

Spectrum Sensing over κ - μ Fading Channel

Diogo Sanders, Fábio von Glehn, and Ugo Silva Dias

Abstract—This paper presents the performance analysis of energy detection over generalized fading channels, modeled by the κ - μ distribution. Receiver operation characteristics (ROC) are obtained under several different fading environments, including low, typical, and very high severe fading conditions, with combining both multipath clusters and line-of-sight scenarios. In addition, the improvement in detection capability is evaluated when cooperative spectrum schemes are employed under κ - μ fading channel. Comparisons are performed against Rayleigh model and a great flexibility is found in spectrum sensing techniques when their formulations consider the use of the κ - μ fading model.

Keywords—Cognitive radio, energy detection, fading channels, spectrum sensing, κ - μ distribution.

Resumo—Este artigo apresenta a análise de desempenho de detecção de energia em canais generalizados de desvanecimento, modelados pela distribuição κ - μ . As características de operação do receptor são obtidas considerando diversos ambientes em desvanecimento, incluindo condições severas, típicas e suaves, com a combinação de cenários em linha de visada e *clusters* de multipercorso. Além disso, a melhoria na capacidade de detecção é avaliada quando os esquemas de cooperação espectral são empregados dentro de canais em desvanecimento κ - μ . Comparações são realizadas entre modelos de Rayleigh e κ - μ , e uma grande flexibilidade é encontrada nas técnicas de sensoriamento de espectro quando as suas formulações consideram o uso do modelo de canal κ - μ .

Palavras-Chave—Rádio cognitivo, detecção de energia, canais em desvanecimento, sensoriamento de espectro, distribuição κ - μ .

I. INTRODUCTION

NEW standards for wireless networks are emerging rapidly and further increasing the demand for spectrum with wider bands [1]. Although the spectrum is almost all booked for high speed wireless communication, the occupancy rate of its channels are pretty low in most of the time [2], [3]. Several studies present the inefficiently usage of the spectrum in wireless networks [3]–[6]. Some frequency bands are overused suffering from excessive data and voice traffics. On the other hand, there are a lot of bands with low or zero usage that could allow an improvement of the quality and capacity of services in which the frequencies allocated are insufficient.

To solve the problem of spectrum scarcity and spectrum underutilization, cognitive radio technology has arisen as a promising solution due to its ability to rapidly and autonomously adapt operating parameters to changing requirements and conditions. Recently [7], [8], energy detection over fading channels has been presented, in which detection probability is derived for some traditional models, such as Rayleigh, Rice, Nakagami, and Hoyt. However, studies concerning generalized

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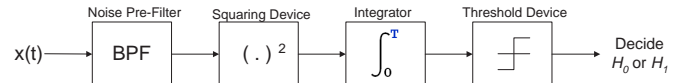


Fig. 1. Energy detector.

fading environments in which those traditional models do not fit well, still lack in the literature. In [9], a general physical fading model, namely the κ - μ model, has been proposed which describes the small-scale variations of the fading signal under a light-of sight (LOS) condition. The κ - μ distribution includes as special cases important other distributions such as Rice (Nakagami- n) and Nakagami- m [9]. Therefore, One-Sided Gaussian and Rayleigh also constitute special cases of it. Its flexibility renders it suitable to better fit field measurements data in a variety of scenarios, both for low- [9] and high-order statistics [10].

In this work, assuming a κ - μ fading scenario, the performance analysis of the spectrum sensing activity under generalized fading channels is presented and investigated. More specifically, receiver operation characteristics (ROC) are obtained under several different fading environments modeled by κ - μ distribution. Moreover, the improvement in detection capability is evaluated when cooperative spectrum schemes are employed under κ - μ fading channel. Comparisons are also performed considering the use of κ - μ and Rayleigh models in spectrum sensing techniques. To the best of the authors' knowledge, this is the first time that the cognitive radio characteristics is investigated under κ - μ fading channels.

The remainder of this paper is structured as follows. In Section II, performance analysis of local spectrum sensing under κ - μ fading channel is presented and discussed, varying the number of multipath clusters and dominant components. Detection probability of the κ - μ distribution is also found and investigated. In Section III, cooperative spectrum sensing is performed under κ - μ fading scenarios and the detection characteristics is analyzed with different numbers of collaborative sensors. Finally, in Section IV, conclusions remarks are drawn.

