Effect of Channel Estimation Errors on Adaptive Modulation Systems Subject to Rayleigh Fading

Rafael Fernandes Lopes, Carlos Danilo Miranda Regis, Wamberto José Lira de Queiroz,
Waslon Terllizzie Araújo Lopes and Marcelo Sampaio de Alencar

***Resumo*— A adoção do esquema de modulação adaptativa torna possível a realização de transmissões robustas e espectralmente eficientes. No entanto, o desempenho dos sistemas de modulação adaptativa é afetado pela precisão das informações do estado de canal, o que pode resultar em decisões incorretas do modulador, causando uma degradação da taxa de erro de bit. Neste trabalho, os autores avaliam o efeito dos erros de estimação em sistemas de modulação adaptativa. Um novo arcabouço analítico é proposto para modelar a relação sinal-ruído estimada instantânea. Diferentemente das abordagens anteriores, este modelo não está relacionado a uma técnica de estimativa específica e pode ser usado para comparar estratégias de estimação de canal com diferentes níveis de precisão.**

***Palavras-Chave*— Modulação adaptativa, Erro de estimação do canal, Desvanecimento Rayleigh.**

***Abstract*— The adoption of the adaptive modulation scheme leads to robust and spectrally efficient transmissions. However, the performance of the adaptive modulation systems is affected by the accuracy of the channel state information, which can result in incorrect modulator decisions, causing a bit error rate degradation. In this paper, the authors evaluate the effect of channel estimation errors in adaptive modulation systems. A novel analytical framework is proposed to model the instantaneous estimated signal-to-noise ratio. Different from previous approaches, this model is not related to a specific estimation technique and can be used to compare channel estimation strategies with different accuracy levels.**

***Keywords*— Adaptive modulation, Channel estimation error, Rayleigh fading.**

I. INTRODUCTION

Adaptive modulation was proposed to improve the spectral efficiency of a radio link and to ensure a maximum system BER (Bit Error Rate) [1]. The idea of adapting the modulation and coding schemes to channel conditions firstly appeared in the 1970s, but only after the second half of the 1990s, optimized modulation schemes have been reported in the literature. An important advantage of this modulation scheme is that it can be designed to provide a maximum BER even when the channel presents a varying SNR (Signal-to-Noise Ratio). For those schemes, the average spectral efficiency is enhanced while the maximum BER value is adjusted to satisfy the applications requirements (*i.e.,* not degrade the desired application performance) [2].

For many detection schemes, it is assumed that some channel parameters have been estimated and are available to the receiver. One such parameter is the phase shift of the carrier, which is assumed known for coherent detectors. In some modulation schemes both the amplitude and phase shifts of the received signal are required. The importance of assessing estimation errors of the parameters that characterize the channel model is justified by the impact that the estimation errors may have on the adaptive modulation schemes.

Some studies have been developed in order to evaluate the impact of the channel estimation error in adaptive modulation systems. In [3], for instance, the BER is determined for an M-QAM modulation scheme under flat Rayleigh fading with imperfect channel estimation and the PSAM (Pilot Symbol Assisted Modulation) technique is used to compensate the channel distortions. The imperfect channel estimation is also evaluated in [4], in which the estimation accuracy in switching the transmitter to the different modes of adaptive modulation schemes is presented. A sensitivity analysis of key performance parameters of a link is presented in [5], such as BER, spectral efficiency, average transmitted power and the outage probability estimation errors for short and long term summarized in a new set of mathematical expressions. The impact of the channel estimation errors is also discussed in [6] which presents an expression for the probability density function (pdf) of the maximum likelihood estimator of the SNR, on flat Rayleigh fading for different modulation schemes.

In this context, this paper presents a novel set of probability density functions to model the channel SNR, taking into account the occurrence of estimation errors. Different from the previous approaches, the proposed model is not related to any specific estimation technique, such as PSAM or MMSE (Minimum Mean Square Error) [7]. The proposed method allows the evaluation of the impact of channel estimation errors considering different accuracy levels. The analysis is general and independent of a specific channel estimation technique. In addition, Monte Carlo simulations of the proposed model can be easily implemented, since no specific filter should be modeled. In this case, the estimation error is modeled by a Gaussian random variable, added to the channel amplitude [3].

