

Effect of Propagation Model Calibration on DCM Positioning Accuracy in a Dense Urban Area

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Abstract—In this paper the authors evaluate the effect of propagation model calibration on the positioning accuracy obtained by a *database correlation method* (DCM). The calibration procedure defines a single calibration factor. This factor is added to the predicted path loss to minimize the sum of the squared differences between predicted and measured received signal strength values. The selected DCM algorithm (RF-FING+RTD-PRED) employs a correlation database built using the Okumura-Hata empirical propagation model. Additional diffraction losses are calculated using Epstein-Peterson's method applied upon digital elevation models with building heights with a planar resolution of 10 meters. The experimental evaluation was carried out in a GSM radio access network in a dense urban area.

Keywords—Okumura-Hata, Calibration, Database Correlation Method, Mobile Station, Positioning Accuracy.

I. INTRODUCTION

Mobile station (MS) positioning in wireless networks has been the focus of increasing attention in recent years, mainly due to: **i**) regulatory issues, specifically regarding positioning of emergency calls originated from MS's in cellular networks [1][2]; and **ii**) the possibility of offering added value services based on location. Even though there are several MS's with built-in *Global Positioning System* (GPS) receivers, the use of network-based methods - i.e., methods which do not require any modification or specific features in the MS - is still required because: **i**) network-based methods can locate any MS within the network coverage area; **ii**) network-based methods can act as a *fall-back*, when more accurate methods are unavailable due to system limitations - e.g, in indoor environments or in dense urban areas, where most of the time GPS signal reception is not possible. Network-base methods shall use only information already available at the serving *base transmission station* (BTS) or sent by the MS through *network measurement reports* (NMR's).

The **RF-FING+RTD-PRED** is a network-based *database correlation method* (DCM), originally proposed by the authors in [3]. As a DCM method, it yields a MS position estimate by comparing a *radio-frequency* (RF) fingerprint measured by the MS, with RF fingerprints previously stored in a *correlation database* (CDB). A RF fingerprint is a set of RF parameters measured by the MS and reported through NMR's. Parameters measured by the serving BTS in the reverse link might also be included in the RF fingerprint. The RF fingerprints stored in

the CDB are geo-referenced, i.e., each one is associated with a pair of geographic coordinates.

DCM solutions in the literature usually employ CDB's built from field tests [4][5], as they provide higher precision. However, to keep those CDB's up-to-date, drive tests must be carried out after any change in *radio access network* (RAN) elements, turning this solution into an impractical one. Therefore, the **RF-FING+RTD-PRED** method uses CDBs built from propagation modeling, which allows quick and inexpensive CDB upgrade and prevents accuracy degradation due to the use of out-of-date network parameters in the correlation process.

There is a wide variety of mathematical models for radio propagation prediction, but they can be roughly grouped in two main classes: deterministic and empirical. Deterministic propagation models are based on ray-tracing techniques. They describe the electromagnetic wave propagation using rays launched from the transmitting antenna. These rays are reflected and diffracted at walls and other obstacles. Ray tracing models require a very accurate knowledge of the environment and have a high computational load, resulting in a long computation time for coverage prediction [6]. Empirical propagation models are based on extensive field measurements that, after statistical analysis, produce parametric path loss equations. Those parameters or coefficients can be adjusted, within some predetermined bounds, to better represent a particular propagation environment [7]. Empirical models are less computationally intensive and, even though they are usually less accurate than deterministic propagation models, they still provide an accuracy compatible with the average accuracy of most RF fingerprinting methods for outdoor positioning [3][8]. Therefore, empirical models become the most suitable option to build a CDB with RF propagation modeling in outdoor environments.

This paper evaluates how the calibration of the Okumura-Hata empirical model affects the **RF-FING+RTD-PRED** positioning accuracy in a GSM RAN in a dense urban environment. The calibration process is based on a procedure proposed in [9], and provides calibration factors for each cell.

