Hybrid MCMC and LMMSE Detector for MIMO Frequency-Selective Channels

Guilherme C. G. Fernandes and Marcelo G. S. Bruno

***Abstract*— Optimal detection in multiple-input multiple-output (MIMO) frequency-selective systems is known to have exponential complexity due to the number of transmitter antennas and channel length. In this paper, we model the detection problem using factor graphs and apply the sum-product algorithm (SPA) to derive the optimal detector. Then we adapt the SPA to propose a hybrid suboptimal algorithm based on two known detectors: the Markov chain Monte Carlo (MCMC) detector initialized with the solution from the linear minimum mean square error (LMMSE) detector. The proposed algorithm achieves better performance than any of the two individually while preserving their lower complexity.**

***Keywords*— MIMO detection, MCMC detector, LMMSE detector, factor graphs, sum-product algorithm**

I. Introduction

An important and challenging problem in multiple-input multiple-output (MIMO) communication systems is decoding the received signal [1]. Typically, in optimal Maximum *a Posteriori* (MAP) MIMO detection, multiple interfering symbols transmitted concurrently by multiple antennas have to be jointly detected subject to random noise and interference at the receiver's end. However, the MAP detector was proven to be non-deterministic polynomial-time hard (NP-hard) such that its computational complexity grows exponentially with the number of transmitter antennas [1] and multipath reflections. Therefore, several suboptimal but faster approaches have been studied in order to make MIMO detection scalable [1].

In our approach, we follow the lead in [2], [3] and [4] and use the factor graph (FG) model to graphically represent the factorization of the joint probability distribution of the transmitted symbols, which improves the visualization of the problem. Marginal distributions can be calculated on a FG using the sum-product algorithm (SPA) [2], also known as message-passing.

Variations on the SPA lead to different message-passing based detection algorithms, as shown in [4]. The optimal MAP detector is obtained by the exact application of the SPA and considering discrete probability mass functions for the states containing sets of transmitted symbols. The proposed suboptimal hybrid detector is based both on the known linear minimum mean square error (LMMSE) and Markov chain Monte Carlo (MCMC) detectors. Similarly to the MAP detector, the LMMSE detector uses exact message-passing, but approximates the probability function of the states as multivariate Gaussian distributions. The MCMC detector is based on particle message-passing, which approximates the desired joint probability function obtaining samples from it using the Gibbs sampler (GS) [5], [6].

The paper is divided into 5 sections. Sec. I is this Introduction. In Sec. II, we describe the signal model and its factor graph representation. In Sec. III, we review the SPA. In Sec. III-A, we present the exact equations for the algorithm and optimal detector. In Sec. III-B, we introduce the hybrid suboptimal detector and the hypotheses that lead the SPA to the LMMSE and MCMC detectors. Simulation and results are shown in Sec. IV and conclusions are presented in Sec. V.

Notation: We denote scalars by a or A, vectors by $\mathbf{a}$ and matrices by $\mathbf{A}$. $(.)^T$ and $(.)^H$ denote matrix transpose and hermitian. $\mathbf{I}$ denotes the identity matrix and $\mathbf{0}_{a\times b}$ denotes a zero matrix with a rows and b columns. $[\mathbf{H}]_{a:b,c:d}$ denotes a submatrix of $\mathbf{H}$, selecting the rows from a to b and columns from c to d. We denote random vectors by $\mathbf{A}$, which can be distinguished from deterministic matrices by the context. We denote the probability of an event by $Pr(\mathbf{A})$. If $\mathbf{A}$ is discrete, we denote its probability mass function (pmf) $Pr(\mathbf{A} = \mathbf{a}\})$ by $P(\mathbf{a})$. If $\mathbf{A}$ is continuous, we denote its probability density function (pdf) by $p(\mathbf{a})$.

II. Problem Statement

A. Signal Model of the Communication Problem

We assume a MIMO communication system with N_T transmitter and $N_R \geq N_T$ receiver antennas. The wireless channel is linear, time-variant, frequency-selective with impulse response length L and presents Rayleigh fading. A stream of N_b uncoded bits is transmitted, each Q bits being mapped into $N_s = N_b/Q$ symbols according to some symbol alphabet $\mathcal{A}$ of size 2^Q. The symbols are transmitted over $N = N_s/N_T$ time instants, and received over $M = N + L - 1$ time instants, due to intersymbol interference.

We assume the wireless channel has a coherence time of M, meaning that it will remain constant while the bit stream is being received. The channel is described by its taps $[\mathbf{H}_1 \cdots \mathbf{H}_L]$, where each $[\mathbf{H}_i]_{r,t} \in \mathbb{C}$ is the channel coefficient between the t-th transmitter and the r-th receiver antenna in the i-th reflection, drawn from a complex Gaussian distribution $\mathcal{CN}(0, 1)$. The channel estimation problem is not in the scope of this paper, thus all coefficients are considered *perfectly known* at the receiver.