The remaining sections are organized as follows. Section II presents the system model adopted in this paper. Section III provides a brief overview of the adaptive modulation scheme. An analysis of the effect of channel estimation error on adaptive modulation systems, as well as the derivation of the proposed analytical framework, are described in Section IV. A performance analysis is presented in Section V. Finally, Section VI is devoted to the conclusions.

Rafael F. Lopes, Carlos D. M. Regis, Wamberto J. L. de Queiroz, Waslon T. A. Lopes and Marcelo S. de Alencar, Federal University of Campina Grande (UFCG), Campina Grande, Brazil, and Institute for Advanced Studies in Communications (Iecom), Campina Grande, Brazil. Rafael F. Lopes, Federal Institute of Maranhão (IFMA), São Luís, Brazil. Carlos D. M. Regis Federal Institute of Paraíba (IFPB), Campina Grande, Brazil. E-mails: {rafaelf, danilo, wamberto, waslon, malencar}@iecom.org.br.

II. System and Channel Models

In adaptive modulation systems different modulation schemes are used by the transmitter, based on the estimated SNR. The modulation schemes are selected in order to maximize the spectral efficiency, under a target BER constraint [2].

Figure 1 illustrates the block diagram of an adaptive modulation system. Linear modulation is assumed and the transmission rate adaptation occurs in multiples of the symbol rate $R_s = 1/T_s$, in which T_s is the symbol time. Furthermore, ideal Nyquist pulses ($\mathrm{sinc}[t/T_s]$) are adopted, and the signal bandwidth is defined as $B = 1/T_s$.

The channel model is characterized by a slowly varying flat fading, considered constant during a frame. Thus, the received signal is defined as

$$r(t) = \alpha(t)s(t) + n(t), \tag{1}$$

in which $n(t)$ represents the additive noise, modeled as a complex white Gaussian process, with zero mean and variance $N_0/2$ by dimension. The multiplicative factor $\alpha(t)$ is the stationary and ergodic time-varying channel gain, modeled as a complex Gaussian process with zero mean and variance σ_α^2. Therefore, the fading amplitude is Rayleigh distributed with parameter σ_α.

Considering that the phase shift is perfectly tracked, the channel model can be rewritten as

$$r(t) = g(t)s(t) + n(t), \tag{2}$$

in which $g(t) = |\alpha(t)|$.

Since slow fading is assumed, the channel can be considered AWGN (Additive White Gaussian Noise) with a varying SNR [8]. The instantaneous channel SNR can be defined as $\gamma(t) = \bar{P}_t g^2(t)/N_0B$, $0 \leq \gamma(t) < \infty$, in which $\bar{P}_t$ denotes the average transmitted signal power.

At the receiver, the estimated SNR denoted by $\hat{\gamma}(t)$ is then sent back to the transmitter, in order to perform the adaptation of the modulation scheme. It is assumed that the feedback link is error free and that it presents a negligible delay. Since $\gamma(t)$ is stationary, its pdf is independent of t, and it is denoted $p(\gamma)$. For a Rayleigh channel, $p(\gamma)$ is exponentially distributed with mean $\bar{P}_r = E[g^2] = 2\sigma_\alpha^2$ (representing the mean received power or SNR). The pdf of γ is given by

$$p(\gamma) = \frac{1}{2\sigma_\alpha^2} e^{-\frac{\gamma}{2\sigma_\alpha^2}}, \ \gamma \geq 0, \ \sigma_\alpha > 0. \tag{3}$$

For simplification, in the following sections the actual and estimated instantaneous SNR are denoted, respectively, by γ and $\hat{\gamma}$ (*i.e.*, $\gamma = \gamma(t)$, $\hat{\gamma} = \hat{\gamma}(t)$).

III. Adaptive Modulation on a Perfectly Estimated Channel

The adoption of the adaptive modulation scheme leads to robust and spectrally efficient transmissions [8]. The adaptive modulation technique changes the modulation scheme according to the channel fluctuations in order to keep the BER below some target value. In this section it is assumed that the channel power gain is perfectly estimated by the receiver, *i.e.*, $\hat{\gamma}(t) = \gamma(t)$.