The remainder of this paper is organized as follows: Section II details the building process of a CDB based on propagation modeling; Section III presents the calibration procedure of such a CDB; Section IV briefly presents the **RF-FING+RTD-PRED** core features; Section V presents the results of the tests in a GSM RAN; and Section VI brings a brief conclusion.

II. BUILDING THE CDB FROM PROPAGATION MODELING

The main advantage of using a CDB built from propagation modeling is to allow for easy, fast and inexpensive CDB

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updating. Whenever there are changes in the RAN elements, it is necessary only to re-run the propagation models with the new RAN parameters to obtain an updated CDB.

The Okumura-Hata model [10] provides an empirical formula for propagation loss derived from extensive field measurements in urban areas. This model is applicable to system designs for *Ultra High Frequency* (UHF) and *Very High Frequency* (VHF), under the following conditions: frequency range 100 – 1500 MHz, distance 1 – 20 km, base station antenna height 30 – 200 m, and MS antenna height 1 – 10 m. The Okumura-Hata model is widely used for RF planning in cellular networks.

The basic Okumura-Hata propagation loss formula does not explicitly take into account diffraction over terrain and buildings. In order to do so, the topography of the service area is represented by a matrix \mathbf{H} , also referred to as a *digital elevation model* (DEM) or digital topographical database [11]. Each matrix element $h_{i,j}$ stores the terrain height averaged over a $r_H \times r_H$ m² surface, and is referred to as a *pixel*. Parameter r_H is the \mathbf{H} matrix planar resolution. The \mathbf{H} matrix might also contain, added to the terrain height, the building heights. If the region covers a total surface of $l \times w$ m², then \mathbf{H} has $\left\lceil \frac{l}{r_H} \right\rceil \times \left\lceil \frac{w}{r_H} \right\rceil$ elements (pixels).

To represent the service area surface as a plane, divided into evenly spaced pixels, it is necessary to apply a geographic coordinate system which uses a rectangular cartographic projection. The *Universal Transverse Mercator* (UTM) is an example of such a system. Assume that the UTM system is being used and that $h_{1,1}$, the first element of \mathbf{H} , is placed at the northwest corner of the service area. If the UTM coordinates $[x_1 \ y_1]^T$ of $h_{1,1}$ are known, then the coordinates of $h_{i,j}$ are given by:

$$\begin{bmatrix} x_j \\ y_i \end{bmatrix} = \begin{bmatrix} x_1 + r_H (j - 1) \\ y_1 - r_H (i - 1) \end{bmatrix} \quad (1)$$

where $i = 1, 2, \dots, \left\lceil \frac{w}{r_H} \right\rceil$ and $j = 1, 2, \dots, \left\lceil \frac{l}{r_H} \right\rceil$.

The terrain profile - including the building heights, if available - between the k -th cell and pixel (i, j) is read from the DEM. After obtaining the terrain profile, the diffraction losses must be calculated using a specific model, like Epstein-Peterson, Bullington or Deygout [7]. The Okumura-Hata average propagation loss in dB between the k -th cell in the service area and pixel (i, j) , plus the additional diffraction loss $u_{i,j,k}$, is given by:

$$L_{i,j,k} = c_1 + c_2 \log_{10}(d_{i,j,k}) + c_3 \log_{10}(z_k) + c_4 u_{i,j,k} + c_5 \log_{10}(z_k) \log_{10}(d_{i,j,k}) \quad (2)$$

where z_k is the k -th cell antenna effective height in meters and $d_{i,j,k}$ is the distance in meters between the k -th cell antenna and pixel (i, j) . The model coefficients (c_1, c_2, c_3, c_4 and c_5) depend on the area morphology and transmission frequency. For a network using the 869-881 MHz band, the model is applied at the central frequency of 875 MHz. The coefficients values are $c_1 = -12.1$, $c_2 = -44.9$, $c_3 = -5.83$, $c_4 = 0.5$ and $c_5 = 6.55$. All of these values are the standard Okumura-Hata values for urban environments, except c_4 , which was empirically defined by the authors [12].