Let s_i^t be the symbol on the t-th transmitter antenna on time instant i and $\mathbf{s}_i = [s_i^1 \cdots s_i^{N_T}]^T$ be the vector of all transmitted symbols at instant i. The received signal is then modeled by

$$\mathbf{y}_i = \sum_{k=1}^{L} \mathbf{H}_k \mathbf{s}_{i-k+1} + \mathbf{z}_i \tag{1}$$

Guilherme C. G. Fernandes and Marcelo G. S. Bruno, Institute of Technology of Aeronautics (ITA), São José dos Campos-SP, Brazil, E-mails: gcosta@ita.br, bruno@ita.br.

where $\mathbf{y}_i = [y_i^1 \cdots y_i^{N_R}]^T$, y_i^r denotes the signal received by the r-th receiver antenna on instant i, and $\mathbf{z}_i$ is random white complex Gaussian noise, with $\mathbb{E}[\mathbf{Z}_i] = \mathbf{0}$ and $\mathbb{E}[\mathbf{Z}_i \mathbf{Z}_i^H] = \sigma_z^2 \mathbf{I}$.

The problem of interest is to obtain an estimate $\hat{\mathbf{b}}$ of all transmitted bits $\mathbf{B}$ given the collection $\mathbf{y} = \mathbf{y}_{1:M} = [\mathbf{y}_1^T \cdots \mathbf{y}_M^T]^T$ of all observations $\mathbf{y}_i$ using $P(\mathbf{b}|\mathbf{y})$. The optimal MAP detector expression for each bit is given by

$$\hat{b}_{i,MAP} = \operatorname*{argmax}_{b_i \in \{0,1\}} \sum_{\mathbf{b}/\{b_i\}} P(\mathbf{b}|\mathbf{y}) = \operatorname*{argmax}_{b_i \in \{0,1\}} P(b_i|\mathbf{y}). \quad (2)$$

where $\sum_{\mathbf{b}/\{b_i\}}$ denotes summation over all variables in $\mathbf{b}$, except b_i. Since bits and symbols relate to each other deterministically, the bitwise detector in Eq. (2) can be expressed as an equivalent symbolwise detector based on $P(\mathbf{s}|\mathbf{y})$

$$\hat{s}_{i,MAP}^t = \operatorname*{argmax}_{s_i^t \in \mathcal{A}} \sum_{\mathbf{s}/\{s_i^t\}} P(\mathbf{s}|\mathbf{y}) = \operatorname*{argmax}_{s_i^t \in \mathcal{A}} P(s_i^t|\mathbf{y}), \quad (3)$$

where $\mathbf{S}$ represents the collection of all transmitted symbols.

B. Representation with Factor Graph

The joint probability functions $P(\mathbf{b}|\mathbf{y})$ or $P(\mathbf{s}|\mathbf{y})$ can be factorized and represented by a factor graph, so we can apply the SPA to calculate the marginals needed for detection, in Eqs. (2) and (3). Without loss of generality, applying the Bayes' Law in the probability function in $P(\mathbf{s}|\mathbf{y})$, we obtain

$$P(\mathbf{s}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{s})P(\mathbf{s}). \quad (4)$$

Applying the chain rule in the likelihood function in the right side of Eq. (4), expanding $\mathbf{Y}$ and $\mathbf{S}$ as the collection of all received and transmitted signals:

$$p(\mathbf{y}_{1:M}|\mathbf{s}_{1:N}) = p(\mathbf{y}_M|\mathbf{y}_{1:M-1}, \mathbf{s}_{1:N}) \cdot p(\mathbf{y}_{1:M-1}|\mathbf{s}_{1:N}). \quad (5)$$

From the model in Eq. (1), given the last L transmitted set of symbols, an observation is independent of any other observations or symbols. Thus we can simplify Eq. (5) to

$$p(\mathbf{y}_{1:M}|\mathbf{s}_{1:N}) = p(\mathbf{y}_M|\mathbf{s}_N) \cdot p(\mathbf{y}_{1:M-1}|\mathbf{s}_{1:N}). \quad (6)$$

Sequentially applying the chain rule to the rightmost factor:

$$\begin{aligned} p(\mathbf{y}|\mathbf{s}) =& p(\mathbf{y}_M|\mathbf{s}_N) \cdots p(\mathbf{y}_N|\mathbf{s}_{N-L+1:N}) \\ & \cdots p(\mathbf{y}_L|\mathbf{s}_{1:L}) \cdots p(\mathbf{y}_2|\mathbf{s}_{1:2}) \cdot p(\mathbf{y}_1|\mathbf{s}_1). \end{aligned} \quad (7)$$

Depending on L, this factorization may result in a cyclic FG, which requires iterative SPA until the messages converge [7]. Fig. 1 shows the cycle for $L \geq 3$.

