Radiation Efficiency of Four Layers Slot Resonator with PBG

Humberto Dionísio de Andrade, Anderson Max Cirilo Silva and Humberto César Chaves Fernandes

***Abstract* — This work has as main objective to analyze the rectangular slot resonator with four layers, with photonic materials - PBG, to obtain the complex resonant frequency and radiation efficiency of this structure. The analysis developed in this work was performed using the TTL - Transverse Transmission Line method. Numerical-computational results are presented in graphical form in three dimensions for all analysis performed, and the resonance frequency depending on the length and width of the slot and the radiation efficiency as a function of resonance frequency and height of the layers of substrate structure.**

***Key Worlds* — TTL Method, PBG-Photonic Band Gap, Four layer, Slot antenna, Efficiency.**

I. Introduction

The rectangular slot line resonator with four layers, consist of one rectangular slot line resonator, where there are two layers under and two layers over the patch. This structure is shown in Fig. 1, with width "w" and length "l". By the analysis through the TTL method, the general equations to the electromagnetic fields are obtained. The complex resonant frequency is calculated using double spectral variables, being the same, used in the elaboration of the efficiency and bandwidth's parameters.

By the usage of the system of Cartesian coordinates and the dimensional nomenclatures as presented in Fig. 1 (perspective view), the equations of the electromagnetic fields are obtained, being considered despicable the thickness of the slot line.

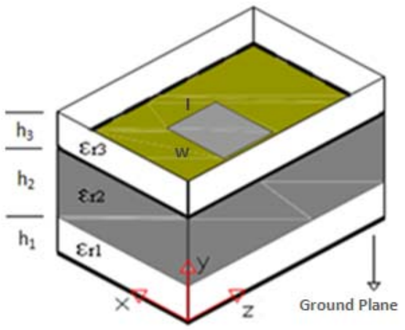

Fig. 1. Perspective view of the four layers slot resonator.

H. D Andrade. A.M.C. Silva and H. C. C. Fernandes. Department of Electrical Engineering - Federal University of Rio Grande do Norte - UFRN -.59078-970-Natal-RN, Brazil. E-mails: humbertodionisio@yahoo.com.br cmaxander@hotmail.com and humbeccf@ct.ufrn.br

II. PBG Structure

One of the problems when working with photonic material is the relative dielectric constant determination as the PBG is a non-homogeneous structure where the incident sign goes at the process of multiple spread.

A solution can be obtained through a numerical process known as homogenization.

The process is based in the theory related to the diffraction of an incident electromagnetic plane wave, imposed by the presence of immerged cylinders of air in a homogeneous material.

For electromagnetic waves propagating in the xy plane these waves have the s polarization (E field parallel to the z axis) and *p* polarization (E field perpendicular to the z axis).

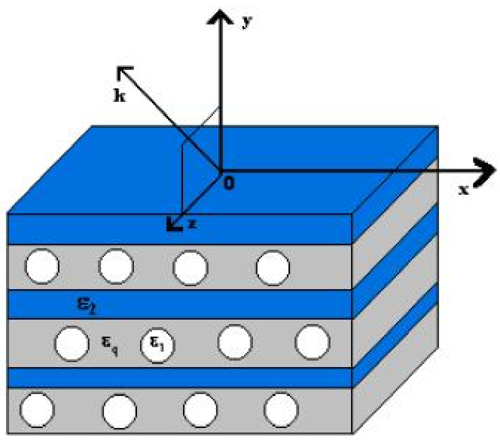

Fig. 2. Homogenized PBG Crystal.

III. Fields Calculation

Due to the limitation in the length, the equations should be used for the analysis in the spectral domain in "x" and "z" directions as function. Therefore the field equations are applied for double Fourier transformed defined as:

$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\cdot e^{j\alpha_n x}\cdot e^{j\beta_k z}\, dx\, dz \quad (1)$$

Where α_n is the spectral variable in the "x" direction and β spectral variable in the "z" direction.