The vertical ϕ and horizontal θ angles between the k -th cell antenna and pixel (i, j) can be calculated using trigonometry,

as the geographic coordinates in space - x , y and z - of both the antenna and the pixel are known. The k -th cell control channel transmission power is assumed to be known, as well as the connector and cable losses between the transmitter and the antenna. Therefore, with the antenna vertical and horizontal radiation patterns, it is possible to estimate the k -th cell control channel *effective isotropic radiated power* (EIRP) in the direction of pixel (i, j) . This direction is defined by angles ϕ and θ . The reference k -th cell control channel RSS in dBm at pixel (i, j) is given by:

$$\text{RSS}_{i,j,k} = \text{EIRP}_{i,j,k} - L_{i,j,k} \quad (3)$$

where $\text{EIRP}_{i,j,k}$ is the k -th cell control channel EIRP in the direction of pixel (i, j) and $L_{i,j,k}$ is the propagation loss between the k -th cell and pixel (i, j) , given by (2).

The RF fingerprint contains not only the RSS values, but also the *round trip delay* (RTD) values. The reference RTD value between the k -th cell and pixel (i, j) can be calculated by:

$$\text{RTD}_{i,j,k} = \left\lfloor \frac{2d_{i,j,k}}{cT_s} \right\rfloor \quad (4)$$

where c is the speed of light in free space in meters per second, T_s is the symbol period in seconds and $d_{i,j,k}$ is the distance in meters between the k -th cell antenna and pixel (i, j) . Equation (4) assumes *line-of-sight* (LOS) conditions between the transmitting antenna and the pixel, but this is hardly the case, especially in dense urban areas. To enhance the accuracy of the reference RTD value, the additional propagation delay due to *non line-of-sight* (NLOS) conditions can be modeled as a random variable [3].

The reference RF fingerprint at (i, j) is completed after $\text{RSS}_{i,j,k}$ and $\text{RTD}_{i,j,k}$ have been calculated for $k = 1, 2, \dots, N_{i,j}$, where $N_{i,j}$ is the number of cells whose predicted RSS values are above a minimum threshold at pixel (i, j) . Note that $1 \leq N_{i,j} \leq N_c$, where N_c is the total number of cells in the service area. The reference RF fingerprint at pixel (i, j) is represented by:

$$\mathbf{S}_{i,j} = \begin{bmatrix} \text{ID}_{i,j,1} & \text{RSS}_{i,j,1} & \text{RTD}_{i,j,1} \\ \vdots & \vdots & \vdots \\ \text{ID}_{i,j,N_{i,j}} & \text{RSS}_{i,j,N_{i,j}} & \text{RTD}_{i,j,N_{i,j}} \end{bmatrix} \quad (5)$$

where $\text{ID}_{i,j,k}$ is the k -th cell identity at pixel (i, j) . The rows are classified in descending order of RSS.

The CDB is complete after $\mathbf{S}_{i,j}$ has been calculated for all pixels in the service area - i.e., for $i = 1, 2, \dots, \left\lceil \frac{w}{r_H} \right\rceil$ and $j = 1, 2, \dots, \left\lceil \frac{l}{r_H} \right\rceil$.

III. CALIBRATING THE CDB

If the CDB is built using propagation modeling, field tests might be used for fine tuning the empirical propagation models. This procedure is expected to enhance the location precision of a RF fingerprinting method using such CDB. Consider that a calibration route is carried out across the service area. At each measurement point - henceforth referred to as *calibration point* - the RSS of each detected cell is collected. The GPS coordinates of the calibration points are also registered, allowing one to identify the CDB pixels where they are located, as shown in Fig. 1. An average of each cell RSS must be calculated for all calibration points located at the same pixel. After that, the test route is complete and each calibration point is identified by the 3-uple (i_n, j_n, \mathbf{M}_n) , as shown in Fig. 2. The pair (i_n, j_n) identifies the pixel where