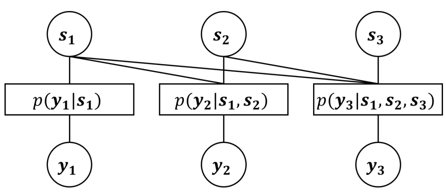

Fig. 1: Cyclic FG resulting from factorization in Eq. (7).

As in [3], in order to remove the cycles, we first introduce the state variables $\mathbf{W}_i$, corresponding to all transmitted symbols that influence the signal received at instant i, shown in Eq. (7), and rewrite the factorization as

$$p(\mathbf{y}|\mathbf{s}) = \prod_{i=1}^{M} p(\mathbf{y}_i|\mathbf{w}_i). \quad (8)$$

Then we add indicator probability functions $P(\mathbf{w}_i|\mathbf{w}_{i-1})$ between consecutive states, which evaluate to 1 when all the common symbols $\mathbf{s}_j$ in $\mathbf{w}_i$ and $\mathbf{w}_{i-1}$ match, and $P(\mathbf{w}_i|\mathbf{s}_i)$ between states and the newly added symbols, which evaluate to 1 when $\mathbf{s}_i \in \mathbf{w}_i$. Thus, Eq. (8) is not modified if written as

$$\begin{aligned} p(\mathbf{y}|\mathbf{s}) =& p(\mathbf{y}_1|\mathbf{w}_1)P(\mathbf{w}_1|\mathbf{s}_1) \\ & \prod_{i=2}^{N} p(\mathbf{y}_i|\mathbf{w}_i)P(\mathbf{w}_i|\mathbf{w}_{i-1})P(\mathbf{w}_i|\mathbf{s}_i) \\ & \prod_{i=N+1}^{M} p(\mathbf{y}_i|\mathbf{w}_i)P(\mathbf{w}_i|\mathbf{w}_{i-1}) \end{aligned} \quad (9)$$

which can be represented by the factor graph in Fig. 2. Fig. 3 details the messages for each state variable, on each time instant i.

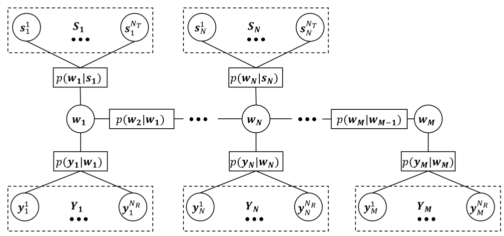

Fig. 2: Cycle-free FG resulting from factorization in Eq. (9).

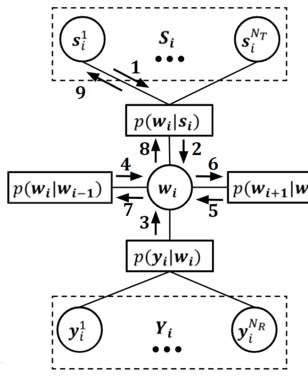

Fig. 3: Factor graph node for symbolwise detection.

III. Sum-Product Algorithm

The sum-product algorithm (SPA), also called message-passing, is described in [2]. Its output is the marginalized probability function for each variable node in the FG. In Subsections III-A and III-B, we explore how the SPA can result in different decoders depending on the chosen assumptions.

A. *Maximum a Posteriori Detector*

The SPA results in the MAP detector when applying the exact message-passing schedule in [2] and the values for the probability distribution of the states is a discrete pmf and calculated individually for each state. The resulting equations are the same as those given in [3] and [4]. All messages are normalized to total probability 1.

The *a priori* knowledge of the symbol distribution is represented by $m_1(s_i^t)$, which usually is considered uniform. Message 3, related to the observation model, is calculated as

$$m_3^{(i)}(\mathbf{w}_i) \propto \exp(-(\mathbf{y}_i - \bar{\mathbf{H}}_i\mathbf{w}_i)^H \Sigma_{z,i}^{-1}(\mathbf{y}_i - \bar{\mathbf{H}}_i\mathbf{w}_i)) \quad (10)$$

where the matrix $\bar{\mathbf{H}}_i$, extended from Eq. (1), is given by

$$\bar{\mathbf{H}}_i = \begin{cases} [\mathbf{H}_i \cdots \mathbf{H}_1] & , i < L \\ [\mathbf{H}_L \cdots \mathbf{H}_1] & , L \le i \le N \\ [\mathbf{H}_L \cdots \mathbf{H}_{i-N+1}] & , i > N \end{cases}$$