After using the Maxwell's equations in the spectral domain, the general equations of the electric and magnetic fields to the method TTL are obtained:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi} \right] \quad (2)$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi} \right] \quad (3)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k \tilde{E}_{yi} \right] \quad (4)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n \tilde{E}_{yi} \right] \quad (5)$$

Where:
i = 1, 2, 3, 4 represent four dielectrics regions of the structure;

$$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \quad (6)$$

Is the constant of the propagation in y direction; α_n is the spectral variable in "x" direction and β_k the spectral variable in "z" direction.

$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ Is the number of wave of i^th term of Dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ Is the dielectric constant relative of the material with losses;

$\omega = \omega_r + j\omega_i$ is the complex angular frequency;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant.

The equations above are applied to the resonator being calculated, the fields Ey and Hy through the solution of the Helmoltz's wave equations in the spectral domain [2]-[4]:

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2 \right)\tilde{E}_y = 0 \quad (7)$$

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2 \right)\tilde{H}_y = 0 \quad (8)$$

The solutions of Helmoltz's equations for the four regions of the structure are given as examples:

Region 2:

$$\tilde{E}_{y2} = A_{2e}\cdot \operatorname{senh}\gamma_2 y + B_{2e}\cdot\cosh\gamma_2 y \quad (9)$$

$$\tilde{H}_{y2} = A_{2h}\cdot \sinh\gamma_2 y + B_{2h}\cdot\cosh\gamma_2 y \quad (10)$$

Region 4:

$$\tilde{E}_{y3} = A_{3e}\cdot e^{-\gamma_3 y} \quad (11)$$

$$\tilde{H}_{y3} = A_{3h}\cdot e^{-\gamma_3 y} \quad (12)$$

Substituting these solutions in the equations of the fields (2) to (5), as function of the unknown constants A_{21}, A_{22}, B_{21} and B_{22} are obtained, for examples, for the region 2:

$$\tilde{E}_{x2} = \frac{-j}{K_2^2 + \gamma_2^2}\begin{bmatrix} (jw\mu_0\beta_k B_{21} + \alpha_n\gamma_2 A_{22})\cosh(\gamma_2 y)) + \\ (jw\mu_0\beta_k B_{22} + \alpha_n\gamma_2 A_{21})\operatorname{senh}(\gamma_2 y)) \end{bmatrix} \quad (13)$$

$$\tilde{\mathrm{H}}_{x2} = \frac{-j}{k_2^2 + \gamma_2^2}\begin{bmatrix} (j\omega\varepsilon_0\beta_k B_{21} + \alpha_n\gamma_2 B_{22})\cosh(\gamma_2 y) + \\ j\omega\varepsilon_0\beta_k A_{22} + \alpha_n\gamma_2 B_{21})\sinh(\gamma_2 y) \end{bmatrix} \quad (14)$$

For the determination of the unknown constants, it is applied the boundary conditions to the 1, 2 and 3 regions:

Regions 1 e 2: y = h1

$$\tilde{E}_{x1} = \tilde{E}_{x2} \quad (15)$$

$$\tilde{E}_{z1} = \tilde{E}_{z2} \quad (16)$$

$$\tilde{H}_{x1} = \tilde{H}_{x2} \quad (17)$$

$$\tilde{H}_{z1} = \tilde{H}_{z2} \quad (18)$$

Regions 2 e 3: y =d ; (g=h1+h2)

$$\tilde{E}_{x2} = \tilde{E}_{x3} = \tilde{E}_{xg} \quad (19)$$

$$\tilde{E}_{z2} = \tilde{E}_{z3} = \tilde{E}_{zg} \quad (20)$$

After several calculations it is obtained, for region two:

$$A_{21} = \frac{\varepsilon_1 \cosh(\gamma_2 y)}{\varepsilon_2 \gamma_1 \operatorname{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \gamma_2 \frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\operatorname{senh}(\gamma_2 g_2)} * \left[j(\alpha_n \tilde{E}_{xg} + \beta_k \tilde{E}_{zg} \right] \tag{21}$$

