

# Outage Analysis of Cooperative Diversity Systems over Generalized Fading Models

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**Resumo**—Neste artigo, o desempenho de outage de sistemas cooperativos amplifica-e-encaminha (AF) de dois saltos sujeito a desvanecimento  $\alpha$ - $\mu$  é investigado. Mais especificamente, os efeitos dos parâmetros de desvanecimento na probabilidade de outage fim-a-fim são examinados. Com esse propósito, resultados de simulação são gerados e revelam que quando os parâmetros de desvanecimento aumentam, o desempenho também melhora.

**Palavras-Chave**—Comunicações cooperativas, probabilidade de outage, desvanecimento  $\alpha$ - $\mu$ .

**Abstract**—In this paper, the outage performance of dual-hop amplify-and-forward (AF) cooperative systems subject to  $\alpha$ - $\mu$  fading is investigated. More specifically, the effects of the fading parameters on the end-to-end outage probability are examined. To this end, simulation results are run and reveal that when the fading parameters increase, the performance also increases.

**Keywords**—Cooperative communications, outage probability,  $\alpha$ - $\mu$  fading.

## I. INTRODUCTION

Cooperative diversity techniques have arisen as promising strategies to be applied in current and future wireless systems, thanks to the increase in connectivity and capacity that they can yield by extending the radio coverage, without the need of high power levels at the transmitters. Depending on the nature and complexity of the relaying technique, cooperative diversity networks can be broadly categorized as either nonregenerative or regenerative. In the former, the relays simply amplify and forward (AF) the received signal, while in the latter the relays decode, encode, and then forward the received signal to the destination node. The AF mode puts less processing burden on the relays and, hence, is often preferable when complexity and/or latency issues are of importance. This paper primarily focuses on AF relaying systems.

In [1], [2], Hasna and Alouini have studied the end-to-end performance of dual-hop transmission systems with regenerative and nonregenerative relays, respectively, over Rayleigh fading channels. The same authors have evaluated in [3] the outage and error performance for nonregenerative channel state information (CSI)-assisted relays over Nakagami- $m$  fading channels. In this paper, differently from previous works, we consider an  $\alpha$ - $\mu$  fading scenario [4] for the investigation of the outage performance of dual-hop AF cooperative systems. More specifically, the effects of the fading parameters on the end-to-end outage probability are examined. To this end, simulation results are run and reveal that when the fading

parameters increase, the performance also increases. To the best of the authors' knowledge, this is the first time that the performance of cooperative systems undergoing  $\alpha$ - $\mu$  fading [4] has been analyzed in the literature.

## II. SYSTEM AND CHANNEL MODELS

### A. System Model

We consider a dual-hop non-regenerative cooperative system where a source node  $S$  communicates, using two time slots, with a destination node  $D$  through the help of an AF relay  $\mathcal{R}$ . All nodes are equipped with single antennas and operate in a half-duplex mode. Also, we assume that the source has a direct link with the destination. Using the time division multiple access (TDMA) mode for performing the communication, the source transmits its data signal to both the relay and destination in a signaling interval, and in the next signaling interval, the relay retransmits the amplified signal to the destination. All channel gains are fixed during one time frame, which corresponds to two time slots, and can vary from one time frame to another, i.e., a block flat-fading channel model is assumed. Incorporating the path loss in the signal propagation, the received signals at the destination and the relay in the first time slot can be written, respectively, as

$$y_{SD} = \sqrt{P_S d_{SD}^{-\eta}} h_{SD} x + n_{SD}, \quad y_{SR} = \sqrt{P_S d_{SR}^{-\eta}} h_{SR} x + n_{SR}, \quad (1)$$