The other messages are calculated from the SPA expressions for variable and function nodes. Message 2 is calculated as

$$m_2^{(i)}(\mathbf{w}_i) = \sum_{\mathbf{s}_i} p(\mathbf{w}_i|\mathbf{s}_i) \prod_{n=1}^{N_T} m_{1,n}^{(i)}(s_i^n). \quad (11)$$

Messages 4 and 6 are calculated sequentially, forming the *forward recursion*.

$$m_6^{(i)}(\mathbf{w}_i) = m_2^{(i)}(\mathbf{w}_i) m_3^{(i)}(\mathbf{w}_i) m_4^{(i)}(\mathbf{w}_i) \quad (12)$$

$$m_4^{(i)}(\mathbf{w}_i) = \sum_{\mathbf{w}_{i-1}} P(\mathbf{w}_i|\mathbf{w}_{i-1}) m_6^{(i-1)}(\mathbf{w}_{i-1}). \quad (13)$$

Messages 5 and 7 are also calculated sequentially, but forming the *backward recursion*

$$m_7^{(i)}(\mathbf{w}_i) = m_2^{(i)}(\mathbf{w}_i) m_3^{(i)}(\mathbf{w}_i) m_5^{(i)}(\mathbf{w}_i) \quad (14)$$

$$m_5^{(i)}(\mathbf{w}_i) = \sum_{\mathbf{w}_{i+1}} P(\mathbf{w}_{i+1}|\mathbf{w}_i) m_7^{(i+1)}(\mathbf{w}_{i+1}). \quad (15)$$

Messages 8 and 9 are calculated to finish the SPA

$$m_8^{(i)}(\mathbf{w}_i) = m_3^{(i)}(\mathbf{w}_i) m_4^{(i)}(\mathbf{w}_i) m_5^{(i)}(\mathbf{w}_i), \quad (16)$$

$$m_{9,n}^{(i)}(s_i^n) = \sum_{w_i} \sum_{\mathbf{s}_i/\{s_i^n\}} P(\mathbf{w}_i|\mathbf{s}_i) m_8^{(i)}(\mathbf{w}_i) \prod_{l=1, l\neq n}^{N_T} m_{1,l}^{(i)}(s_i^l). \quad (17)$$

Substituting Eqs. (12) and (16) in (17), we obtain a simplified expression for message 9

$$m_{9,n}^{(i)}(s_i^n) = \frac{1}{m_{1,n}^{(i)}(s_i^n)} \sum_{\mathbf{w}_i} P(\mathbf{w}_i|\mathbf{s}_i) m_5^{(i)}(\mathbf{w}_i) m_6^{(i)}(\mathbf{w}_i). \quad (18)$$

Finally, detection is done by calculating the marginal distribution of each symbol, $p(S_i^t|\mathbf{y})$, as the product $m_{1,n}^{(i)}(s_i^n) * m_{9,n}^{(i)}(s_i^n)$, and find the maximum according to Eq. (3). If desired, turbo processing is done feeding back $m_{9,n}^{(i)}(s_i^n)$ in $m_{1,n}^{(i)}(s_i^n)$, which can be interpreted as having an improved *a priori* knowledge of the symbol distribution.

On each instant i, the exponential complexity $\mathcal{O}(2^{QLN_T})$ is evident in the messages whose argument is $\mathbf{w}_i$, since the summation must be evaluated on each of the 2^{QLN_T} possible values for $\mathbf{w}_i$.

B. *Suboptimal Hybrid Detector*

The proposed suboptimal detection algorithm is based primarily on the Markov chain Monte Carlo (MCMC) detector. Using the FG approach and the SPA, the MCMC detector is obtained not by calculating a joint probability function as the MAP does, but by approximating the distribution using the Gibbs sampler (GS) to sequentially obtain samples that converge to samples drawn from the true joint probability.

However, it is known that the MCMC detector presents a bit error rate floor at high SNR regimes [8]. This problem is referred to as the Gibbs sampler *stalling*, since the small variance of the received signal makes moving along the Markov chain less likely [9], thus the GS gets stuck in a local optimum. An alternative to improve the MCMC performance mentioned in [10] and [8] is to use a smart initial state for the GS instead of a random state.

In this paper, we initialize the GS with the solution coming from the linear minimum mean square error (LMMSE) detector. The LMMSE solution is obtained by applying the exact message-passing schedule described in Eqs. (11) to (18), but approximating the distribution of the states $\mathbf{W}_i$ as multivariate Gaussians. Although it is a coarse approximation, it simplifies the SPA equations, which become dependent only on the mean vector and covariance matrix parameters instead of a value for each possible state $\mathbf{w}_i$.