$$A_{22} = \frac{\gamma_1 \sinh(\gamma_2 y)}{\gamma_2 \gamma_1 \sinh(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \gamma_2 \frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\sinh(\gamma_2 g_2)} * \left[j(\alpha_n \tilde{E}_{xg} + \beta_k \tilde{E}_{zg}) \right] \tag{22}$$

$$B_{21} = -\frac{\sinh(\gamma_1 g_1)}{\omega\mu_0 \sinh(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\sinh(\gamma_2 g_2)} \left[-\beta_1 \tilde{E}_{xg} + \alpha_n \tilde{E}_{zg} \right] \tag{23}$$

$$B_{22} = -\frac{\gamma_1}{\gamma_2} \frac{\cosh(\gamma_1 g_1)}{\omega\mu_0 \sinh(\gamma_1 g_1)\cosh(\gamma_2 g_2) + \frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\sinh(\gamma_2 g_2)} \left[-\beta_1 \tilde{E}_{xg} + \alpha_n \tilde{E}_{zg} \right] \tag{24}$$

The general equations of the electromagnetic fields are obtained as function of the tangential electric fields on the antenna resonator.

IV. Admittance Matrix Calculation

The following equations (25) and (26) relate the current densities on the sheets ($\tilde{J}_{xt}$ and $\tilde{J}_{zt}$) and the magnetic fields on the interface y = h1+h2:

$$\tilde{H}_{x2} - \tilde{H}_{x3} = \tilde{J}_{zt} \tag{25}$$

$$\tilde{H}_{z2} - \tilde{H}_{z3} = -\tilde{J}_{xt} \tag{26}$$

It has being done the substitutions of the magnetic fields equations, so after some calculations are obtained,

$$Y_{xx}\tilde{E}_{xg} + Y_{xz}\tilde{E}_{zg} = \tilde{J}_{zg} \tag{27}$$

$$Y_{zx}\tilde{E}_{xg} + Y_{zz}\tilde{E}_{zg} = \tilde{J}_{xg} \tag{28}$$

These equations are represented in the matrix form:

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix} \begin{bmatrix} \tilde{E}_{xg} \\ \tilde{E}_{zg} \end{bmatrix} = \begin{bmatrix} \tilde{J}_{zg} \\ \tilde{J}_{xg} \end{bmatrix} \tag{29}$$

The "Y" admittance terms are the Green's dyadic functions to the antenna and they are represented by:

$$Y_{xx} = -\frac{j}{\varpi\mu 0(\gamma 2^2 + k2^2)}\left[-\beta k^2 \gamma 2.E + k2^2 \alpha n^2.F\right] + \frac{j}{\varpi\mu 0\gamma 3}\left[\alpha n^2 k3^2 E - \beta k^2 \gamma 3^2.D\right] \tag{30}$$

$$Y_{xz} = \frac{-j\alpha n\beta k}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[A + k2^2.(B)\right] - \frac{\alpha n\beta k}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[k3^2.C + \gamma 3^2.D\right] \tag{31}$$

$$Y_{zx} = \frac{-j\alpha n\beta k}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[A + k2^2.(B)\right] - \frac{\alpha n\beta k}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[k3^2.C + \gamma 3^2.D\right] \tag{32}$$

$$Y_{zz} = \frac{j}{\varpi\mu 0\left(\gamma 2^2 + k2^2\right)}\left[\alpha n^2.A - \beta k^2.k2^2.B\right] - \frac{j}{\varpi\mu 0\gamma 3\left(k3^2 + \gamma 3^2\right)}\left[\alpha n^2 \gamma 3^2.C - \beta k^2 \gamma 3^2.D\right] \tag{33}$$

