Simple and Robust Method for OFDM Performance Improvement with Nonlinear Amplification

Robson F. de Moraes, Roberto C. G. Porto and Ernesto L. Pinto

***Abstract*— This work proposes a new technique to be used in orthogonal frequency division multiplexing (OFDM) systems subject to nonlinear power amplification, aiming to obtain better bit error rate (BER) performance with reduced computational complexity and low peak-to-average power ratio (PAPR). The core of the proposed technique is a new metric to choose a modified OFDM symbol to be transmitted among several alternatives with the aim of reducing the deleterious effects of nonlinear amplification. The proposed metric is independent of a power amplifier model and may be associated with different approaches to generate the modified OFDM symbols. Simulation results are presented, using a Partial Transmit Sequence (PTS) approach, which show that the proposed scheme provides BER performance similar to other known techniques at a significantly lower computational cost (about 33% less real multiplications and 66% less real additions). Estimates of the PAPR complementary cumulative distribution function (CCDF) are also presented and show that the proposed method does not degrade this performance index substantially compared to those techniques.**

***Keywords*— bit error rate (BER) performance, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), solid state power amplifier (SSPA), partial transmit sequence (PTS).**

I. Introduction

Several peak-to-average power ratio (PAPR) reduction techniques in orthogonal frequency division multiplexing (OFDM) systems have been proposed over the years [1], [2], [3], [4], with the ultimate goal of improving performance. They include, among others, Selected Mapping (SLM), Partial Transmit Sequence (PTS) and Clipping and Filtering. However, in spite of being a good parameter to evaluate the high power amplifiers (HPA) non-linear distortions, PAPR minimization is not enough to guarantee best bit error rate (BER) performance [2], [3].

A few methods have been recently developed with the aim of achieving good balances between PAPR reduction and BER performance. Such techniques use a mathematical model of the HPA to calculate the metric used for selecting the modified OFDM symbols to be transmitted [2], [3]. The use of such models leads to two problems: computational complexity increase and sensitivity to model mismatch.

The paper presents a new technique to perform this selection without any HPA model, thereby achieving considerable gains of computational complexity in comparison with state-of-the-art methods. A simulation-based performance comparison is presented, using the PTS technique to generate the modified OFDM symbols. The results here reported indicate that this complexity reduction is obtained without sacrificing the improvement of PAPR and BER performance previously achieved in [2].

The rest of this paper is organized as follows. In section II the OFDM system model and the mathematical models of the HPA are presented. Section III describes the proposed method. In Section IV its computational complexity is evaluated and compared with other known techniques. Section V shows the simulation results and, finally, in Section VI the conclusions are drawn.

Robson F. de Moraes. Defense Engineer Program, Instituto Militar de Engenharia, Rio de Janeiro, Brasil, E-mail: moraes@ime.eb.br. Roberto C. G. Porto e Ernesto L. Pinto. Electrical Engineer Department, Instituto Militar de Engenharia, Rio de Janeiro, Brasil, E-mails: camara@ime.eb.br, ernesto@ime.eb.br. This work was partially supported by CNPq (461895/2014-5).

II. Background

To establish a common notation to be used in this paper, consider N subcarriers modulated by a complex symbols sequence $\mathbf{d} = [d_0, d_1, \ldots, d_{N-1}]$ from a M-ary modulation constellation. Let x be the resulting OFDM signal obtained by applying an inverse fast Fourier transform (IFFT) to $\mathbf{d}$, so its nth element can be expressed as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k e^{j\frac{2\pi nk}{N}}, n = 0, 1, \ldots, N-1. \tag{1}$$

The OFDM signal PAPR can be estimated as

$$PAPR = \frac{max\{|x[n]|^2\}}{P_{in}}, n = 0, 1, \ldots, N-1, \tag{2}$$

where P_{in} denotes the average power per OFDM symbol.