II. SPECTRUM SENSING OVER FADING CHANNELS

Usual and well-known schemes for primary signal detection are based on low power signal detection, with local observation for each secondary terminal, in compliance with a decision process with two hypothesis:

$$\begin{aligned} H_0 : y(t) &= n(t), \\ H_1 : y(t) &= hx(t) + n(t), \end{aligned} \quad (1)$$

in which $y(t)$ is the signal detected by the secondary user, and $x(t)$ is the primary transmitted signal. The channel gain is

represented by h and additive white gaussian noise (AWGN), by $n(t)$.

According to the hypothesis model (1), three detection schemes may be mentioned [11]: (i) Matched Filter Detection, which requires a priori knowledge of the primary user signal; (ii) Energy Detection, which is adequate when the receiver cannot gather sufficient information about the primary user signal; and (iii) Cyclostationary Feature Detection, which can perform better than an energy detector, but is computationally complex and requires long observation time. In this work, the energy detection scheme is applied.

Using an energy detector, as depicts block-diagram of Figure 1, the signal which serves as decision statistic is the output of an integrator. We denote it Y , and it may be shown to have the following distribution [12],

$$\begin{aligned} H_0 &: Y \chi_{2TW}^2 \\ H_1 &: Y \chi_{2TW}^2(2\gamma) \end{aligned} \quad (2)$$

in which γ is the signal-to-noise rate (SNR), χ_{2TW}^2 and $\chi_{2TW}^2(2\gamma)$ are chi-square distributions, centered and non-centered, respectively, with $2TW$ degrees of freedom and non-centrality parameter 2γ for the second distribution. TW is the product time-bandwidth, which is assumed to be an integer, denoted u .

In order to eliminate interference and collisions, the state of the channel ought to be detected in a reliable manner by secondary users. In non-fading environments, where the channel gain h is deterministic, detection probability using energy detectors equals the conditional probability that the signal Y is above the threshold λ , while the channel is indeed occupied. Detection probability is given by [8] as

$$P_d = P\{Y > \lambda \mid H_1\} = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}), \quad (3)$$

in which λ is a comparison threshold, above which the output Y indicates the presence of signal, and $Q_u(\cdot, \cdot)$ is the generalized Marcum Q-function [13], defined as follows,

$$Q_u(a, b) = \int_b^\infty \frac{x^u}{a^{u-1}} \exp\left(-\frac{x^2 + a^2}{2}\right) I_{u-1}(ax) dx, \quad (4)$$

in which I_{u-1} is the modified Bessel function, with order $u-1$. The missing probability is then defined as $P_m = 1 - P_d$. False alarm probability is given by

$$P_f = P\{Y > \lambda \mid H_0\} = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)}, \quad (5)$$

where $\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ are the complete and incomplete gamma functions, respectively.

In fading environments, where h varies, (3) gives the conditional detection probability, for a given instantaneous signal-to-noise ratio γ . Detection probability is then obtained, averaging the conditional probability over SNR probability distribution function (PDF), $f_\Gamma(\cdot)$, as follows,

$$P_d = \int_x Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) f_\Gamma(x) dx. \quad (6)$$

Performance of energy detector, given the average SNR,

$\bar{\gamma}$, and the time-bandwidth, u , may be characterized by the complementary receiver operating characteristics (ROC) curves, which plots the missing probability versus the false-alarm probability (P_m vs. P_f). In what follows we study performance under κ - μ fading channel and under κ - μ Extreme fading model. Comparisons are performed between these two distributions and Rayleigh model.

A. The κ - μ Fading Channel

The κ - μ distribution is a general fading distribution that can be used to represent the small variation of the fading signal under line-of-sight (LOS) conditions. It includes as special cases important other distributions such as Rice (Nakagami- n) and Nakagami- m [9]. Therefore, One-Sided Gaussian and Rayleigh also constitute special cases of it. As its name implies, it is written in terms of two physical parameters, namely κ and μ . The parameter $\kappa > 0$ concerns the ratio between the total power of the dominant components and the total power of the scattered waves, whereas the parameter $\mu > 0$ is related to the multipath clustering.

