Distributed Resource Allocation for Wireless Service Provision in a Competitive Scenario

José Mairton B. da S. Júnior, F. Rafael M. Lima, Tarcísio F. Maciel and F. Rodrigo P. Cavalcanti

***Resumo*— Devido ao constante avanço das redes móveis celulares, operadoras tornaram-se novas participantes no competitivo mercado de serviços de dados. Aspectos econômicos como o preço cobrado pelo serviço fazem parte da maioria das estratégias de *marketing* atualmente. O foco deste artigo é estudar o problema da alocação de recursos e definir preços em um cenário onde operadoras competem por usuários. A solução centralizada para o problema é revisada e uma solução distribuída utilizando informações locais de qualidade do canal é proposta. Os resultados indicam que a solução proposta possui uma pequena degradação em relação a solução centralizada.**

***Abstract*— With the continuous development of mobile wireless networks, they have become a new player in the competitive market of data service provision. Economical aspects such as pricing strategy are part of most of today's market strategies. In this article, we focus on the problem of resource allocation and price definition in a competitive scenario with mobile wireless providers. In this context we review the centralized solution of the welfare maximization problem and propose a distributed solution that uses only local channel information. The results show that the proposed solution presents a small error when compared to the centralized solution.**

***Keywords*— Resource Allocation, economics, wireless and distributed algorithms.**

I. Introduction

With the Third Generation (3G) advent, mobile wireless networks have become a new player in the competitive market of data services provision. As an example of this new reality we can mention the High Speed Packet Access (HSPA) system that has been experiencing a widespread adoption in the world. The perspective is even better with the advent of Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (WiMAX) networks and the standardization of LTE-Advanced that will meet the requirements of International Mobile Telecommunications - Advanced (IMT-A), i.e., the so called Fourth Generation (4G).

With the popularity of mobile data services we have also witnessed the increased competition among system providers in order to expand their customer base. Technological aspects such as availability of terminals, amount of spectrum, spectrum efficiency, coverage and the portfolio of supported services are not able to make the difference in this competition since they are in general similar among the majority of the providers. On the other hand, economical aspects such as the pricing strategy are part of most of today's market strategies. Some works have focused on the study of the impact of economical aspects in wireless networks. *Badia et al.* in [1]–[3] have shown how concepts from microeconomics are employed to set the pricing strategy of a provider selling wireless data access. Such strategy takes into account the utility or Quality of Service (QoS) perceived by users and their willingness to pay for that utility. In [4] we have studied how economical aspects can be taken into account in Radio Resource Management (RRM) design in order to control the churn rate in these networks, i.e., the relative fraction of customer defection in a given time basis.

Some works have studied the providers' price competition to attract users [5]–[8]. In [5]–[7], the authors consider the problem of association among providers and users in which purchasing a unit of resource from different providers brings the same amount of utility to the user. This is in general not the case in the wireless setting since users experience different channel quality conditions to different providers. This aspect is taken into account in [8] that models heterogeneous users in both utility function and perceived quality for each provider. In that article, the authors consider a scenario in which the users are not contractually tied by the providers. Specifically, the users are free to switch in real time to the provider with best cost-rate trade-off. The main contribution of [8] is the study of the providers' competition problem as a two-stage game that is shown to have a zero efficiency loss at the equilibrium point, that is, neither the providers nor the users have unilaterally incentive to deviate their strategy with the centralized solution of the welfare maximization problem.

In this article, we review the problem studied in [8] and propose a distributed solution to the providers' competition problem. Our proposed solution uses only local information of channel quality conditions. The remaining of this article is organized as follows: in Section II we present the main assumptions about the system modelling used in this paper; the centralized solution to the studied problem is reviewed in Section III; the main contribution of this article, the distributed solution to the studied problem, is presented in Section IV; finally, we present the simulation results in Section V and the main conclusions and perspectives in Section VI.

II. System Modelling

We consider the downlink of a system that has a set $\mathcal{J} = \{1, \ldots, J\}$ of wireless service providers and a set $\mathcal{I} = \{1, \ldots, I\}$ of users. Each provider has a certain amount of resources Q_j that can be split into smaller resources and be

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purchased by different users. These resources can be transmit power, or time slots in a Time Division Multiple Access (TDMA) system, or subcarriers in a Orthogonal Frequency Division Multiple Access (OFDMA) system, for example. Without loss of generality we consider that the available resources are time slots. A user i can purchase resources from one or more providers simultaneously.

We assume that the time is split into frames that is composed of several time slots and at the beginning of a frame each provider j defines the price per unit of resource, p_j. Once the prices per unit of resource are defined, each user i determines the demand or the amount of resources q_{ij} this user will purchase from provider j. Under this model, we call an undecided user the one that purchases resources from two or more providers. We define $\mathbf{p} = [p_1, \ldots, p_J]^T$ as a vector with the prices of all providers. Also, we consider that $\mathbf{q}_i = [q_{i1} \ldots q_{iJ}]^T$ is a vector with the demand of user i for each provider and $\mathbf{q} = [\mathbf{q}_1{}^T \ldots \mathbf{q}_I{}^T]^T$ is a vector with the demand of all user for all providers.