the n -th calibration point is located. Note that $1 \leq n \leq N'$, where N' is the number of points in the calibration route. Matrix \mathbf{M}_n is the set of RSS measurements collected at the n -th point, and is given by:

$$\mathbf{M}_n = \begin{bmatrix} \text{ID}_{n,1} & \text{RSS}_{n,1} \\ \vdots & \vdots \\ \text{ID}_{n,N_n} & \text{RSS}_{n,N_n} \end{bmatrix} \quad (6)$$

Note that \mathbf{M}_n is a $N_n \times 2$ matrix, where N_n is the number of cells detected at the n -th point, and that $1 \leq N_n \leq N_c$. The rows are classified in descending order of RSS.

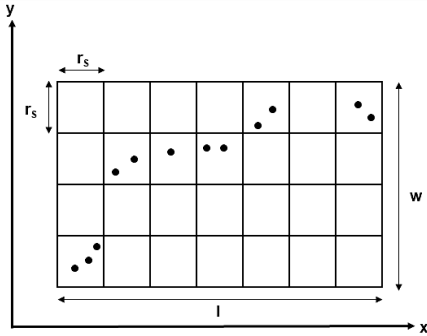


Fig. 1. Calibration route.

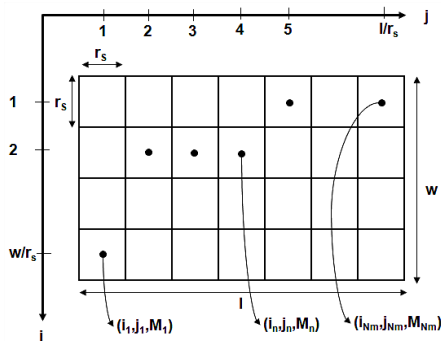


Fig. 2. Calibration route after averaging.

At the n -th point in the calibration route, the difference between the predicted and measured RSS of the k -th cell is given by:

$$b_{n,k} = \mathbf{M}_n(k', 2) - \mathbf{S}_{i,j}(k'', 2) \quad (7)$$

for

$$\mathbf{M}_n(k', 1) = \mathbf{S}_{i,j}(k'', 1) = \text{ID}_k \quad (8)$$

and

$$(i_n, j_n) = (i, j) \quad (9)$$

where ID_k is the k -th cell identity, where $1 \leq k \leq N_c$. Observe that k' is the index of the line in \mathbf{M}_n whose cell identity is equal to the k -th cell identity, i.e., ID_k . An analogous observation can be made regarding k'' and $\mathbf{S}_{i,j}$. Note that $1 \leq k' \leq N_n$ and $1 \leq k'' \leq N_{i,j}$.

Calculating $b_{n,k}$ for all points on the calibration route where the k -th cell has been detected, one obtains a $N_k \times 1$ matrix \mathbf{B}_k . The parameter N_k is the number of calibrating points where the k -th cell was detected. Note that $1 \leq N_k \leq N'$.

The propagation model calibration is done on a per cell basis, defining a calibration factor C_k that shall be added to the path loss, given by (2), between the k -th cell and any pixel (i, j) in the service area. The C_k factor minimizes the sum of the squared differences between the measured and predicted k -th RSS values along the calibration route points where the k -th cell was detected. The C_k factor in dB can be estimated using *least-squares* (LS) [13], as follows:

$$C_k = (\mathbf{V}^T \mathbf{V})^{-1} \mathbf{V}^T \mathbf{B}_k \quad (10)$$

where \mathbf{V} is a $N_k \times 1$ matrix of ones [9].

If a breakpoint is defined as a radial distance from the cell transmitting antenna, two calibration factors might be calculated per cell: one for the near region (before the breakpoint) and another for the far region (beyond the breakpoint). This procedure might help to better adjust the calibration to the different propagation conditions in the immediate vicinity of the antenna (where usually line of sight conditions occur) and in areas farther away from the antenna.