where  $P_S$  stands for the transmit power of the source,  $h_{SD}$  and  $h_{SR}$  represent the channel coefficients of the  $\alpha$ - $\mu$  channels between  $S$ - $D$  and between  $S$ - $\mathcal{R}$ , respectively, which are assumed to have zero mean and unity variance,  $d_{SD}$  and  $d_{SR}$  are the distances between  $S$ - $D$  and between  $S$ - $\mathcal{R}$ ,  $\eta$  is the path loss exponent,  $x$  denotes the transmitted symbol and,  $n_{SD}$  and  $n_{SR}$  are the additive white Gaussian noise (AWGN) components at the relay and destination, which are mutually independent with zero mean and variance  $N_0$ . In the second time slot the relaying node forwards  $y_{RD}$  to the destination after multiplying it with a gain  $G$ . Hence, the received signal at  $D$  from the relay is given by

$$y_{RD} = \sqrt{P_{\mathcal{R}} d_{RD}^{-\eta}} h_{\mathcal{R},D} G \left( \sqrt{P_S d_{SR}^{-\eta}} h_{SR} x + n_{SR} \right) + n_{RD}, \quad (2)$$

where  $P_{\mathcal{R}}$  denotes for the transmit power of the relay,  $h_{\mathcal{R},D}$  denotes the fading coefficient of the  $\alpha$ - $\mu$  channel between  $\mathcal{R}$ - $D$ , having zero mean and unity variance,  $d_{RD}^{-\eta}$  is the distance between  $\mathcal{R}$ - $D$ , and  $n_{RD}$  represents the AWGN component at

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the destination with zero mean and variance  $N_0$ . One choice for the relay gain was given in [1] as

$$G^2 = \frac{1}{P_S |h_{SR}|^2 + N_0}. \quad (3)$$

By substituting (3) into (2) and after some algebraic manipulations, the end-to-end instantaneous signal-to-noise ratio (SNR) from the relying path can be written as

$$\gamma_{SRD} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \quad (4)$$

where  $\gamma_1 = \frac{P_S |h_{SR}|^2 d_{SR}^{-\eta}}{N_0}$  and  $\gamma_2 = \frac{P_R |h_{RD}|^2 d_{RD}^{-\eta}}{N_0}$ . The destination receives two copies from the signal  $x$  through the source link and relay link. Assuming the knowledge of the channel coefficients, the destination combines these copies using a maximal-ratio combiner (MRC), which is the optimal technique that maximizes the end-to-end SNR. Then, the end-to-end SNR at the output of the MRC can be written as

$$\gamma_{\text{end}} = \gamma_{SD} + \gamma_{SRD}, \quad (5)$$

where  $\gamma_{SD} = \frac{P_S |h_{SD}|^2 d_{SD}^{-\eta}}{N_0}$ .

### B. Channel Model

As aforementioned, the channel coefficients of the system model under study are assumed to follow an  $\alpha$ - $\mu$  distribution. Therefore, herein we will briefly revisit the  $\alpha$ - $\mu$  fading distribution proposed in [4]. This distribution is a general fading distribution that can be used to better represent the small-scale variation of the fading signal in a non line-of-sight fading condition. It includes as special cases important other distributions such as Nakagami- $m$  and Weibull. (Therefore, the Negative Exponential, One-Sided Gaussian, and Rayleigh are also special cases of it). As its name implies, it is written in terms of two physical parameters, namely  $\mu$  and  $\alpha$ . Roughly speaking, the parameter  $\alpha$  is related to the non-linearity of the environment, whereas the parameter  $\mu$  is associated to the number of multipath clusters.