According to the standard statistical procedure of transformation of variates, the PDF of γ can be obtained from [9, Eq. 11] as

$$f_\Gamma(\gamma) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp[\kappa\mu] \sqrt{\gamma\bar{\gamma}}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\mu}{2}} \times \exp\left[-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right] I_{\mu-1}\left[2\mu\sqrt{\kappa(\kappa+1)\frac{\gamma}{\bar{\gamma}}}\right] \quad (7)$$

where $I_\nu(\cdot)$ is the modified Bessel function of the first kind and order ν [14, Eq. 9.6.20]. Interestingly, by setting some specific fading parameters values, the PDF of γ of the κ - μ distribution reduces to the traditional models as: Rice ($\mu = 1$), Nakagami- m ($\kappa \rightarrow 0$), and Rayleigh ($\mu = 1$ and $\kappa \rightarrow 0$). In the same way, by setting $\mu = 0.5$ and $\kappa \rightarrow 0$, the PDF of γ of the One-Sided Gaussian distribution can be also obtained.

Using (7) in (6), the detection probability of the κ - μ distribution is found, unfortunately, in this case, not in closed form. However, the required integral can be straightforwardly and accurately implemented in largely used mathematical tools, and the results are generated at a negligible computational cost.

B. The κ - μ Extreme Distribution

The κ - μ Extreme distribution was also proposed in [9] and arises as a particular case of κ - μ distribution, for which the fading parameters assume extreme values, i.e., $\kappa \rightarrow 0$ and $\mu \rightarrow \infty$. Accordingly to [9], the PDF of the κ - μ Extreme envelope can be written in terms of the Nakagami- m fading parameter, $m = \frac{\mu(1+\kappa)^2}{1+2\kappa}$. Such result shows that for a given m , an infinite number of curves of κ - μ distribution can be obtained for appropriate values of κ and μ , rendering it well suited to field measurements in LOS conditions with very severe fading scenarios [9], [15].

In the same way of (7), the PDF of γ of the κ - μ Extreme distribution can be obtained from [9, Eq. 16] as

$$f_{\Gamma}(\gamma) = \frac{2mI_1(4m\sqrt{\gamma/\bar{\gamma}})}{\sqrt{\gamma/\bar{\gamma}}\exp[2m(1+\gamma/\bar{\gamma})]} + \frac{\exp(-2m)}{2\sqrt{\gamma/\bar{\gamma}}}\delta(\sqrt{\gamma/\bar{\gamma}}), \quad (8)$$

where $\delta(\cdot)$ is the Dirac delta function. Using (8) in (6), the P_d for the κ - μ Extreme fading channel is found (after some manipulation), unfortunately, not in closed form. Again, the required integral can be straightforwardly and accurately implemented in largely used mathematical tools, and the results are generated at a negligible computational cost.

C. Results and Discussions

Figures 2 and 3 depict the complementary ROC under the κ - μ fading scenarios. u and $\bar{\gamma}$ values are assumed to be 5 and 10 dB, respectively. The Rayleigh fading case are also depicted for comparison. It can be clearly observed the flexibility of the κ - μ distribution, which may model several different fading scenarios (LOS condition and multipath clustering) unlike Rayleigh which contemplates only one single case. Figure 2 shows that increasing the effect of the multipath clustering, μ , the missing probability decreases, which can benefit signal detection and possibly leading to a more favorable scenario than that with Rayleigh fading. Figure 3 illustrates the effect of the dominant component, described by κ fading parameter, on detection characteristics. We can see that this parameter has a very significant impact on the analysis, since a scenario with a low parameter value has a low probability of detection with a very high missing probability. Note that, when the κ parameter increases, i.e. when the dominant components prevail, the probability of detection also increases, leading to more favorable scenarios.

Figure 4 presents the ROC characteristics under very high severe fading scenarios modeled by the κ - μ Extreme distribution. As before, u and $\bar{\gamma}$ values are assumed to be 5 and 10 dB, respectively. It can be seen that some curves indicate the existence of non-nil detection probability for false alarm probability nulls. This particular behavior occurs due to impulse at the origin of the κ - μ Extreme PDF of γ (8). As already expected, increasing the predominance of the multipath clustering, the detection probability also increases for a given false alarm probability. Observe that κ - μ fading channel is able to report a larger number of detection characteristics, with higher and lower missing probability for a given false alarm probability, allowing the wireless receiver to operate with a most suitable detection probability in different environments.

III. COOPERATIVE SPECTRUM SENSING

The opportunistic usage of spectrum by a secondary network should cause interference to the licensed network. Thus, spectrum sensing activity must be performed in a reliable manner. If a secondary terminal tries to detect a primary signal experiencing deep fading with respect to the primary transmitter, its transmission could cause tremendous interference. To account for possible losses due to the channel between the primary transmitter and a secondary terminal, an increased sensitivity would be required among cognitive radios [16].