Where,

$$A = \frac{\gamma 1 \cdot \gamma 2}{\gamma 2 tgh(\gamma 1 \cdot h1) + \gamma 1 tgh(\gamma 2 h2)} + \frac{\gamma 2^2}{\frac{\gamma 2}{tgh(\gamma 2 \cdot h2)} + \frac{\gamma 1}{tgh(\gamma 1 \cdot h1)}} \tag{34}$$

$$B = \left(\frac{\varepsilon 1}{\gamma 1 \cdot \varepsilon 2 tgh(\gamma 1 \cdot h1) + \gamma 2 \cdot \varepsilon 1 tgh(\gamma 2 \cdot h2)} + \frac{\gamma 1 \cdot \varepsilon 2}{\frac{\gamma 1.\gamma 2.\varepsilon 2}{tgh(\gamma 2.h2)} + \frac{\gamma 2^2.\varepsilon 1}{tgh(\gamma 1.h1)}} \right) \tag{35}$$

$$C = \left(\frac{\frac{\gamma 3 \cdot \varepsilon 4}{\gamma 4 \varepsilon 3} + tgh(\gamma 3 h3)}{1 + \frac{\gamma 3 \varepsilon 4.tgh(\gamma 3 h3)}{\gamma 4 \varepsilon 3}} \right) \tag{36}$$

$$D = \left(\frac{\frac{\gamma 4}{\gamma 3} + tgh(\gamma 3 h3)}{1 + \frac{\gamma 4.tgh(\gamma 3 h3)}{\gamma 3}} \right) \tag{37}$$

$$E = \left(\frac{\gamma 1 + \gamma 2.tgh(\gamma 1 h1).tgh(\gamma 2 h2)}{\gamma 2 tgh(\gamma 1 h1) + \gamma 2.tgh(\gamma 2 h2)} \right) \quad (38)$$

The tangential electric fields in the interface have being expanded using base functions [3], [5]:

$$\tilde{E}_{xg} = \sum_{i=1}^{n} a_{xi} \cdot \tilde{f}_{xi}(\alpha_n, \beta_k) \quad (39)$$

$$\tilde{E}_{zg} = \sum_{j=1}^{m} a_{zj} \cdot \tilde{f}_{zj}(\alpha_n, \beta_k) \quad (40)$$

Where a_{xi} and a_{zj} are constant unknown and the terms n and m are numbers integer and positive that can be done equal to 1, as seen in equations (41) and (42) following:

$$\tilde{E}_{xg} = a_x \cdot \tilde{f}_x(\alpha_n, \beta_k) \quad (41)$$

$$\tilde{E}_{zg} = a_z \cdot \tilde{f}_z(\alpha_n, \beta_k) \quad (42)$$

The Fourier transformed of the base functions chosen are [6]:

$$\tilde{f}_x(\alpha_n) = \pi \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (43)$$

$$\tilde{f}_x(\beta_k) = \frac{2\pi l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \quad (44)$$

$$\tilde{f}_x(\alpha_n, \beta_k) = \frac{2\pi^2 l \cdot \cos\left(\frac{\beta_k l}{2}\right)}{\pi^2 - (\beta_k l)^2} \cdot J_o\left(\alpha_n \frac{w}{2}\right) \quad (45)$$

Where J_0 is the Bessel's function of first specie and zero order. The Garlekin method is applied to (29), to eliminate current densities and the new equation in matrix form is obtained [5], [7].

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (46)$$

Where,

$$K_{xx} = \sum_{-\infty}^{\infty} \tilde{f}_x \cdot Y_{xx} \cdot \tilde{f}_x^{*} \quad (47)$$

The solution to the characteristic equation of the determinant (14) it supplies the resonant frequency.

V. Radiation Efficiency

For rectangular slot resonator, the theoretical development to determine the radiation efficiency takes into account the losses of reflection, conduction and dielectric. The radiation efficiency is defined as the ratio between the power delivered, to the radiation resistance and input power delivered to R_L and Rr [8]:

$$\eta(\%) = \frac{R_t}{R_r} 100 \quad (48)$$

The total resistance, R_t (Ω) is calculated from the following equation:

$$\frac{1}{R_t} = \frac{1}{R_r^{'}} + \frac{1}{R_c} + \frac{1}{R_d} \quad (49)$$

VI. Numerical Results

The Figure 3 shows the 3D result of the resonant frequency as function of the slot length and width. The frequency increases when the width and the length decrease.