In this adaptive modulation system the number of binary symbols in the constellation diagram is changed to maintain the BER below some target value. The choice of the modulation scheme is based on the definition of N decision regions (or fading regions), $R_i = [\gamma_i, \gamma_{i+1})$, $i = 0, ..., N-1$, in which γ_i is an SNR decision threshold defined to achieve some performance level (in terms of BER), with $\gamma_N = \infty$ and $\gamma_0 \geq 0$. A constellation with M_i symbols (each one equivalent to $k_i = \log_2 M_i$ bits) is used when $\gamma(t) \in R_i$ (*i.e.* the instantaneous SNR value belongs to region i).

The most important performance measures of the adaptive modulation scheme are the average spectral efficiency and the average BER [2]. The spectral efficiency can be calculated as follows [9], [10]

$$\bar{\eta} = \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma. \tag{4}$$

The average performance, in terms of BER, is computed using the following expression [9], [10]

$$\overline{\mathrm{BER}} = \frac{1}{\bar{\eta}} \sum_{i=0}^{N-1} k_i \int_{\gamma_i}^{\gamma_{i+1}} \mathrm{BER}_i(\gamma) p(\gamma) d\gamma, \tag{5}$$

in which $\mathrm{BER}_i(\gamma)$ refers to the BER function for modulation i in AWGN channels at some received SNR γ.

The average performance of the adaptive modulation system depends on the choice of the decision thresholds γ_i. Those thresholds should be carefully determined to ensure that the system remains operating at or below a certain maximum target BER. Lower threshold values leads to a high throughput. On the other hand, a high decision threshold implies a lower BER for the adaptive modulation scheme [11].

IV. Effect of the Channel Estimation Error on Adaptive Modulation

In the previous section, the adaptive modulation system was presented considering an ideal CSI (Channel State Information), *i.e.,* the channel power gain is perfectly known. However, in a practical implementation, the channel gain is not known and should be estimated at the receiver. This estimated value is used to select the modulation scheme.

The channel gain estimation process evaluates the fading distortion introduced by the channel in the transmitted data symbols. This process is commonly based on the periodic transmission of known training symbols (denoted by $C(t)$). The model for the received training symbols is

$$r_c(t) = g(t)C(t) + n(t), \tag{6}$$

in which $r_c(t)$ represents the received training symbols.

Since $C(t)$ is known by the receiver, the estimated channel gain is

$$\hat{g}(t) = \frac{r_c(t)}{C(t)} \Rightarrow \hat{g}(t) = g(t) + \epsilon(t), \tag{7}$$

in which $g(t)$ is the actual Rayleigh distributed instantaneous channel gain, with parameter $\sigma_\alpha = \sqrt{\frac{\bar{P}_r}{2}}$, and $\epsilon(t)$ is a

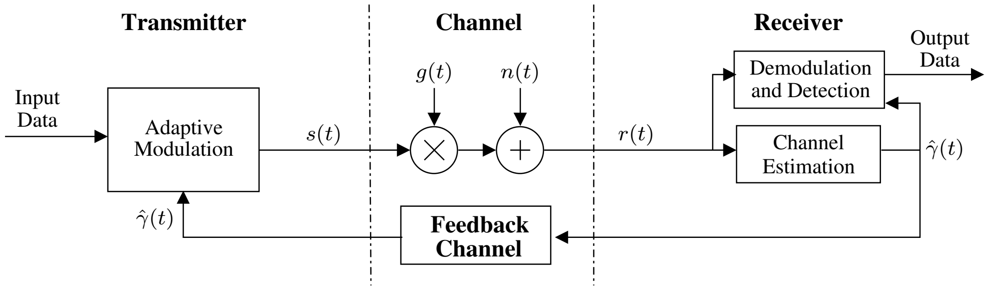

Fig. 1. Adaptive modulation system model.

Gaussian estimation error with zero mean and variance σ_{ϵ}^2. It is assumed that $g(t)$ and $\epsilon(t)$ are independent, since the former is the channel gain and the later is a function of the noise.

Based on the model defined by Equation 7, the estimated received SNR is given by

$$\hat{\gamma}(t) = [g(t) + \epsilon(t)]^2. \tag{8}$$

The presence of the channel estimation error affects the overall performance of the adaptive modulation system. Although the system is subject to the SNR γ, the modulation scheme is selected according to the estimated SNR $\hat{\gamma}$, leading to incorrect decisions about the selected modulation.