A memoryless model is assumed for the nonlinear amplifier so its output y_n is expressed as

$$y_n = A(\rho_n) e^{j[\theta_n + \phi(\rho_n)]},\ \rho_n \triangleq |x_n|, \theta_n \triangleq arg\,(x_n) \tag{3}$$

where $A(\rho_n)$ and $\phi(\rho_n)$ denote the AM/AM and AM/PM conversion, respectively.

For traveling wave tube amplifiers (TWTA) the memoryless Saleh model is used [5], and the AM/AM and AM/PM conversion can be respectively expressed as

$$A(\rho_n) = \alpha_a \frac{\rho_n}{1 + \beta_a \rho_n^2},\ \phi(\rho_n) = \alpha_\varphi \frac{\rho_n^2}{1 + \beta_\varphi \rho_n^2}. \tag{4}$$

The Rapp model [6] is a widely used model for solid state power amplifier (SSPA). For this model, the AM/AM conversion is given by

$$A(\rho_n) = \rho_n \left[1 + \left(\frac{\rho_n}{A_0}\right)^{2p}\right]^{-\frac{1}{2p}}, \tag{5}$$

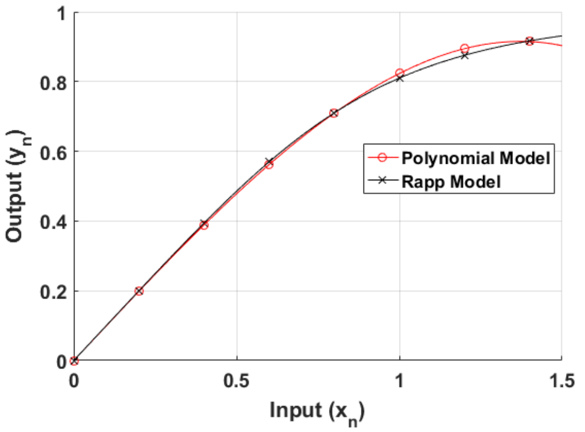

Fig. 1. AM/AM response for HPA models.

where A_0 is the maximum amplifier output, and the parameter p is the smoothness factor determining the curve inclination when transitioning from the linear to the saturation region.

Following a common trend in modeling SSPA, the phase conversion is set as $\phi(\rho_n) = 0$, once this type of amplifier typically has an almost flat phase deviation for large amplitude variation of its input power.

Another possibility to mimic a HPA is the use of polynomial models [7] which can be applied both to the AM/AM and AM/PM measurements.

Considering an odd third order nonlinearity the SSPA behavior is expressed by

$$y_n = \alpha_1 x_n + \alpha_3 x_n |x_n|^2 \tag{6}$$

It is usual to assume $\alpha_1 = 1$, while α_3 can be obtained by curve fitting the normalized AM/AM response.

Figure 1 depicts the SSPA AM/AM response for $A_0 = 1$, $2p = 3.286$, $\alpha_1 = 1$ and $\alpha_3 = -0.1769$ as in [2]. These values have been obtained by curve fitting, for a practical SSPA described in [8].

Figure 2 gives a general view of the technique here addressed. The first block provides different modifications of the OFDM symbol to be transmitted. The second block selects one of these modified symbols by optimizing an specific metric aiming lower PAPR and better BER performance. Typically, side information is sent to inform the receiver about the modification introduced in the selected symbol.

Some current symbol selection approaches can be summarized:

A. PAPR

In the conventional PTS [1], the phase sequence $\mathbf{b}$ which generates $\mathbf{x}_i$ with the lowest PAPR is selected. From this point on this technique will be referred to as PAPR method.

B. Cross Correlation

In [2] an approach that uses the cross correlation (CORR method) $R_{xy}^{(0)} = \sum_{n=0}^{N-1} x[n]\, y^*[n]$ as the optimization metric to select the best modified OFDM symbol is presented, where $y[n]$ represents the amplifier output.