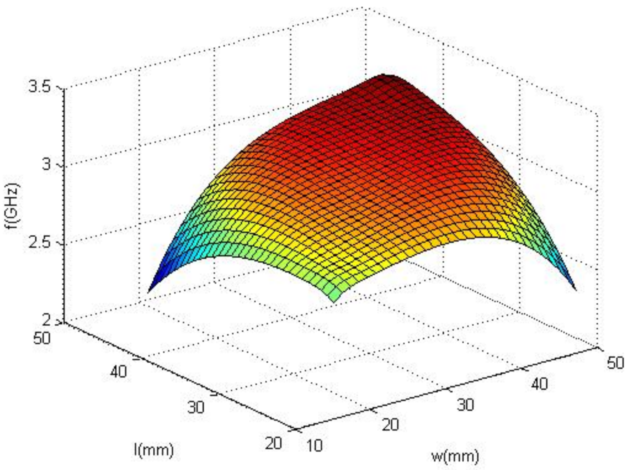

Fig.3. Resonant Frequency, (GHz), as function of the width (mm) and length (mm) patch.

Figure 4 shows a graphic comparing the curve of the radiation efficiency as a function of resonance frequency, for different thicknesses of substrate. For the results in Fig. 4 was considered permittivity as follows: εr1 = 12.0, εr2 = 8.702 (p polarization), εr3 = 12.0, εr4 = 1. The air is considered as the fourth layer. For the p polarization, was considered a height of substrate, h_1 = 3.302 mm, h_3=1.27 mm, h_2 = 2.54 mm, h_2 = 3.302 mm and h_2 = 5.842 mm, respectively. The width 35.56 mm and the length is 71.12 mm. Therefore there is an improvement of radiation efficiency of the structure under study, and an increase in resonant frequency when the efficiency increases.

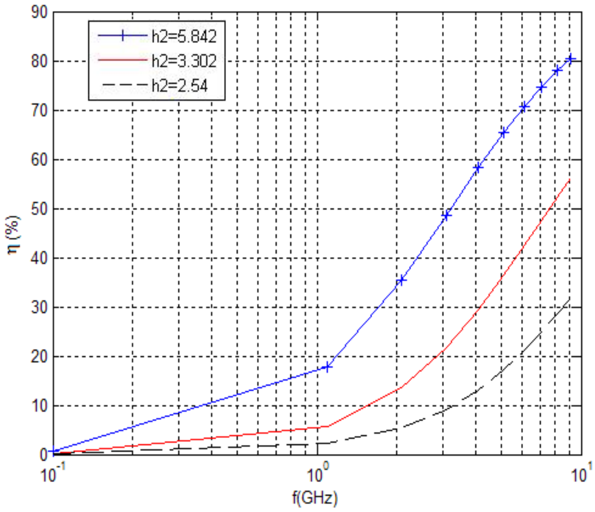

Fig.4. Radiation efficiency (η) as function of the frequency for p polarization.

For the results in Fig. 5 was considered permittivities for the following: εr1 = 12, εr2 = 10.233 (s polarization), εr3 = 12, εr4 = 1 (air). The thicknesses employed for this analysis are: h_1 = 3.302 mm, h_3=1.27 mm, h_2 = 2.540 mm, h_2 = 3.302 mm and h_2 = 5.842 mm, respectively. The width is 35.56 mm and the length is 71.12 mm.

