreases, for a given displacement of the mobile receiver, and vice-versa. The physical phenomena involved concern the following. When the magnitude of the nonlinearities of the propagation medium increases, the magnitude of the multipath clustering decreases, possibly in an attempt to keep the variance of the power constant meaning that the process is stationary. It must be said that the peak values (highest values assumed by α and correspondingly lowest values assumed by μ) occur when the dominant component appear in the propagation path. Conversely, the lowest values for α (and correspondingly highest values for μ) occur when the dominant component is weak and there is a prevalence of multipath clusters. These interesting behaviors among the two fading parameters, as predicted by its theoretical model [4] is now confirmed in practice.

Figures 4, 5, and 6 present some sample curves of the empirical α and μ PDFs for the same environments of the Figures 1, 2, and 3. Theses PDFs of both α and μ have similar shapes, but they are centered in different values and they seem to approximately follow a positive gaussian PDF. The average estimated median and mean values of α are respectively 2.43 and 2.58, with standard deviations of 0.42. For the μ cases, these are respectively 0.84 and 0.89, with standard deviations of 0.33. The range of possible practical values of α, as found here, vary from 1.6 to 6.2. And for the μ case, these are from 0.1 to 3.8. The knowledge of possible practical levels of α and μ and their expected values are very important to better estimate the fading margin in wireless systems.

Figures 7, 8, and 9 depict sample plots of the empirical autocorrelation functions of the α and μ fading parameters for different indoor and outdoor environments. Interestingly, observe how the α and μ curves tend to keep track of the changes of the concavity of each other with slight shifts. In most measured cases, the autocorrelation curves of α is found above the curves of μ, as expected. The sharply decreasing curves shown in Figure 7 report field measurements performed in an anisotropic environment, whereas the slightly decreasing curves sketched in Figures 8 and 9 present measurements under isotropic scenarios.

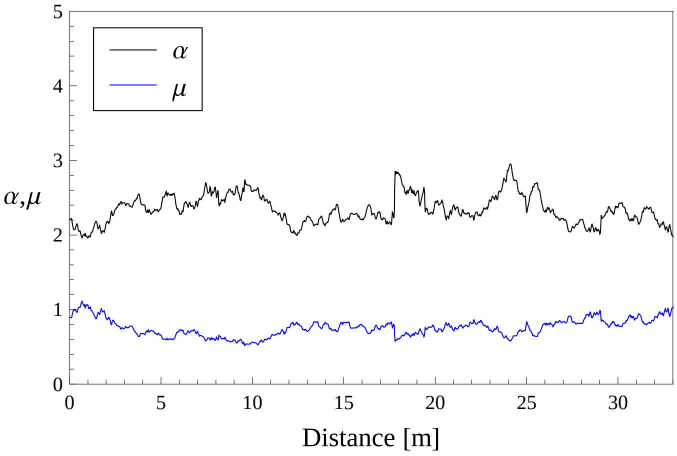

Fig. 1. α and μ magnitudes with distance. Indoor measurements.

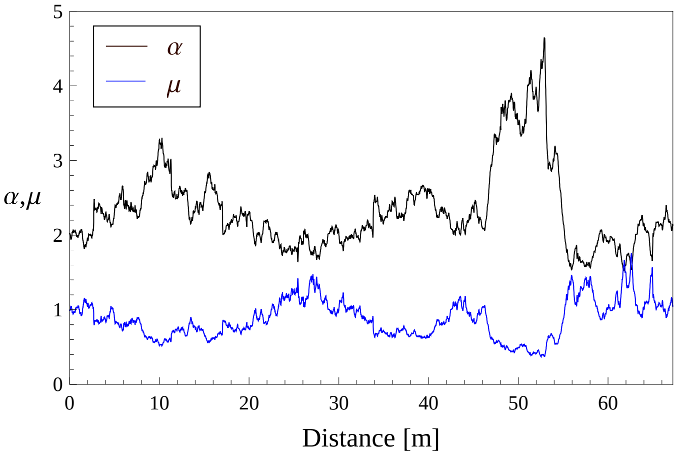

Fig. 2. α and μ magnitudes with distance. Outdoor measurements.