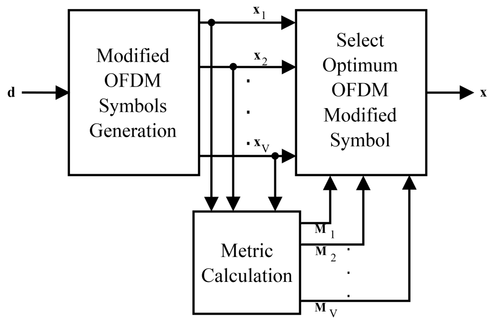

Fig. 2. General view of techniques for performance improvement over nonlinear amplification.

C. Mean Squared Error

Alternatively, [3] transmits the modified OFDM symbol with lowest mean squared error (MSE) metric, given by $MSE = \sum_{n=0}^{N-1} |x[n] - y[n]|^2$. This technique is referred to as MSE method.

It is worth to observe that both CORR and MSE techniques have to calculate an estimate of the HPA output $y[n]$ to obtain the corresponding metrics. This leads to increased computational complexity and model dependence, since a mathematical model of the nonlinear amplifier is necessary.

III. Proposed Method

Defining a sum of the largest powers (SLP) as

$$SLP = \sum_{k=0}^{M-1} x_{max}[k] \tag{7}$$

where $x_{max}[k]$ is the kth element of a vector containing the sorted elements $|x[n]|^2, n = 0, 1, ..., N-1$ in descend order, and M is the number of largest powers considered.

The method here proposed selects the modified OFDM symbol with minimum SLP. With $M = 1$ this is the minimum PAPR criterion, but as the value of M increases, the selected symbol tends to present degraded PAPR and improved BER performance, since the tendency is to choose the option that offers the lowest amount of variation above the HPA operation point. The expected behavior is to foresee a better performance for lower values of M because for larger values the summation in (7) includes also values closest to or below the operation point.

A methodology for determining M as a function of the characteristics of the system has not yet been established being chosen in an ad-hoc way.

IV. Computational Complexity

A complexity evaluation and comparison is made among the methods SLP, CORR, MSE and PAPR.

The computational complexity (CC) is measured by the number of real multiplication (RM) and real additions (RA).

The CC comparison may be achieved by evaluating only the amount of operations implied in the choice of the modified symbol, so the difference among the methods can be reduced

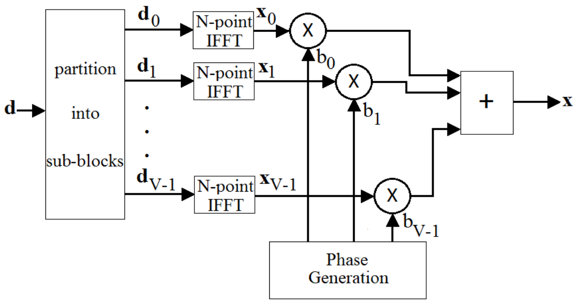

Fig. 3. PTS-based modified symbol generation block diagram.

just to the optimization metrics. Any calculus that doesn't impact the metrics is therefore left aside, such as the IFFT and other operations.

A. SLP

The metric defined in (7) is based on calculating $|x[n]|^2$ that takes 2 RMs and 1 RA. A total of $2N$ RMs and N RAs is used. In order to evaluate the summation additional M RAs are required. Disregarding the comparisons and memory accesses to sort $|x[n]|^2$ values, the complexity measure is given by

$$CC_{SLP} = 2N \text{ RMs} + (N + M) \text{ RAs}. \tag{8}$$

B. PAPR

Computing $|x[n]|^2$, $n = 0, 1, \ldots, N-1$ requires a total of $2N$ RMs and N RAs. Once knowing $|x[n]|^2$, it takes additional N RAs to find P_{in} in (2). Disregarding the comparisons to find $max\{|x[n]|^2\}$, the total complexity for this method hence is $2N$ RMs + $2N$ RAs + 1 RD, where RD states for real division, which can be ignored for simplicity.