In Subsections III-B.1 and III-B.2, we start by detailing the steps for the LMMSE detector, then the steps for the MCMC detector as in [3] and [4]. Finally, we describe the algorithm for the final hybrid detector.

1) Linear Minimum Mean Square Error Detector: A multivariate Gaussian pdf is parametrized by its mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, whose sizes in the largest case for state $\mathbf{w}_i$ are LN_T and $LN_T \times LN_T$, respectively.

We use the fact that the product of two Gaussian densities $\mathcal{N}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$ and $\mathcal{N}(\boldsymbol{\mu}_2, \boldsymbol{\Sigma}_2)$, after normalization, is also Gaussian, with equivalent density $\mathcal{N}(\boldsymbol{\mu}_{eq}, \boldsymbol{\Sigma}_{eq})$ such as that

$$\boldsymbol{\Sigma}_{eq}^{-1} = \boldsymbol{\Sigma}_1^{-1} + \boldsymbol{\Sigma}_2^{-1}, \quad \boldsymbol{\mu}'_{eq} := \boldsymbol{\Sigma}_{eq}^{-1}\boldsymbol{\mu}_{eq} = \boldsymbol{\mu}'_1 + \boldsymbol{\mu}'_2. \quad (19)$$

Modeling the communication problem in Eq. (1) as a state-space model, we can apply the results in [7] on Eqs. (11) to (18) to obtain the equations for the LMMSE detector [3].

$$\mu_{1,n}^{(i)} = \mathbb{E}[S_i^n], \quad \sigma_{1,n}^{2(i)} = \mathbb{E}[|S_i^n - \mu_{1,n}^{(i)}|^2] \quad (20)$$

For message 2, $m_2^{(i)}(\mathbf{w}_i)$, the parameters are

$$\boldsymbol{\Sigma}_2^{(i)^{-1}} = \textbf{diag}(0_{a\times 1}^T, \sigma_{1,1}^{-2(i)}, \cdots, \sigma_{1,N_T}^{-2(i)}), \quad (21)$$

$$\boldsymbol{\mu}_2'^{(i)} = [0_{a\times 1}^T, \mu_{1,1}^{(i)}\sigma_{1,1}^{-2(i)}, \cdots, \mu_{1,N_T}^{(i)}\sigma_{1,N_T}^{-2(i)}] \qquad (22)$$

where $a = (i-1)N_T, i = 1$ to L and $a = (L-1)N_T, i > L$ and $\textbf{diag}(v)$ is a diagonal matrix $\mathbf{A}$ with $\mathbf{A}_{i,i} = v_i$.

For message 3, $m_3^{(i)}(\mathbf{w}_i)$, $i = 1$ to M, Eq. (10) shows it has a Gaussian distribution with parameters

$$\boldsymbol{\Sigma}_3^{(i)^{-1}} = \bar{\mathbf{H}}_i^H \boldsymbol{\Sigma}_{z,i}^{-1} \bar{\mathbf{H}}_i, \quad \boldsymbol{\mu}_3'^{(i)} = \bar{\mathbf{H}}_i^H \boldsymbol{\Sigma}_{z,i}^{-1} \mathbf{y}_i. \qquad (23)$$

For messages 4 and 6 in the forward recursion,

$$\boldsymbol{\Sigma}_6^{(i)^{-1}} = \boldsymbol{\Sigma}_2^{(i)^{-1}} + \boldsymbol{\Sigma}_3^{(i)^{-1}} + \boldsymbol{\Sigma}_4^{(i)^{-1}}, \qquad (24)$$

$$\boldsymbol{\mu}_6^{(i)} = \boldsymbol{\Sigma}_6^{(i)}\big(\boldsymbol{\mu}_2'^{(i)} + \boldsymbol{\mu}_3'^{(i)} + \boldsymbol{\mu}_4'^{(i)}\big), \qquad (25)$$

$$\boldsymbol{\Sigma}_4^{(i)^{-1}} = \begin{bmatrix} [\boldsymbol{\Sigma}_6^{(i-1)^{-1}}]_{b:c,b:c} & \mathbf{0}_{c-d+1\times N_T} \\ \mathbf{0}_{N_T\times c-d+1} & \mathbf{0}_{N_T\times N_T}, \end{bmatrix} \qquad (26)$$

$$\boldsymbol{\mu}_4^{(i)} = \Big[[\boldsymbol{\mu}_6^{(i-1)}]_{b:c}^T \quad \mathbf{0}_{N_T}\Big], \qquad (27)$$

where for $i = 1$ to L, $b = 1$ and $c = (i-1)N_T$, and for $i = L+1$ to N, $b = N_T + 1$ and $c = LN_T$. For messages 5 and 7:

$$\boldsymbol{\Sigma}_7^{(i)^{-1}} = \boldsymbol{\Sigma}_2^{(i)^{-1}} + \boldsymbol{\Sigma}_3^{(i)^{-1}} + \boldsymbol{\Sigma}_5^{(i)^{-1}}, \qquad (28)$$

$$\boldsymbol{\mu}_7^{(i)} = \boldsymbol{\Sigma}_7^{(i)}\big(\boldsymbol{\mu}_2'^{(i)} + \boldsymbol{\mu}_3'^{(i)} + \boldsymbol{\mu}_5'^{(i)}\big), \qquad (29)$$

$$\boldsymbol{\Sigma}_5^{(i)^{-1}} = \begin{cases} \begin{bmatrix} \mathbf{0}_{N_T\times N_T} & \mathbf{0}_{N_T\times d} \\ \mathbf{0}_{d\times N_T} & [\boldsymbol{\Sigma}_7^{(i+1)^{-1}}]_{1:d,1:d} \end{bmatrix} & , i \geq L \\ [\boldsymbol{\Sigma}_7^{(i+1)^{-1}}]_{1:d,1:d} & , i < L, \end{cases} \qquad (30)$$

$$\boldsymbol{\mu}_5^{(i)} = \begin{cases} \Big[\mathbf{0}_{N_T\times N_T} \quad [\boldsymbol{\mu}_7^{(i+1)^T}]_{1:d}\Big] & , i \geq L \\ [\boldsymbol{\mu}_7^{(i+1)^T}]_{1:d} & , i < L, \end{cases} \qquad (31)$$

where $d = (N + L - i - 1)N_T$ for $i = N+1$ to $M, (L-1)N_T$ for $i = L+1$ to N and $(i-1)N_T$ for $i = 1$ to L.

To calculate message 9, first we denote the marginal distribution of $\mathbf{w}_i$ as message 11, with parameters

$$\boldsymbol{\Sigma}_{11}^{(i)^{-1}} = \boldsymbol{\Sigma}_5^{(i)^{-1}} + \boldsymbol{\Sigma}_6^{(i)^{-1}}, \quad \boldsymbol{\mu}_{11}^{(i)} = \boldsymbol{\Sigma}_{11}^{(i)}\big(\boldsymbol{\mu}_5'^{(i)} + \boldsymbol{\mu}_6'^{(i)}\big). \qquad (32)$$

Then, $m_{9,n}^{(i)}(s_i^n)$ is calculated extracting the corresponding entry from message 11 and subtracting the *a priori* info from message 1.

$$\sigma_{9,n}^{-2(i)} = [\boldsymbol{\Sigma}_{11}^{(i)^{-1}}]_{e+n,e+n} - \sigma_{1,m}^{-2(i)} \qquad (33)$$

$$\mu_{9,n}^{(i)} = \sigma_{9,n}^2\big([\boldsymbol{\Sigma}_{11}^{(i)^{-1}}]_{e+n,e+n}[\boldsymbol{\mu}_{11}^{(i)}]_{e+n} - \sigma_{1,n}^{-2(i)}\mu_{1,n}^{(i)}\big) \qquad (34)$$

with $e = \min\{(i-1)N_T, (L-1)N_T\}$. The message is converted to a probability for each symbol according to

$$m_{9,n}^{(i)}(s_i^n) \propto \exp\big(-\sigma_{9,n}^{-2(i)}|s_i^n - \mu_{9,n}^{(i)}|^2\big). \qquad (35)$$

Note that inverting a $LN_T \times LN_T$ matrix is the critical step on each instant, resulting in a complexity of $\mathcal{O}(L^3N_T^3)$.

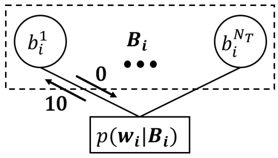

Fig. 4: Modification on Fig. 3 for MCMC bitwise detection.

2) Markov Chain Monte Carlo Detector: With the Gibbs sampler [5], [6], decoding is done using the bitwise detector in Eq. (2). The GS iteratively generates R samples from $P(\mathbf{b}|\mathbf{y})$ by drawing samples from all conditional probabilities $P(b_k|\mathbf{b}/\{b_k\}, \mathbf{y})$. Using Bayes' Law, they are proportional to the likelihood functions $p(\mathbf{y}|b_k, \mathbf{b}/\{b_k\})$, which are multivariate Gaussian pdf's depending on b_k.