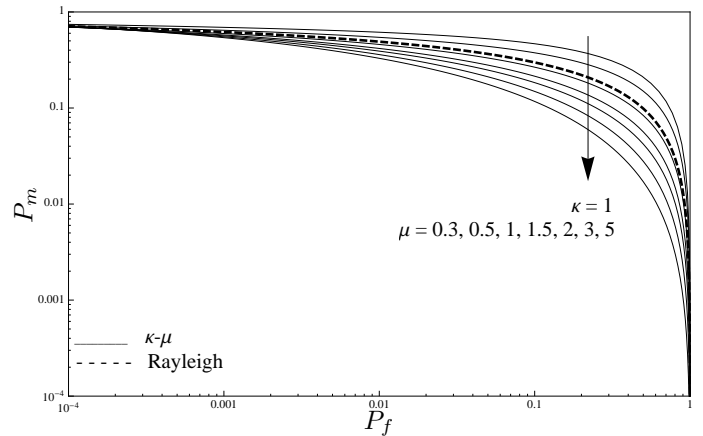


Fig. 2. Complementary ROC under κ - μ fading ($\bar{\gamma} = 10$ dB, $u = 5$). Rayleigh curve is depicted for comparison.

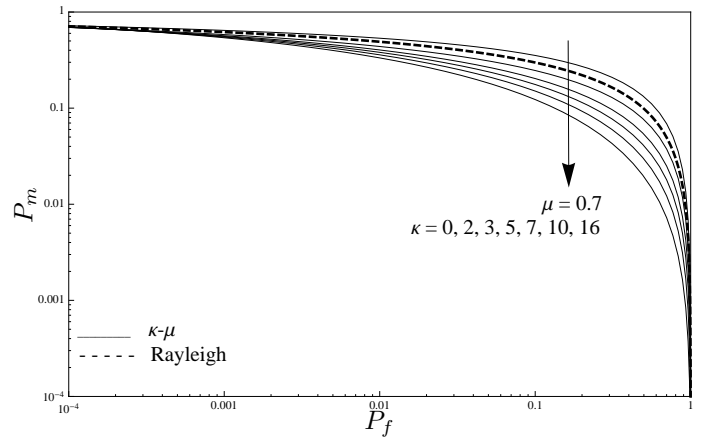


Fig. 3. Complementary ROC under κ - μ fading ($\bar{\gamma} = 10$ dB, $u = 5$). Rayleigh curve is depicted for comparison.

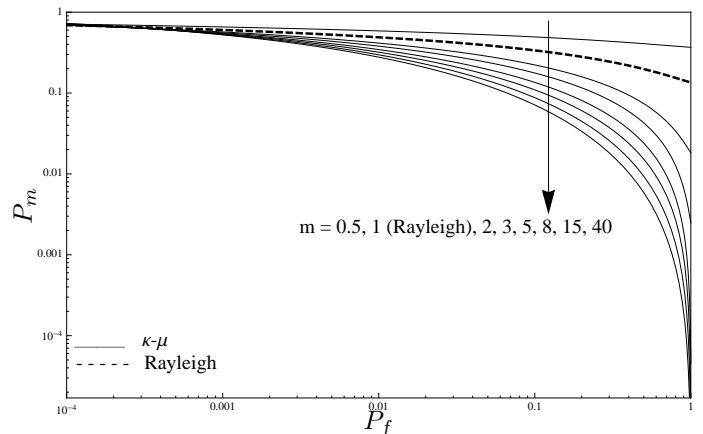


Fig. 4. Complementary ROC under κ - μ Extreme fading ($\bar{\gamma} = 10$ dB, $u = 5$). Rayleigh curve is depicted for comparison.

More robust spectrum sensing can be achieved using multiple collaborative users, with reduced sensitivity. In this case, multiple realizations of related random variables will be available, and the probability of all users experience deep fade is low. Thus, cooperative spectrum sensing provides confidence to secondary decisions, reducing the probability of an erroneous channel occupation. Besides the protection of prior users from the primary network, a throughput gain is also obtained, as many undesirable collisions are avoided. Detection time is also reduced, resulting an agility gain to the secondary network [17].

Despite the many advantages of cooperative spectrum sensing, its implementation presents some restrictions. Such as the increase of bandwidth for the communications among secondary users, the standardization of a band manager, in which local measurements should be processed into a decision, and the establishment of tradeoffs related to the reliability of the links and services.