Assuming that P_{in} is constant in each symbol $\mathbf{x}$, since the HPA operation point is fixed, the computational complexity is obtained as

$$CC_{PAPR} = 2N \text{ RMs} + N \text{ RAs}. \tag{9}$$

C. CORR

Using cross correlation and considering the polynomial model for the HPA defined in (6), the metric can be expressed as

$$R_{xy}^{(0)} = \sum_{n=0}^{N-1} x[n]\, y^*[n] = \alpha_1 \sum_{n=0}^{N-1} |x[n]|^2 + \alpha_3 \sum_{n=0}^{N-1} |x[n]|^4. \tag{10}$$

Computing $\alpha_1 \sum_{n=0}^{N-1} |x[n]|^2$ takes, as mentioned, $2N$ RMs and $2N-1$ RAs, since $\alpha_1 = 1$. Knowing $|x[n]|^2$, it takes additional $N+1$ RMs and $N-1$ RAs to calculate $\alpha_3 \sum_{n=0}^{N-1} |x[n]|^4$. Therefore the CC for the CORR is given by

$$CC_{CORR} = (3N + 1) \text{ RMs} + (3N - 1) \text{ RAs}. \tag{11}$$

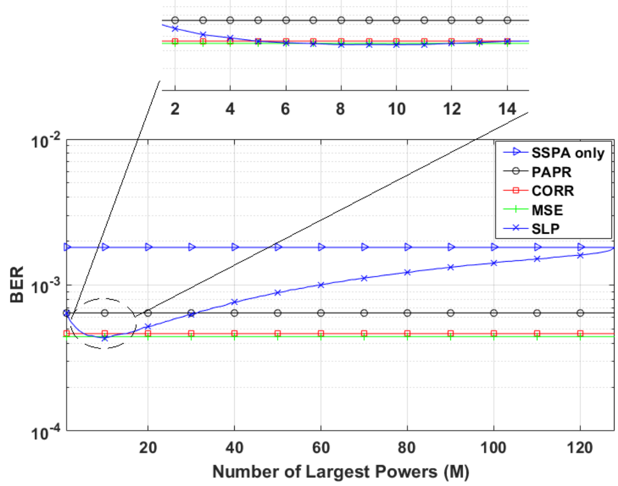

Fig. 4. Proposed method BER performance for different M values, $V = 4$, $N = 128$, $SNR = 25$ dB and $16-QAM$ modulation.

D. MSE

Adopting MSE as the optimization metric and substituting the polynomial model(6) the following expression is obtained

$$MSE = \alpha_3^2 \sum_{n=0}^{N-1} |x[n]|^6. \tag{12}$$

Using a similar calculation to the above one, the amount of operations necessary to evaluate the MSE is $4N+1$ RMs and $2N-1$ RAs, since α_3^2 can be previously calculated. Therefore, for the MSE metric the CC is expressed as

$$CC_{MSE} = (4N + 1) \text{ RMs} + (2N - 1) \text{ RAs}. \tag{13}$$

V. Numerical Results

Simulation results of performance evaluation were obtained with the proposed method SLP, PAPR, CORR, and MSE. For this evaluation the modified OFDM symbols were generated by a PTS approach as illustrated in Figure 3.

A set of 10^5 random OFDM symbols have been simulated to obtain each performance estimate, with $N = 128$ and $N = 512$ subcarriers with $16-QAM$ and $4-PSK$ modulation constellation.

The number of data vector partitions (V) used within the PTS method was set to 4 and 16. For $V = 4$ all eight possible phase vectors $\mathbf{b}$ with elements obtained from $\{-1, 1\}$ were considered. With $V = 16$, 64 different phase vectors $\mathbf{b}$ were randomly chosen to each OFDM symbol to be transmitted over an additive white Gaussian noise (AWGN).

The performance measures of a system without any improvement technique were also evaluated and included in the following figures as references.