The probability density function (PDF) of  $\lambda_i = |h_i|^2$  ( $i \in SD, SR, RD$ ) can be expressed according to [4] as

$$f_{\lambda_i}(\lambda) = \frac{\alpha_i \mu_i^{\mu_i} \lambda^{\frac{\alpha_i \mu_i}{2} - 1}}{2 \hat{r}_i^{\alpha_i \mu_i} \Gamma(\mu_i)} \exp\left(-\mu_i \frac{\lambda^{\frac{\alpha_i}{2}}}{\hat{r}_i^{\alpha_i}}\right), \quad (6)$$

where  $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$  is the Gamma function,  $\alpha_i$  and  $\mu_i$  denote the non-linearity parameter and the number of multipath clusters of the respective links, and  $\hat{r}_i = \sqrt{E[R_i^{\alpha_i}]}$ , with  $E[\cdot]$  denoting expectation and  $R_i$  the fading envelope of the links. By setting  $\alpha_i = 2$ , the Nakagami- $m$  fading scenario is attained, whereas for  $\mu_i = 1$  we get a Weibull environment. For further details about the fading parameters, please check [4]. In what follows, without any loss of generality and for simplicity purposes, we will assume that the channel coefficients are independent and identically distributed (i.i.d.)  $\alpha$ - $\mu$  random variables (i.e.,  $\alpha_i = \alpha$  and  $\mu_i = \mu$ ).

### III. OUTAGE PROBABILITY

The outage probability is defined as the probability that the received signal falls below a given threshold  $\gamma_{\text{th}}$ . This threshold

is a protection value for the SNR, above which the quality of service is deemed satisfactory. Specifically, such metric can be determined as the cumulative distribution function of  $\gamma_{\text{end}}$  evaluated at  $\gamma_{\text{th}}$ .

### IV. SIMULATION RESULTS

In this Section, Monte Carlo simulation results are presented in order to examine the effects of the fading parameters  $\alpha$  and  $\mu$  on the end-to-end outage performance. In the plots, we have normalized the total distance between the source and the destinations to unity so that  $d_{SR} = 0.5$  and  $d_{RD} = 0.5$ , we set  $\eta = 3$ , and  $\hat{r}_i = 1$ . Moreover, equal powers are assumed at the source and the relay, i.e.,  $P_S = P_R = P$ .

Fig. 1 portrays the outage probability as a function of the system SNR  $P/N_0$  for  $\gamma_{\text{th}} = 0\text{dB}$ . Two plots are shown. For each case, different system configurations were employed. First, note that most of the curves overlap each other when  $P/N_0$  is small, indicating that the fading scenarios do not affect considerably the outage performance for small values of the system SNR. In addition, it can be seen that when  $P/N_0$  increases, the outage probability drops sharply due to the effect of fading. As expected, low severity of fading implies in low values of the outage probability. In other words, when either  $\alpha$  or  $\mu$  increase, the performance improves. This is because of these parameters describe the severity of the fading. Observe also that, performing the flexibility of the  $\alpha$ - $\mu$  fading model in contrast to the traditional models (Rayleigh, Nakagami- $m$ ), it is possible to better estimate the fading severity in general environments.

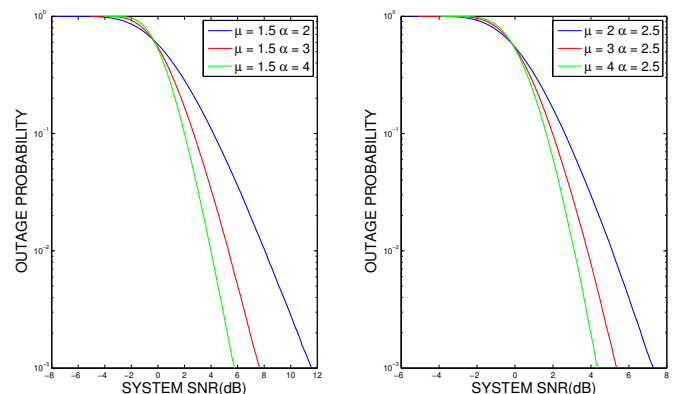


Fig. 1. Outage probability of cooperative diversity systems subject to  $\alpha$ - $\mu$  fading ( $d_{SR} = 0.5$ ,  $d_{RD} = 0.5$ ,  $\eta = 3$ ,  $\gamma_{\text{th}} = 0\text{dB}$ ).

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