In this context, we consider now a secondary network with n collaborating users, sensing all the desired frequency band in a periodic regime. For simplicity we assume that all n users experience independent and identically distributed (iid) fading with same average SNR. A fundamental result in distributed binary hypothesis testing is that when sensors are conditionally independent (as in our case), optimal decision rule for individual sensors is likelihood ratio test (LRT) [18]. However, optimum individual thresholds are not necessarily equal and it is generally hard to derive them. We assume that all users employ energy-detection rather than LRT and use the same decision rule (i.e. same threshold λ). While these assumptions render our scheme sub-optimum, they facilitate analysis as well as practical implementation.

A secondary user receives decisions from $n - 1$ others terminals and decides H_1 if any of the total n individual decisions is H_1 . This fusion rule is known as the OR-rule or 1-out-of- n rule [18]. Thus, the detection and the false alarm probabilities for the collaborative scheme (denoted by Q_d and Q_f , respectively) may be written as follows,

$$Q_d = 1 - (1 - P_d)^n, \quad (9)$$

$$Q_f = 1 - (1 - P_f)^n, \quad (10)$$

where P_d and P_f are the detection and false alarm probabilities of each collaborating secondary user, as defined previously in (6) and (5), respectively. It may be observed from (9) and (10) that although both detection and false alarm probabilities are increased when the cooperation scheme is applied, the network performance is indeed improved when users share information, cooperating to each other.

Figures 5 and 6 show the detection characteristics for different number of collaborating users under a typical case of κ - μ fading channel (with $\kappa = 1.5$ and $\mu = 1.75$), and under a very high severe fading scenario modeled by the κ - μ Extreme distribution (with $m = 5$), respectively. Observe how the cooperative spectrum sensing can improve the detection characteristics, as expected, reducing the overall missing probability. Note that a high number of collaborative terminals can be easily found in practice, for example, in a sensor network.

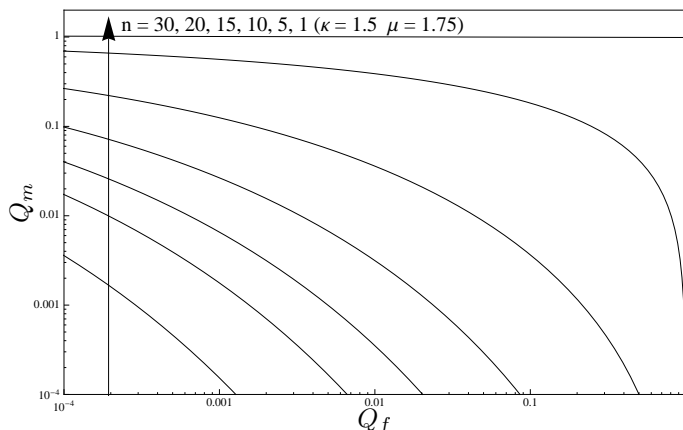


Fig. 5. Q_m vs Q_f under κ - μ fading for different number of collaborative spectrum sensors. ($\bar{\gamma} = 10$ dB, $u = 5$)

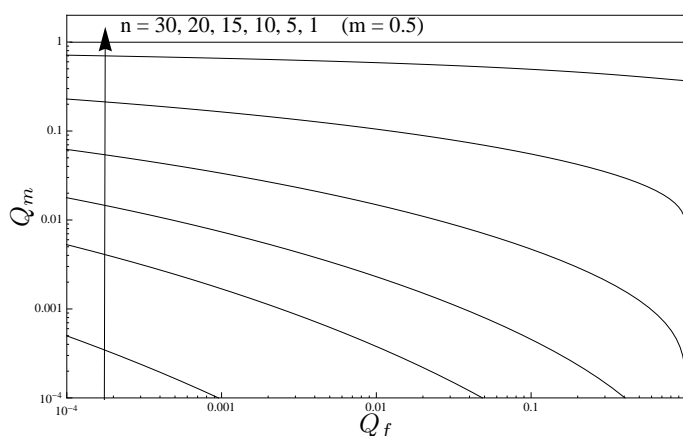


Fig. 6. Q_m vs Q_f under κ - μ Extreme fading for different number of collaborative spectrum sensors. ($\bar{\gamma} = 10$ dB, $u = 5$)

Even if few terminals are cooperating in the network, the result is a relevant gain in the probability of detection. Comparing Figures 5 and 6, we observe the differences caused by the effect of very high severe fading described by the κ - μ Extreme distribution. This difference is clearly noticed in cases where are few numbers of collaborating users. However, increasing the number of collaborative users, such difference decreases. Again, it can be seen in Figure 6 that some curves indicate the existence of non-nil detection probability for false alarm probability nulls.