The system was initially simulated using the Rapp Model defined in (3). For the CORR and MSE methods the polynomial model (6) was used for the metrics as defined in (10), (12), using α_1 and α_3 as described in section II. This way the difference between the HPA and its mathematical model was

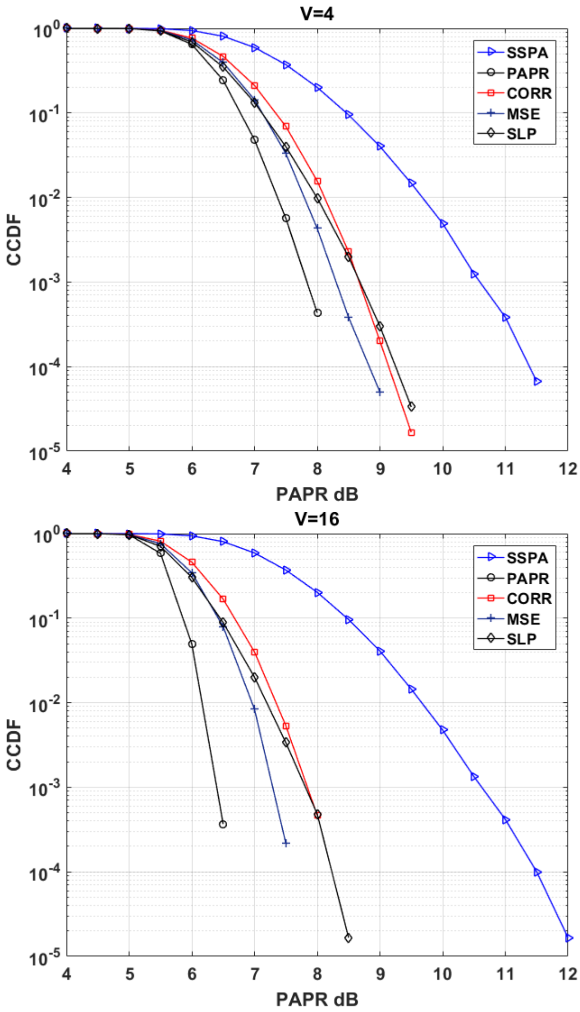

Fig. 5. PAPR CCDF comparison for $V = 4$ and $V = 16$, $N = 128$, $M = 6$ and $16 - QAM$ modulation with Rapp Model.

taken into account. The amplifier operation point is set by the input back-off (IBO) or the output back-off (OBO), defined as

$$IBO = 10log_{10}\frac{A_{sat}^2}{P_{in}} \quad OBO = 10log_{10}\frac{A_0^2}{P_{out}}, \qquad (14)$$

where P_{out} is the average power per OFDM symbol at the HPA output, and A_{sat} is the input amplifier saturation point. The input back-off was set at 3 dB, with $A_{sat} = 1$.

The effect of the M parameter on the performance of the SLP method is illustrated in Figure 4, which shows the BER performance for different values of M in comparison to the other methods. For this case 25 dB SNR, $V = 4$, $16 - QAM$ modulation and $N = 128$ subcarriers were assumed. It is worth to note that for $M = 1$ the performance is identical to the PAPR method while $M = N$ leads to a BER equivalent to a system without any improvement.

The proposed method presents a stable BER performance as M varies from 4 to 12, while maintaining equivalent performance to the CORR and MSE schemes.

Figures 5 and 6 depict, respectively, the PAPR complementary cumulative distribution function (CCDF) and BER performance for the different methods considering $M = 6$,

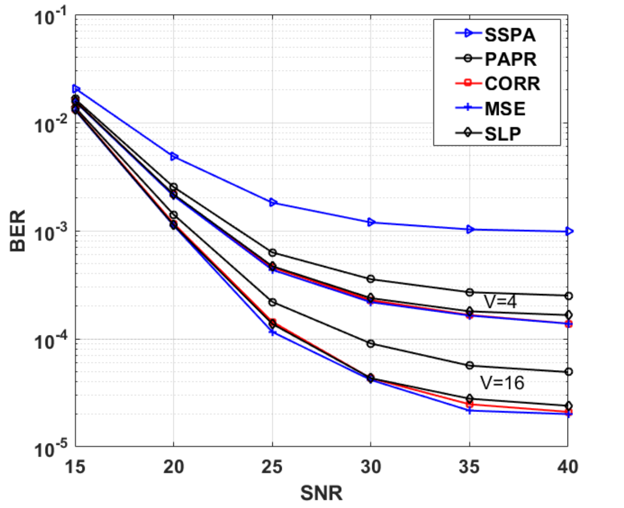

Fig. 6. BER performance comparison for $V = 4$ and $V = 16$, $N = 128$, $M = 6$ and $16 - QAM$ modulation with Rapp Model.