Using just one calibration factor per cell - or two, if a breakpoint is defined - instead of calibrating Equation (2) coefficients, eliminates the necessity of using a high resolution DEM during the calibration process. After changing any of the Equation (2) coefficients, it is necessary to re-run the RF coverage predictions. But, particularly for c_4 , any changes also demand re-calculation of the additional diffraction losses, which requires the test area DEM.

IV. RF-FING+RTD-PRED:RF FINGERPRINT, CORRELATION FUNCTION AND DETERMINISTIC FILTERING

Any DCM method yields a MS position estimate by comparing or correlating a target RF fingerprint with reference RF fingerprints. A target RF fingerprint is the RF fingerprint associated with the MS which is to be localized, i.e. it contains signal parameters measured by the MS or by its anchor cells. The reference RF fingerprints are the RF fingerprints stored in the CDB. Each reference RF fingerprint is associated with a unique set of geographic coordinates. The target RF fingerprint used by **RF-FING+RTD-PRED** is given by the $N_a \times 3$ matrix:

$$\mathbf{F} = \begin{bmatrix} \text{ID}_1 & \text{RSS}_1 & \text{RTD}_1 \\ \vdots & \vdots & \vdots \\ \text{ID}_{N_a} & \text{RSS}_{N_a} & \text{RTD}_{N_a} \end{bmatrix} \quad (11)$$

where N_a is the number of anchor cells within range of the MS, ID_i and RSS_i are the cell identity and measured *received signal strength* (RSS) from the i -th anchor, respectively. RTD_i is the RTD between the MS and the i -th anchor cell. The rows are sorted in descending order of RSS. In GSM RAN's, the maximum number of anchor cells which can be reported through the NMR is 7, comprising the serving cell and up to 6 neighbor cells. Also in GSM RAN's, the RTD value is available only for the best server [14], which usually is the anchor cell with the highest RSS.

It is not feasible to compare the target RF fingerprint to all reference RF fingerprints stored in the CDB, as this would result in a very large computational load and in a correspondingly long time to produce a position fix. The **RF-FING+RTD-PRED** reduces the search space within the CDB through a process called *deterministic filtering* [3]. This process consists of three successive filtering steps, the first of

which restrains the search space to the pixels whose predicted best server is equal to the serving sector listed in \mathbf{F} , i.e., ID_1 .

The **RF-FING+RTD-PRED** returns as the MS position estimate the coordinates of the pixel whose reference RF fingerprint has the smallest Euclidean distance in relation to the target RF fingerprint. If *K-nearest neighbors* (KNN) [15] is used, it returns the arithmetic average of the coordinates of the K pixels with the smallest Euclidean distances in relation to \mathbf{F} . The Euclidean distance between the target RF fingerprint \mathbf{F} and the reference RF fingerprint $\mathbf{S}_{i,j}$ in the N -dimensional RSS space is given by:

$$d_{i,j} = \sqrt{\sum_{k=1}^N \left(\left| \frac{\mathbf{S}_{i,j}(n_k, 2) - \mathbf{F}(k, 2)}{\delta} \right| \right)^2} \quad (12)$$

where n_k is the index of the line in $\mathbf{S}_{i,j}$ whose cell identity is equal to the cell identity in the k -th line in \mathbf{F} , i.e., $\mathbf{S}_{i,j}(n_k, 1) = \mathbf{F}(k, 1)$, with $n_k \in [1, N_{i,j}]$. Parameter $N_{i,j}$ is the number of rows in $\mathbf{S}_{i,j}$. Parameter δ represents the MS inherent RSS measurement inaccuracy in dB units [12]. In (12), any difference between target and reference RSS values which is smaller than δ is considered to be zero. Note that $1 \leq N \leq N_a$ [3].