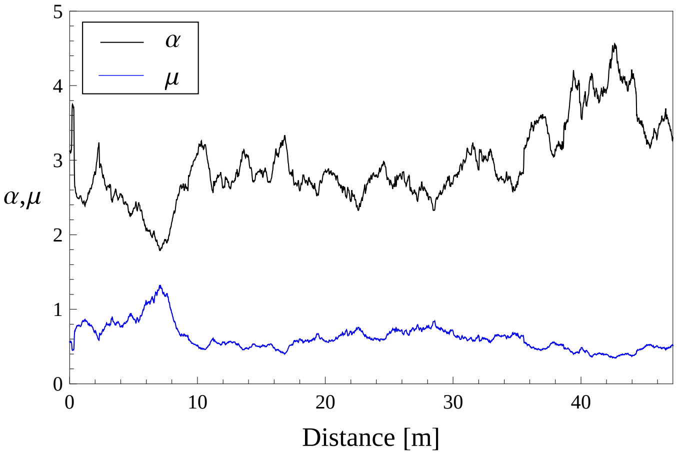

Fig. 3. α and μ magnitudes with distance. Outdoor measurements.

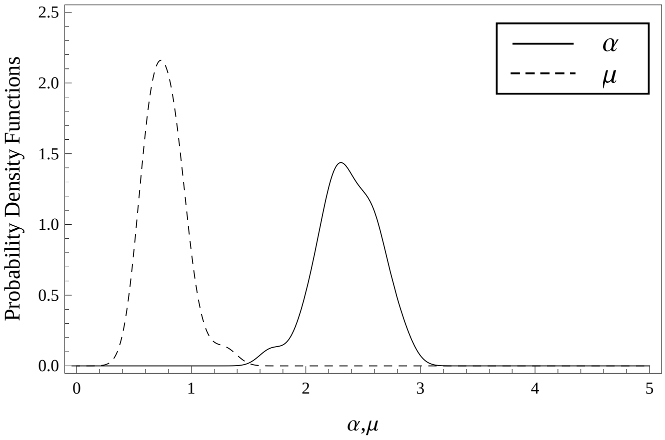

Fig. 4. Empirical PDFs of the α and μ fading parameters. Indoor measurements.

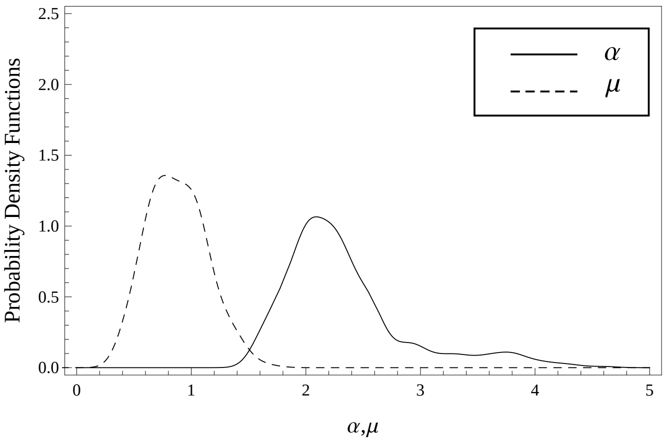

Fig. 5. Empirical PDFs of the α and μ fading parameters. Outdoor measurements.

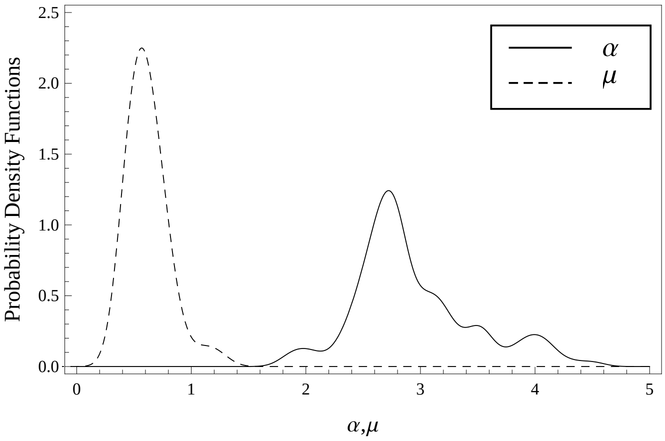

Fig. 6. Empirical PDFs of the α and μ fading parameters. Outdoor measurements.