The FG model is similar to that in Fig. 3, changing the symbol nodes to the bit nodes shown in Fig. 4. On the r-th iteration, for the k-th bit in $\mathbf{b}$, denoted as $b_k^{(r)}$, we obtain its conditional distribution given the bits $\mathbf{b}_{1:k-1}^{(r)}$ and $\mathbf{b}_{k+1:N_b}^{(r-1)}$, apply a simplified SPA, shown in [4] and below, and draw a sample from its marginal to be used in the following iterations. The notation $\hat{m}$ refers to messages on each iteration, whereas m refers to the posterior averaging of messages $\hat{m}$.

$$\hat{m}_3^{(i)(r)}(\mathbf{w}_i(b_k)) \propto \exp[-(\mathbf{y}_i - \bar{\mathbf{H}}_i\mathbf{w}_i(b_k))^H \Sigma_{z,i}^{-1}(\mathbf{y}_i - \bar{\mathbf{H}}_i\mathbf{w}_i(b_k)] \qquad (36)$$

$$\hat{m}_7^{(i)(r)}(\mathbf{w}_i(b_k)) \propto \hat{m}_3^{(i)(r)}(\mathbf{w}_i(b_k))\hat{m}_5^{(i)(r)}(\mathbf{w}_i(b_k)) \qquad (37)$$

$$\hat{m}_5^{(i)(r)}(\mathbf{w}_i(b_k)) \propto \hat{m}_7^{(i+1)(r)}(\mathbf{w}_i(b_k)) \qquad (38)$$

$$\hat{m}_{10,n,q}^{(i)(r)}(b_k) \propto \hat{m}_3^{(i)(r)}(\mathbf{w}_i(b_k))\hat{m}_5^{(i)(r)}(\mathbf{w}_i(b_k)) \qquad (39)$$

$$b_k^{(r)} \sim m_{0,n,q}^{(i)}(b_k) * \hat{m}_{10,n,q}^{(i)(r)}(b_k) \qquad (40)$$

where the indexes represent the q-th bit of the symbol transmitted by the n-th antenna on instant i in the r-th iteration.

In order to obtain the true distribution $m_{10,n,q}^{(i)}$, we drop the first b samples before averaging, called *burn-in samples*, since they usually do not present good convergence properties.

$$m_{10,n,q}^{(i)}(b_k) \approx \frac{1}{R-b}\sum_{r=b+1}^{R} \hat{m}_{10,n,q}^{(i)(r)}(b_k). \qquad (41)$$

On each instant i, there are Q transmitted bits on each antenna. For each bit, the GS iterates R times and there is a message-passing schedule with complexity of $\mathcal{O}(L)$, resulting in a complexity of $\mathcal{O}(RQLN_T)$ for the MCMC.

Finally, the proposed hybrid detector is obtaining initializing the GS with the LMMSE solution, as shown in Algorithm 1.

Algorithm 1 Hybrid MCMC and LMMSE Detector

Step 1: LMMSE Detection
Require all $m_{1,n}^{(i)}$
for $i = 1$ to N **do** ▷ Fwd Recursion
  Compute $m_2^{(i)}$, $m_3^{(i)}$, $m_4^{(i)}$ and $m_6^{(i)}$
end for
for $i = M$ to 1 **do** ▷ Bwd Recursion
  Compute $m_3^{(i)}$, $m_5^{(i)}$ and $m_7^{(i)}$
end for
for $i = 1$ to N **do** ▷ Detection
  Calculate $m_{9,n}^{(i)}$, decode bits in s_i^n
end for ▷ Partial output: $\hat{\mathbf{b}}_{\mathbf{MMSE}}$

Step 2: MCMC Detection
Initialize $\mathbf{b}^{(0)} = \hat{\mathbf{b}}_{\mathbf{MMSE}}$, require R and b
for $r = 1$ to R **do** ▷ Each GS iteration
  for $k = 1$ to N_b **do** ▷ Each transmitted bit
    for $l = i(k)$ to $i(k) + L - 1$ **do** ▷ Bwd Recursion
      Calculate $\hat{m}_3^{(l)(r)}$, $\hat{m}_5^{(l)(r)}$, $\hat{m}_7^{(l)(r)}$ and $\hat{m}_{10}^{(l)(r)}$
      ▷ $i(k)$ is the state $\mathbf{w}_i$ containing bit b_k
    end for
    Draw a sample $b_k^{(r)} \sim m_0 * \hat{m}_{10}$
  end for
end for
Calculate $m_{10,n}^{(k)} = \sum_{r=b+1}^{R} \hat{m}_{10,n}^{(r)(k)} / (R - b)$

if not last iteration **then**
  Feedback $m_{10,n}^{(k)}$ into $m_1^{(k)}$ and $m_0^{(k)}$, go to **Step 1**
end if
Decode $\hat{\mathbf{b}}$ from all $m_{10,n}^{(k)}$

IV. Simulation Results

We compare the bit error rate (BER) of the MAP, hybrid and pure LMMSE and MCMC detectors. Setup is done with $N_T = 2, N_R = 2, L = 2$. For each coherence time block, the Rayleigh channel coefficients $[\mathbf{H}_i]_{r,t}$ are drawn independently from a $\mathcal{CN}(0, 1)$ distribution. A total of 2^{20} BPSK modulated bits are transmitted on blocks, each containing 2^5 bits. The GS is set with $R = 15$ and $b = 5$. We also simulated a second iteration with feedback of the output message, m_9 (or m_{10}), in the *a priori* information message, m_1 (or m_0). Results are shown in Fig. 5.