$V = 4$ and 16, $16-QAM$ modulation and $N = 128$ subcarriers. The BER performances of SLP, CORR, and MSE methods are shown to be equivalent and superior to the BER performance of the PAPR method. This last one as expected presents the best CCDF PAPR performance. The proposed method performs as well as the CORR scheme for lower and medium values of PAPR, and approaches the performance of the MSE technique for higher values.

Figures 7 and 8 depict the results obtained by changing the subcarriers number to $N = 512$ and using $M = 12$. The same relative behaviors are observed by comparison of the PAPR and BER performances of the methods here considered.

Figure 9 shows the results obtained using $IBO = 3$ dB, $M = 6$, $V = 4$, $4 - PSK$ modulation and $N = 128$ subcarriers when using the Saleh Model defined in (4), assuming $\alpha_a = 2$, $\beta_a = 1$, $\alpha_\varphi = 1$, and $\beta_\varphi = 1$. For the CORR and MSE methods, the polynomial model parameters were the same previously used. The resulting PAPR CCDF and BER performance of the evaluated techniques maintain the same relative behavior observed when using an SSPA model.

VI. Conclusions

This paper presented a new technique to mitigate the nonlinear effects of power amplifiers on OFDM systems, improving the BER performance without degrading the CCDF PAPR.

In contrast to CORR and MSE methods recently proposed with the same goal, this technique is independent of any HPA mathematical model, being therefore intrinsically insensitive to model mismatch.

The proposed method selects a modified OFDM symbol to amplify based on the summation of a number (M) of its highest power samples. Simulation experiments have shown that the BER performance provided by the technique is robust regarding the choice of M parameter.

Besides, a comparison of BER and PAPR performances showed that the proposed methods essentially performs as well as the CORR and MSE techniques, despite being computationally much simpler.

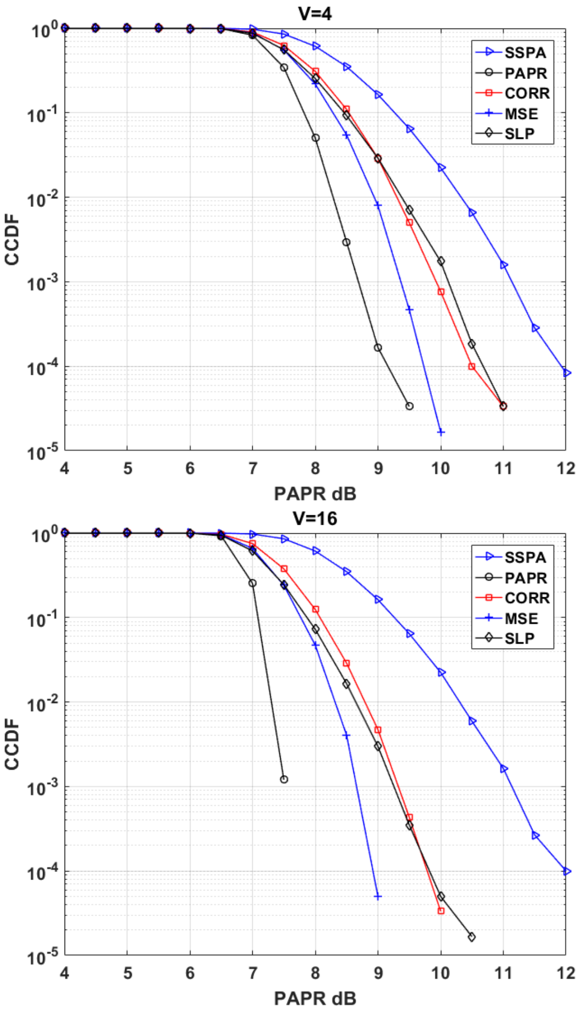

Fig. 7. PAPR CCDF comparison for $V = 4$ and $V = 16$, $N = 512$, $M = 12$ and $16 - QAM$ modulation with Rapp Model.