V. TRIALS IN A GSM RAN

Field tests were performed in a GSM 875 MHz network in the downtown region of Rio de Janeiro city. The region is a 2.2×2.2 km² dense urban area with 114 cells [3]. The DEM used to represent the test area has a planar resolution $r_H = 10$ meters and includes building heights, which increases the accuracy of the propagation modeling used to build the CDB. The test set was composed of a GSM phone and a GPS receiver, both connected to a laptop placed inside a moving vehicle. The MS was in active mode and for each transmitted NMR the current location was calculated by the GPS receiver. The NMR contains the cell ID and RSS of the best server and up to the six strongest neighbor cells. The RTD values, known as *Timing Advance* (TA) in GSM systems, were registered every time a NMR was sent. Two routes were performed: the first, during which 1279 NMR's were collected, was used to calibrate the Okumura-Hata propagation model on a per cell basis; the second, during which 4500 NMR's were collected, was used to evaluate the **RF-FING+RTD-PRED** method positioning accuracy. The second route is depicted in Fig. 3. The GPS location was assumed to be the reference position, so, for each NMR, the positioning error is the Euclidean distance in meters between the GPS position and the location provided by **RF-FING+RTD-PRED**.

The **RF-FING+RTD-PRED** is used with $N = 5$ and $\delta = 6$ dB [14]. A *moving average filter* with length 20 is used to eliminate abrupt variations in location estimates between adjacent position fixes along the test route, so the current MS location estimate is given by the arithmetic mean of the previous 20 estimated positions. KNN has also been used, with $K = 5$ and $K = 2$, before and after calibration, respectively. It has been observed that, the better the propagation model tuning, the lower the optimum K value.

Fig. 4 shows the *cumulative distribution function* (CDF) of **RF-FING+RTD-PRED** positioning error, before and after

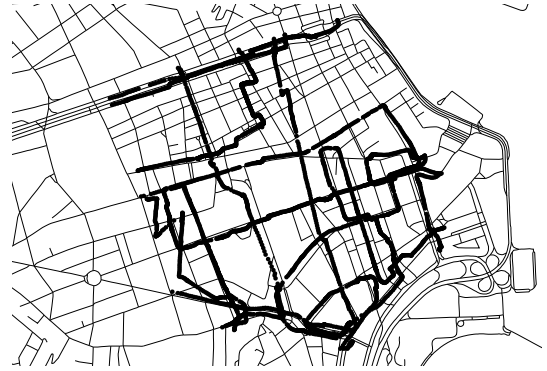


Fig. 3. Test route using GPS as reference. The street vectors are also shown.

the propagation model calibration on a per cell basis. After calibration, the median error suffers a 21% reduction, from 98 to 78 meters. However, the calibration route points are not uniformly distributed throughout the test area. Therefore, among the 114 cells in the test area, only 60 were detected during the calibration route. In this subset, only 33 have more than 50 valid RSS measurements (a RSS measurement was considered valid for calibration purposes if it was above -104 dBm). The low average number of valid RSS measurements per cell prevents the propagation model calibration from improving the overall positioning accuracy even more.

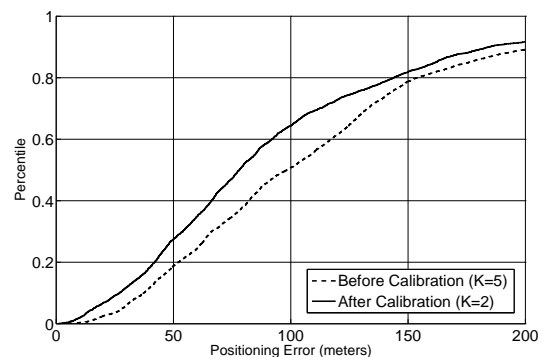


Fig. 4. CDF of DCM Positioning Error in Test Area.