In this context, it is necessary to define an extension of the standard BER formulas for the AWGN channel. A simple model for the BER functions subject to estimation errors is defined as

$$\text{BER}(\gamma, \hat{\gamma}) = \sum_{j=0}^{N-1} \text{BER}_j(\gamma)[\text{u}(\hat{\gamma} - \hat{\gamma}_j) - \text{u}(\hat{\gamma} - \hat{\gamma}_{j+1})], \tag{9}$$

in which $\text{u}(x)$ is the unit step function, $\hat{\gamma}_j$ represents the decision thresholds specified as regions of the estimated SNR values, with $j = 0, ..., N-1$, $\hat{\gamma}_N = \infty$, $\hat{\gamma}_0 \geq 0$ and $\text{BER}_j(\gamma)$ is the AWGN BER function used when $\hat{\gamma}$ falls into the interval $[\hat{\gamma}_j, \hat{\gamma}_{j+1})$. The average BER for a given estimated channel SNR can be evaluated as follows [2], [6]

$$\text{BER}(\hat{\gamma}) = \int_0^{\infty} \text{BER}(\gamma, \hat{\gamma}) p(\gamma|\hat{\gamma}) d\gamma, \tag{10}$$

in which $p(\gamma|\hat{\gamma})$ denotes the conditional pdf of the actual SNR γ given an estimated SNR $\hat{\gamma}$.

Considering the estimation error, the average spectral efficiency expression becomes [2], [6]

$$\bar{\eta} = \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} p(\hat{\gamma}) d\hat{\gamma}, \tag{11}$$

in which $p(\hat{\gamma})$ is the pdf of the estimated SNR. Similarly, the average BER performance is given by [2], [6]

$$\overline{\text{BER}} = \frac{1}{\bar{\eta}} \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} \text{BER}(\hat{\gamma}) p(\hat{\gamma}) d\hat{\gamma}. \tag{12}$$

Substituting Equation 10 into Equation 12 and using Bayes' rule, one obtains

$$\overline{\text{BER}} = \frac{1}{\bar{\eta}} \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} \int_0^{\infty} \text{BER}(\gamma, \hat{\gamma}) p(\gamma, \hat{\gamma}) d\gamma d\hat{\gamma}. \tag{13}$$

After performing the substitution of Equation 9 into Equation 13, it can be seen that, for all $i \neq j$, $\text{BER}(\hat{\gamma}) = 0$ (since the decision regions are disjoint). Thus,

$$\overline{\text{BER}} = \frac{1}{\bar{\eta}} \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} \int_0^{\infty} \text{BER}_i(\gamma) p(\gamma, \hat{\gamma}) d\gamma d\hat{\gamma}. \tag{14}$$

In order to evaluate the spectral efficiency and the BER of an adaptive modulation system (Equations 11 and 14, respectively), the pdfs $p(\hat{\gamma})$ and $p(\gamma, \hat{\gamma})$ are required. In this context, the authors propose a novel analytical framework for modeling the pdfs of the channel SNR, considering the occurrence of estimation errors. Different from previous approaches, such as [3], [4], [5] and [6], this model is not related to a specific estimation technique, and can be used to evaluate the performance of various adaptive modulation configurations or even to compare channel estimation techniques with different accuracy levels. The following sections present the derivation of the required pdfs.

A. Probability density function of the estimated SNR $p(\hat{\gamma})$

The pdf $p(\hat{\gamma})$ can be obtained by the transformation of random variable presented in Equation 8. The transformation is based on two steps: first, obtain the pdf of the sum of g and ϵ (*i.e.,* the convolution of the Rayleigh and Normal pdfs), and square the resultant pdf. After the mathematical manipulation, Equation 15 is obtained.

B. Joint probability density function of the SNR and estimated SNR $p(\gamma, \hat{\gamma})$

In order to find the joint pdf of the SNR and estimated SNR $p(\gamma, \hat{\gamma})$, it is necessary to perform a bivariate transformation, according to the following model

$$\begin{cases} \gamma(t) = [g(t)]^2, \\ \hat{\gamma}(t) = [g(t) + \epsilon(t)]^2, \end{cases} \tag{16}$$

in which $g(t)$ is the channel gain (Rayleigh distributed with parameter σ_{α}) and $\epsilon(t)$ is the estimation error (Gaussian