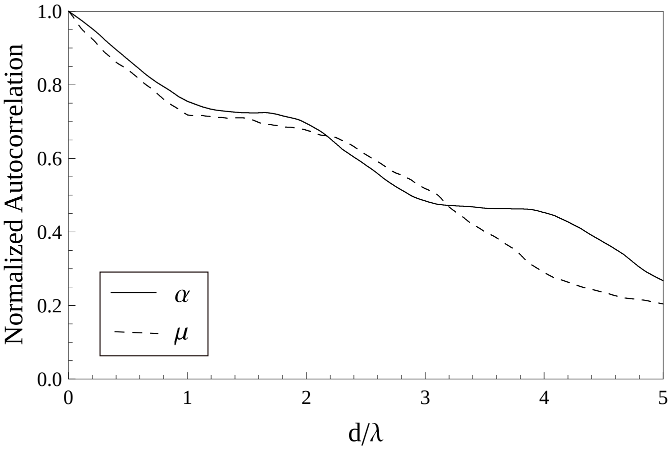

Fig. 7. Empirical autocorrelations of the α and μ fading parameters. Indoor measurements.

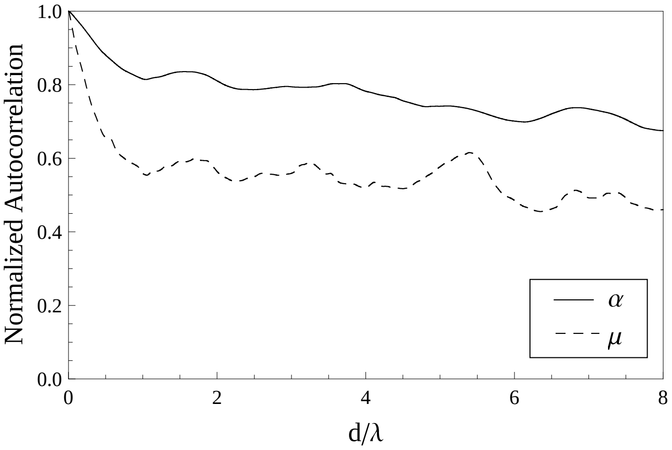

Fig. 8. Empirical autocorrelations of the α and μ fading parameters. Outdoor measurements.

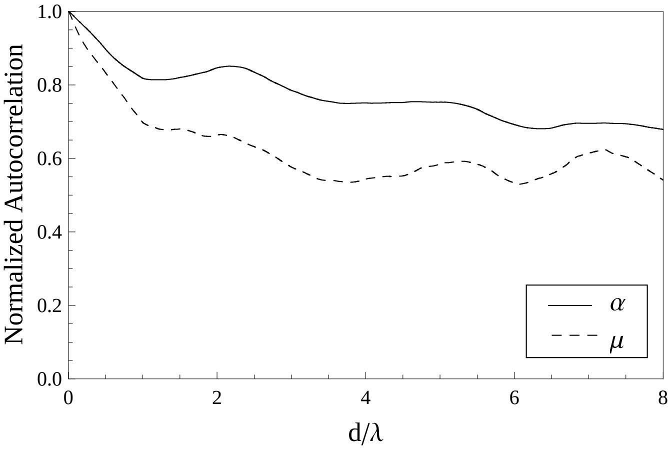

Fig. 9. Empirical autocorrelations of the α and μ fading parameters. Outdoor measurements.

IV. CONCLUSIONS

In this paper, we reported the results of field trials aimed at investigating first- and second-order statistics of the fading parameters of the α-μ distribution. More specifically, the empirical probability density functions and the autocorrelation functions of the α and μ fading parameter were obtained. Moreover, the ranges of possible practical values for the α and μ fading parameter were estimated from the empirical data, and the instantaneous variations of their magnitudes were evaluated considering the displacement of the mobile receiver in indoor and outdoor environments. The results provided important information about the practical usefulness of the α-μ fading model in mobile communication systems.

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