In order to better evaluate the effect of the propagation model calibration on the positioning accuracy, the two cells with the highest number of RSS measurements in the calibration route were selected: cell ID9, with 249 measurements, and cell ID53, with 241 measurements. Then, the points in the test route whose best server (as informed in the NMR's) was either cell ID9 or ID53 were selected, and the CDF of the positioning error of those two sets was built, as depicted in Fig. 5 and Fig. 6. The low-pass moving average filter was not used, as the selected points might not be adjacently distributed along the calibration route.

The improvement in positioning accuracy due to the propagation calibration is now more evident. In both cells best server areas, the median positioning error decreased approximately 42%. In cell ID9 best server area, the median error diminished

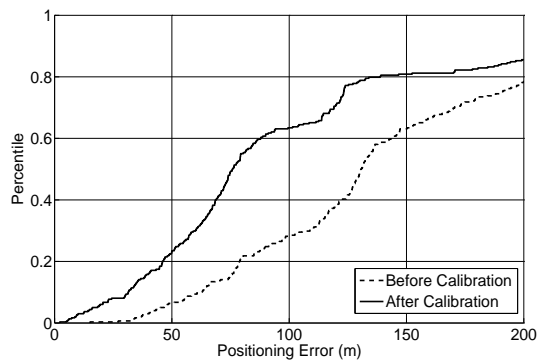


Fig. 5. CDF of DCM Positioning Error in Cell ID9 Best Server Area.

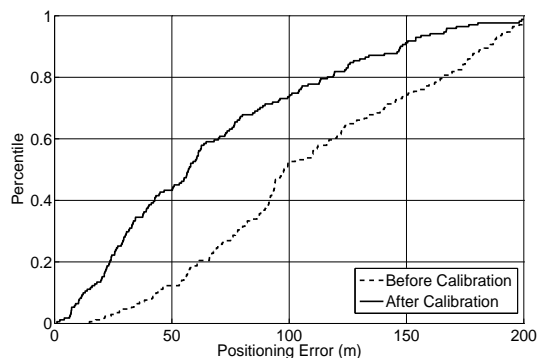


Fig. 6. CDF of DCM Positioning Error in Cell ID53 Best Server Area.

from 130 to 75 meters. In cell ID53 best server area, the median error diminished from 98 to 57 meters.

Due to the relatively small density of calibration points per cell coverage area, defining a breakpoint resulted in no precision improvement in relation to the condition without a breakpoint. Therefore, only one calibration factor was calculated per cell. If a given cell had less than 5 valid RSS measurements in the calibration route, no calibration factor was calculated. So, from the 114 cells in the test area, only for 57 the calibration factors were calculated. Their distribution is depicted in Fig. 7.

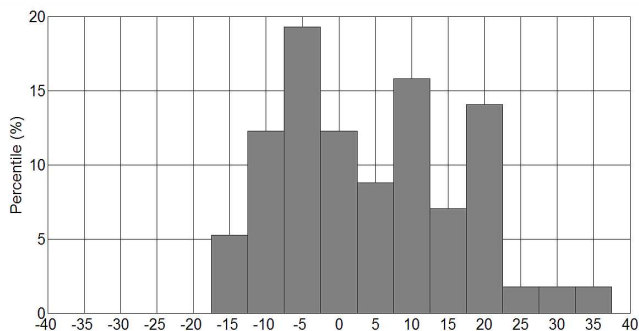


Fig. 7. Distribution of Calibration Factors Values.

VI. CONCLUSION

This paper evaluated the effect propagation model calibration on the positioning accuracy of a DCM algorithm. After the calibration process, a calibration factor was added to the Okumura-Hata equation. The calibration factors were defined for each sector in the test area which was detected in the calibration route. The calibrated empirical propagation model was then used to built a CDB to be used by the DCM algorithm to produce MS position estimates. Experimental evaluation was carried out in a GSM network in a dense urban area, and the results showed that propagation model calibration significantly reduces the DCM positioning error, particularly on those cells with a high number of valid calibration measurements.

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