$$p(\hat{\gamma}) = \frac{\sigma_\epsilon}{\sqrt{2\pi\hat{\gamma}}(\sigma_\alpha^2+\sigma_\epsilon^2)} \exp\left[-\frac{\hat{\gamma}}{2\sigma_\epsilon^2}\right] + \frac{\sigma_\alpha}{2(\sigma_\alpha^2+\sigma_\epsilon^2)^{3/2}} \exp\left[-\frac{\hat{\gamma}}{2\sigma_\epsilon^2}\left(1-\frac{\sigma_\alpha^2}{\sigma_\alpha^2+\sigma_\epsilon^2}\right)\right] \operatorname{erf}\left[\frac{\sigma_\alpha\sqrt{\hat{\gamma}}}{\sigma_\epsilon\sqrt{2(\sigma_\alpha^2+\sigma_\epsilon^2)}}\right], \hat{\gamma}, > 0. \quad (15)$$

$$p(\gamma,\hat{\gamma}) = \frac{1}{2\sqrt{2\pi\hat{\gamma}}\sigma_\alpha^2\sigma_\epsilon} \exp\left[-\frac{1}{2}\left(\frac{\gamma}{\sigma_\alpha^2}+\frac{\hat{\gamma}+\gamma}{\sigma_\epsilon^2}\right)\right] \cosh\left[\frac{\sqrt{\gamma\hat{\gamma}}}{\sigma_\epsilon^2}\right], \gamma \geq 0, \hat{\gamma} > 0. \quad (18)$$

distributed with zero mean and variance σ_ϵ^2). The bivariate transformation is then applied to the joint distribution of $g(t)$ and $\epsilon(t)$, defined as

$$p(g,\epsilon) = \frac{g}{\sigma_\alpha^2\sigma_\epsilon\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{g^2}{\sigma_\alpha^2}+\frac{\epsilon^2}{\sigma_\epsilon^2}\right)\right]. \quad (17)$$

After the bivariate transformation of Equation 17, the joint pdf $p(\gamma,\hat{\gamma})$ is obtained (Equation 18).

V. Performance Analysis and Results

This section presents a numerical analysis and Monte Carlo simulations used to validate and evaluate the impact of channel estimation techniques with different precision levels on adaptive modulation systems[1]. Four modulation schemes were considered in the experiments ($N = 4$), according to Table I.

TABLE I
MODULATION SCHEMES ADOPTED IN THE EXPERIMENTS.

i	Modulation	M_i	k_i	Decision region
0	BPSK	2	1 bit/symbol	$\gamma_0 \leq \gamma(t) < \gamma_1$
1	QPSK	4	2 bits/symbol	$\gamma_1 \leq \gamma(t) < \gamma_2$
2	16-QAM	16	4 bits/symbol	$\gamma_2 \leq \gamma(t) < \gamma_3$
3	64-QAM	64	6 bits/symbol	$\gamma(t) \geq \gamma_3$

In the experiments, three different performance profiles were designed, each one associated with a maximum target BER (10^{-2}, 10^{-3} and 10^{-4}). Based on the target BER of each profile, appropriate decision regions were obtained by a numerical inversion of the adopted AWGN BER functions (the occurrence of outage is not considered in this analysis). The defined decision thresholds are presented in Table II.

TABLE II
DECISION THRESHOLDS FOR THE THREE DESIGNED PROFILES.

Profile	Target BER	γ_0	γ_1	γ_2	γ_3
High loss	10^{-2}	0.00	7.31	13.89	19.72
Medium loss	10^{-3}	0.00	9.78	16.52	22.52
Low loss	10^{-4}	0.00	11.38	18.21	24.28

Incorrect decisions about the selected modulation schemes can make the system more vulnerable to the noise, leading to a severe BER degradation (*e.g.,* if the system incorrectly chooses 16-QAM instead of BPSK, the BER increases). In this context, each performance profile is differently impacted by the channel estimation errors. Configurations that frequently select low order modulation schemes are more impacted by estimation errors than the others (since a wrong choice from a high order to a low order modulation decreases the system BER).

The performance obtained with the "High loss" profile is presented in Figure 2 (considering a perfectly estimated channel and different values for the variance of the channel estimation error). As can be seen in the figure, the impact of the estimation error is significantly decreased after its γ_3 value (19.72 dB), from which the highest order modulation (64-QAM) is often selected.