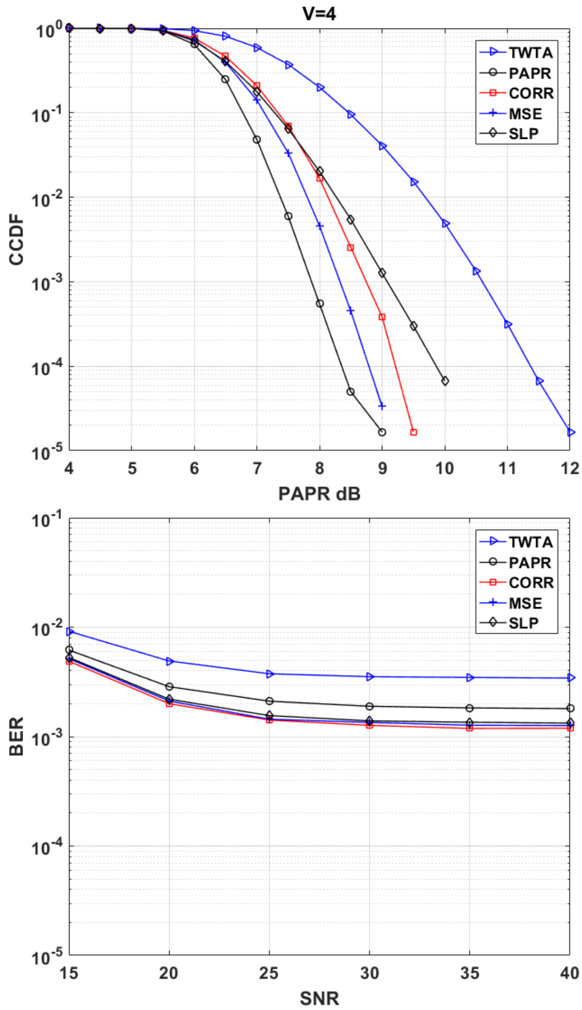

Fig. 9. PAPR CCDF and BER performance comparison for $V = 4$, $N = 128$, $M = 6$ and $4 - PSK$ modulation with TWTA.

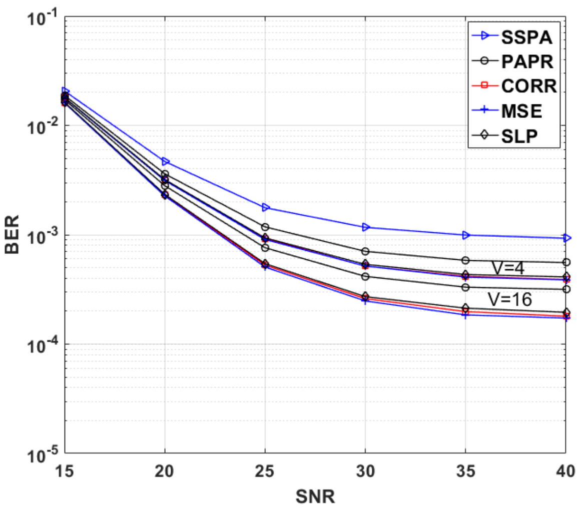

Fig. 8. BER performance comparison for $V = 4$ and $V = 16$, $N = 512$, $M = 12$ and $16 - QAM$ modulation with Rapp Model.

A proposal for future work is the search for an analytical method to determine the optimal M value to optimize the system performance. Another possibility is a study regarding the relation between M and N.

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