As expected, the MAP detector presented best performance and the fastest BER decaying rate. Both the LMMSE and MCMC detectors have worse BER and decaying rates than the MAP, the MCMC performing worse than the LMMSE and stalling at high SNR.

However, the error floor disappeared in the hybrid detector, even improving the LMMSE solution by a constant factor. The extra processing time in the hybrid detector is not restrictive, since the complexity of MCMC is inferior to that of LMMSE. At low SNR, the detectors present similar performance, especially the three proposed suboptimal detectors.

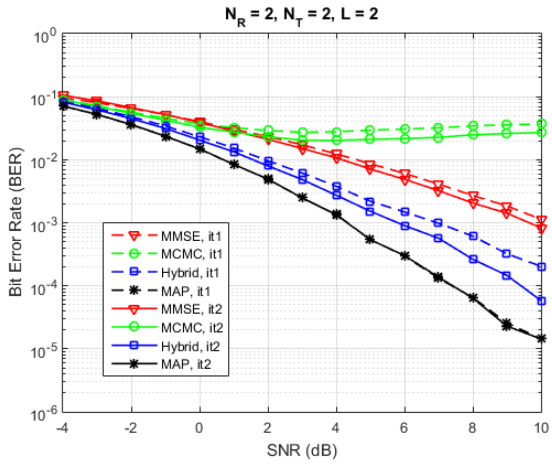

Fig. 5: BER performance comparison of the four detectors.

V. Conclusions

In this work we presented a factor graph approach to the problem of MIMO detection in frequency-selective channels and reviewed the MAP, LMMSE and MCMC detectors by assuming different hypotheses when applying the SPA. In addition, we proposed a hybrid detector by sequentially applying the LMMSE and MCMC methods, which presented better bit error rate than each detector individually while maintaining the LMMSE complexity. It is important to note that the alternatives to improve the MCMC detector performance usually determine the new detector complexity. Further work can be done on novel approaches to reduce the error floor on MCMC or to represent the distribution of the states.

Acknowledgements

The first author's work was supported by a Master's scholarship from CAPES, Brazil.

References

[1] S. Yang, L. Hanzo, "Fifty Years of MIMO Detecton: The Road to Large-Scale MIMOs," *IEEE Comm. Surv. Tut.*, vol. 17, pp. 1941-1988, 2015.
[2] F. R. Kschischang and B. J. Frey and H. A. Loeliger, "Factor Graphs and the Sum-Product Algorithm," *IEEE Trans. Inform. Theory*, vol. 47, pp. 498-519, Feb. 2001.
[3] B. Etzlinger, W Haselmayr and A. Springer, "Equalization Algorithms for MIMO Communication Systems Based on Factor Graphs," *in Proc. IEEE Int. Conf. on Comm. (ICC 2011)*, June 2011.
[4] B. Etzlinger, W Haselmayr and A. Springer, "Message Passing Methods for Factor Graph Based MIMO Detection," *Wireless Advanced*, pp. 132-137, June 2011
[5] A. Doucet and X. Wang, "Monte Carlo Methods for Signal Processing," *IEEE Sig. Proc. Mag.*, vol. 22, pp. 152-170, Nov. 2005.
[6] D. Gamerman, *Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference*. Chapman and Hall, 1997.
[7] H. A. Loeliger and J. Dauwels and J. Hu and S. Korl and L. Ping and F. R. Kschischang, "The Factor Graph Approach to Model-Based Signal Processing," *Proc. of the IEEE*, vol. 95, pp. 1295-1322, June 2007.
[8] S. Akoum, R. Peng, R. R. Chen and B. Farhang-Boroujeny, "Markov Chain Monte Carlo Detection Methods for High SNR Regimes," *Proc. IEEE Int. Conf. on Comm. (ICC 2009)*, June 2009.
[9] R. R. Chen, R. Peng and B. Farhang-Boroujeny, "Markov Chain Monte Carlo: Applications to MIMO detection and channel equalization," *Inform. Theory and App. Workshop*, Feb. 2009
[10] X. Mao and P. Amini and B. Farhang-Boroujeny, "Markov Chain Monte Carlo MIMO Detection Methods for High Signal-to-Noise Ratio Regimes," *IEEE GLOBECOM*, pp. 3979-3983, Nov. 2007.