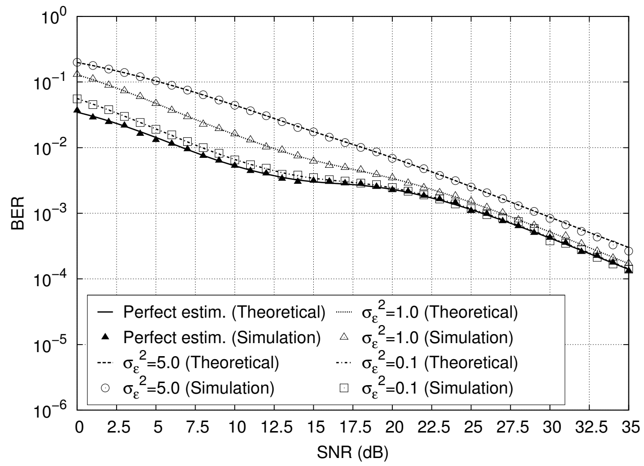

Fig. 2. Average BER as a function of the actual SNR for the "High loss" profile, considering different values for the variance of the estimation error.

As can be seen in Figure 2, depending on the variance of the estimation error, the system presents different losses in terms of average BER. For the values of $\sigma_\epsilon^2 = 0.10$, 1.00 and 5.00, the system loses, respectively, 0.20 dB, 1.20 dB and 3.60 dB to achieve a BER value of 10^{-3} (this BER value is achieved approximately at 25.67 dB on a perfectly estimated channel). However, higher BER losses occur at low SNR values, and decreases as the SNR increases.

Figure 3 shows the results for the "Medium loss" profile. Assuming the average BER value of 10^{-3}, it can be seen that this target BER is achieved by the system (with perfect estimation) at 15.08 dB. However, depending on the variance of the estimation error (0.10, 1.00 and 5.00), the system loses, respectively, 0.68 dB, 3.53 dB and 8.43 dB.

One can verify that those losses overcome the obtained with the "High loss" profile. For example, the BER value 10^{-3} is achieved at 15.08 dB, a smaller value than the required SNR for the "High loss" profile (25.67 dB). Furthermore, this SNR is also smaller than γ_2, making the system more vulnerable to

[1]Python language and the Mpmath library [12] were used in the numerical evaluation, while the C language was adopted to implement the Monte Carlo simulations.

incorrectly chosen high-order modulations schemes (16-QAM and 64-QAM).

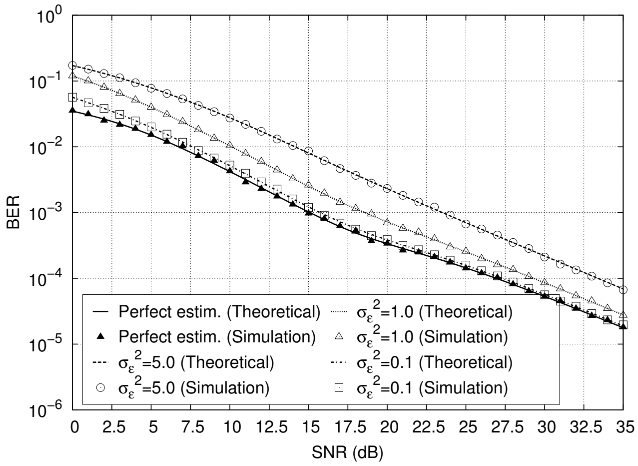

Fig. 3. Average BER for the "Medium loss" profile, considering different values for the variance of the estimation error.

Similar losses were obtained in the presence of estimation errors when using the "Low loss" profile (Figure 4). Also for BER $= 10^{-3}$ (achieved at 15.13 dB), the system loses 0.58 dB, 2.24 dB and 5.72 dB (respectively, for $\sigma_\epsilon^2 = 0.10$, 1.00 and 5.00), smaller values than with the "Medium loss" profile. For example, with the "Low loss" profile, the difference from the evaluated SNR (15.13 dB) and its range border γ_3 (18.21 dB) is larger than in the "Medium loss" (from 15.08 dB to 16.52 dB), reducing the number of wrong shifts from QPSK to 16-QAM.