Figures 7 and 8 show the detection probability versus average signal-to-noise ratio for different cooperative schemes under κ - μ fading model and κ - μ Extreme fading channel, respectively. For each curve, decision threshold, λ , is chosen such that $Q_f = 10^{-1}$. Time-bandwidth product, u , is set to 5 as before. Note that cooperation reduces the average SNR required to achieve the desired detection probability in both scenarios. Looking at figure 7, for a probability of detection equal to 0.9, with $n = 2$, local spectrum requires $\bar{\gamma} \simeq 8$ dB, while collaborative sensing with $n = 8$ only needs an average SNR of 3.9 dB for individual users under κ - μ fading scenario. Analyzing Figure 8, for a $m = 0.5$ and $n = 2$ it will be necessary a SNR way beyond normal parameters. And with $n = 8$ the average SNR required is about 4 dB. Here again we

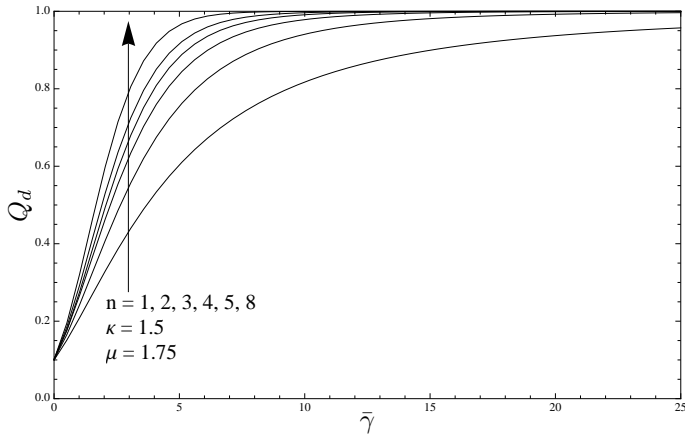


Fig. 7. Q_d vs. $\bar{\gamma}$ under κ - μ fading for different number of collaborative spectrum sensors ($Q_f = 10^{-1}$, $u = 5$).

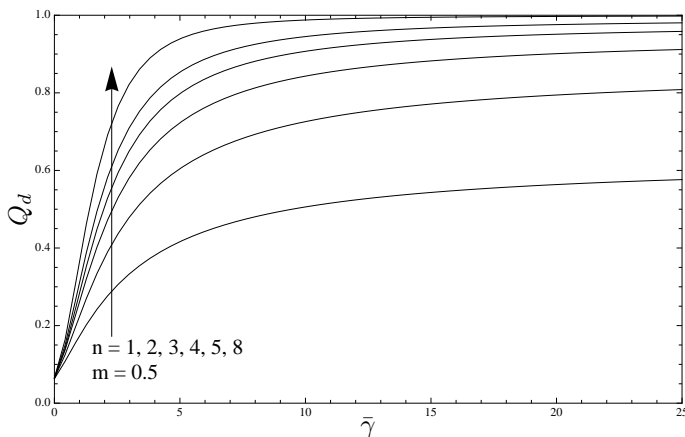


Fig. 8. Q_d vs. $\bar{\gamma}$ under extreme κ - μ fading for different number of collaborative spectrum sensors ($Q_f = 10^{-1}$, $u = 5$).

observe, in the κ - μ Extreme fading, that for a small number of cooperative users the necessary SNR required to obtain some desired detection probability is pretty higher than with the equivalent on the typical case κ - μ fading model. This effect is suppressed with the increase of collaborative terminals, confirming the great importance of cooperation between users to perform a reliable spectrum sensing.

IV. CONCLUSIONS

This paper presented the performance analysis of energy detection for an unknown transmit signal over generalized fading channels, modeled by the κ - μ distribution. Comparisons have been performed between κ - μ and Rayleigh fading channels, and a great flexibility was found in spectrum sensing techniques when the formulations considered the use of the κ - μ fading model. We have obtained the receiver operation characteristics under different fading scenarios, including low, typical and very high severe fading conditions, and considering the influence of both combining multipath clustering and the line-of-sight scenarios. We have also evaluated and quantified the improvement in detection capability when receive diversity schemes are employed. The results presented and analyzed

here are timely for emerging applications involving wideband wireless systems and cognitive radio technologies.

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