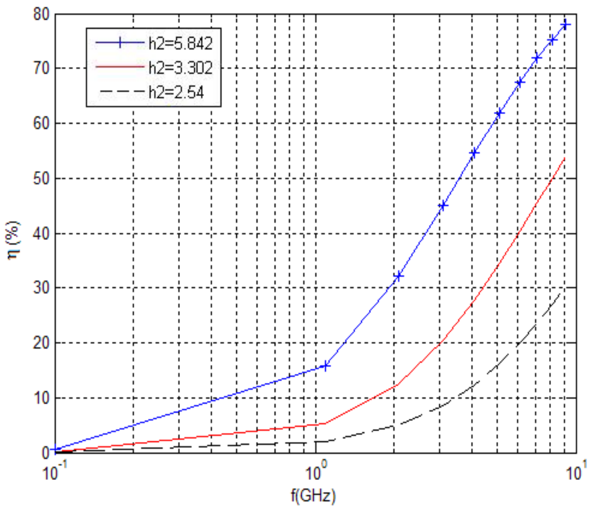

Fig.5. Radiation efficiency (η) in function of frequency for s polarization.

VII. Conclusion

The Transverse Transmission Line – TTL method was used, in the analysis, to obtain the numeric results of the four layers slot line resonator. According to the concise and effective procedures the calculus of the complex resonant frequency was obtained with accuracy. The rectangular slot resonator with four dielectric layers has its results from a complex resonant frequency. The development of the calculation of radiation efficiency as presented here is consistent with current literature.

Acknowledgment

This work was partially supported by CNPQ and INCT-CST.

References

[1] A. K. Agrawal and B. Bhat, *Resonant Characteristics and End Effects of a Slot Resonator in Unilateral Fin Line*, Proc. IEEE, Vol. 72, pp. 1416-1418, Oct. 1984.

[2] H. C. C. Fernandes, S. A. P Silva and José P. Silva, "*Coupling Analisys at the Coupler and Unilateral Edge-Coupled Fin Line*," Int. Conference on Millimeter and Submillimeter Waves and Applications II, SPIE's 1998 International Symposium on Optical Science, Engineering and Instrumentation, San Diego, CA, USA. Conf. Proc. pp. 53-54, Jul. 1998.

[3] H. C. C. Fernandes and Sidney A. P. Silva "*Asymmetric and Unilateral Thick Edge-Coupled Fin Line and Coupling Analysis"*, in PIERS 1999 - The Progress in Electromagnetics Research Symposium, Taipei, Taiwan, Republic of China. Conf. Proc. P. 90, Mar. 1999.

[4] H. C. C. Fernandes, S. A. P. Silva and Costa, O.S. Delfino, "*3D Complex Propagation of Coupled Unilateral and Antipodal Arbitrary Finlines"*, Brazilian CBMAG'96-Congress of Electromagnetism, Ouro Preto-MG, pp. 159-162 , Nov. 1996.

[5] B. Baht and S. K. Koul, "*Analysis, design and applications of finlines"*, Artech House, 1987.

[6] S. A. P. Silva and H.C.C. Fernandes, "*Functions of basis in analysis of the coupled unilateral fin line coupler"*, IV SPET-Symposium of Research and Extension in Technology, Natal-RN, Annals pp. 79-81, Nov. 1998.

[7] H. C. C. Fernandes e M.Bonfim Aquino Neto, "Four layers slot resonator", Proc. of the WSEAS2004, CSCC, CD-ROM, Atenas, Grécia, 5p., Jul. 2004.

[8] C. A. Balanis, "Antena Theory: analysis and design" Jonh Wiley & Sons, 1997.

[9] H. C. C. Fernandes, H. D. Andrade, M.B.L. Aquino and D.B. Brito, "Rectangular Slot Resonator with Four Dielectrics Layers". PIERS2007- Progress In Electromagnetics Research Symposium, , Beijin, China.. V. 1. p. 2121-2127, 2007.

[10] H. C. C. Fernandes, H.D. Andrade, "Photonic Rectangular Slot Resonator with Four Dielectrics Layers". ICAM – Int. Conference on Advances Materials, Rio de Janeiro: SBPMAT, 2009. V. I. p. 301-301, 2009.