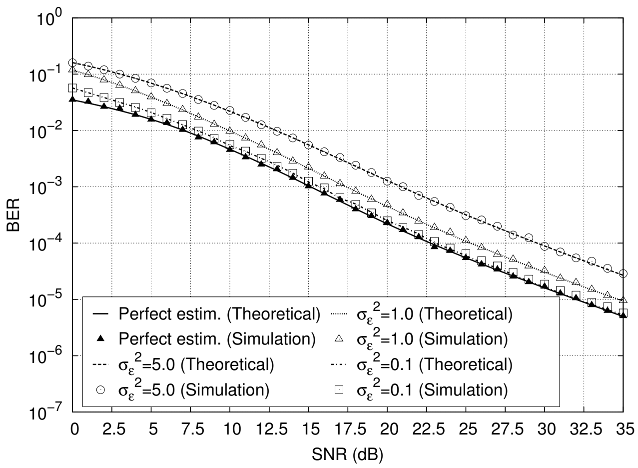

Fig. 4. Average BER for the "Low loss" profile, considering different values for the variance of the estimation error.

Based on the experiments one can conclude that increasing the decision thresholds makes the system more robust to errors in the transmitted data, but also makes the system more vulnerable to estimation errors. This can be observed in Figures 2, 3 and 4, in which the overall loss caused by a $\sigma_\epsilon^2 = 5.00$, significantly increases from the "High loss" to the "Low loss" profile (mainly in high SNR values). However, the analysis for intermediate SNR values must take into consideration the difference from those values and their next superior decision threshold.

Another important result from the experiments is that they have confirmed the validity of the proposed analytical model. This allows the use of the expressions presented in Section IV to evaluate the impact of estimation errors in adaptive modulation systems.

VI. Conclusions

This paper presents a study of the impact of imperfect channel estimation on adaptive modulation systems. A new analytical framework for modeling the instantaneous SNR subject to channel estimation errors was derived and validated by Monte Carlo simulations. An important feature of the novel proposed expressions is that they do not depend on a specific channel estimation technique, allowing the comparision of adaptive modulation systems with different channel estimators. This work shows that robust adaptive modulation systems (in terms of average BER) may have their performance seriously damaged by the channel estimation errors. Based on the derived expressions, the parameters of the adaptive modulation system can be adjusted in order to improve its performance, even in the presence of channel estimation errors.

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Referências

[1] A. Goldsmith and S.-G. Chua, "Adaptive coded modulation for fading channels," *IEEE Transactions on Communications*, vol. 46, no. 5, pp. 595 –602, may 1998.

[2] A. Svensson, "An introduction to adaptive QAM modulation schemes for known and predicted channels," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2322–2336, Dec. 2007.

[3] X. Tang, M.-S. Alouini, and A. J. Goldsmith, "Effect of channel estimation error on M-QAM BER performance in Rayleigh fading," *IEEE Transactions on Communications*, vol. 47, no. 12, pp. 1856–1864, Dec. 1999.

[4] F. Y. Long and K. T. Lo, "Measurements for performance degradation of adaptive modulation scheme caused by channel quality estimation error," *IEE Proceedings on Communications*, vol. 150, no. 2, pp. 80–84, April 2003.

[5] J. F. Paris, M. C. Aguayo-Torres, and J. T. Entrambasaguas, "Impact of channel estimation error on adaptive modulation performance in flat fading," *IEEE Transactions on Communications*, vol. 52, no. 5, pp. 716–720, May 2004.

[6] M. Mohammad and R. M. Buehrer, "On the impact of SNR estimation error on adaptive modulation," *IEEE Communications Letters*, vol. 9, no. 6, pp. 490–492, Jun. 2005.

[7] F. R. P. Cavalcanti and S. Andersson, Eds., *Optimizing wireless communication systems*. New York, NY, USA: Springer, 2009.

[8] A. Goldsmith, *Wireless Communications*. New York, NY, USA: Cambridge University Press, 2005.

[9] C. Wong and L. Hanzo, "Upper-bound performance of a wide-band adaptive modem," *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 367–369, Mar. 2000.

[10] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Transactions on Communications*, vol. 49, no. 9, pp. 1561–1571, Sep. 2001.

[11] J. M. Torrance and L. Hanzo, "Performance upper bound of adaptive QAM in slow Rayleigh-fading environments," in *ICCS'96/ISPAC'96*, vol. 3, November 1996, pp. 1653–1657.

[12] Mpmath, "Python library for arbitrary-precision floating-point arithmetic," April 2011. [Online]. Available: http://code.google.com/mpmath