We model the perceived QoS of a user i by a utility function, $u_i(\cdot)$ that depends on the transmit data rate and should be differentiable, strictly increasing and strictly concave. We defined the utility function of user i as

$$u_i = a_i \log\left(1 + \sum_{j=1}^{J} q_{ij} c_{ij}\right), \tag{1}$$

where a_i's are the individual willingness to pay of each user and

$$c_{ij} = W \log\left(1 + \frac{P_j |h_{ij}|^2}{\sigma_i^2}\right), \tag{2}$$

represents the maximum data rate that provider j can transmit to user i assuming that that this user gets allocated all the time slots. Also, W is the available frequency bandwidth of provider j, P_j is the fixed transmit power, σ_i is the noise power for user i and h_{ij} is the coefficient of the channel transfer function of the wireless link between user i and provider j given by

$$h_{ij} = \sqrt{(d_{ij})^{-\alpha}}, \tag{3}$$

where α is the path-loss exponent and d_{ij} is the distance between the user i and the base station of provider j. We assume that the frequency bandwidth of all providers are orthogonal and, therefore, there is no interference between different providers. Furthermore, the channel coefficients are assumed to be constant during a frame.

The objective of provider j is to set the price for unit of resource, p_j, so as to maximize its revenue $p_j \sum_{i=1}^{I} q_{ij}$ subject to the resource constraint $\sum_{i=1}^{I} q_{ij} \leq Q_j$. Specifically, a provider maximizes its revenue when all resources are purchased by users. On the other hand, a user i should define the demand for each provider, $\mathbf{q}_i$, with the objective of maximizing his/her payoff that is defined as the difference between the utility and the payment shown as follows

$$v_i = a_i \log\left(1 + \sum_{j=1}^{J} c_{ij} q_{ij}\right) - \sum_{j=1}^{J} p_j q_{ij}. \tag{4}$$

The social welfare maximization problem aims at maximizing the summation of the users' payoffs and the providers' revenues. It can be easily observed that the welfare maximization problem is equivalent to maximizing the sum of users' utility, because the payments and revenue are system internal transfers and cancel out. It is useful to define a vector of weighted resources as $\mathbf{x} = [x_1 x_2 \ldots x_I]^T$, where $x_i = \sum_{j=1}^{J} q_{ij} c_{ij}$. In this way, the welfare maximization problem is given by

$$\max_{\mathbf{x}} \sum_{i=1}^{I} u_i(x_i) \tag{5a}$$

subject to

$$\sum_{j=1}^{J} q_{ij} c_{ij} = x_i \;\; \forall i \in \mathcal{I} \tag{5b}$$

$$\sum_{i=1}^{I} q_{ij} = Q_j, \;\; \forall j \in \mathcal{J} \tag{5c}$$

$$q_{ij} \geq 0 \;\; \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{5d}$$

The constraint (5b) specifies the relationship between x_i and q_{ij}. The constraint (5c) indicates that a provider cannot sell more resources than the available one, and finally, the constraint (5d) assures that the amount of resources demanded by users are non-negative.

III. Centralized Solution

In [8] the authors have considered the problem depicted in Section II and modelled the interaction between the providers and the users as a two-stage multi-leader-follower game (see [9] and [10]). In this model the providers are the leaders and users are the followers. It is assumed that the channel gains of all users are roughly constant for the duration of the game (frame), and furthermore, that they are known to all game participants (e.g. each provider collects channel condition information to each user, and then broadcasts this information to all users and providers).

The authors in [8] show that the solution of the social welfare optimization problem (5) has a unique maximizing solution $\mathbf{x}^*$, with a unique Lagrange multiplier vector $\boldsymbol{\lambda}^*$. We can see in the constraint (5b) that $\mathbf{q}_i$ uniquely determines x_i and, consequently, $\mathbf{q}$ uniquely determines $\mathbf{x}$. On the other hand, it is not guaranteed that a given $\mathbf{x}$ can be mapped to a unique $\mathbf{q}$. However, in [8] the authors shows by using a bipartite graph representation that for this specific problem a simple algorithm can be derived that uniquely determines $\mathbf{q}$ from $\mathbf{x}$. In this way, the solution of (5) is unique in $\mathbf{q}$.

According to the solution of problem (5) the users can be divided into three categories: decided users, undecided users and non-connected users. The decided users purchase from only one provider, while the undecided users purchase from several providers. Other users may decide not to connect to any provider having zero demand for all providers.

Other interesting result found in [8] is that the optimum solution of problem (5) is also the subgame perfect Nash equilibrium of the two-stage game where the prices are equal to the Lagrange multipliers $\mathbf{p}^* = \boldsymbol{\lambda}^*$. In other words, the problem has zero efficiency loss at the equilibrium because at the maximizing solution $\mathbf{q}^*$ and $\mathbf{p}^*$ neither the providers nor the users have unilaterally incentive to deviate their strategy despite the selfish nature of the providers and users.

In order to find the solution of problem (5) it is assumed that all the information about channel quality, users and providers should be concentrated in a central node. We call this solution as centralized solution. In Figure 1 we represent the association between providers, users and the centralized node. The centralized solution is not practical since it requires a central node and the complexity of solving the problem (5) may require a large computational effort. In the next section we propose a distributed solution to this problem.

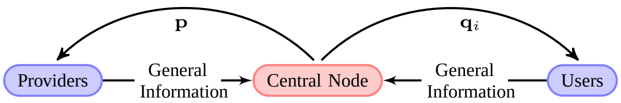

Fig. 1. Information diagram in Centralized Algorithm

IV. Proposed Distributed Solutions

Distributed algorithms have the advantage of decentralizing the processing in the system. This results in the division of the main task in many low-complexity subtasks. Moreover, in distributed solutions pieces of information that previously had to be available at the central node are not necessary anymore, due to the local processing.

The objective of our proposed distributed solution is to find a good solution to the problem (5) that, as shown before, is also the equilibrium solution. In this way, the user and providers iteratively solve small problems based on the information exchanged among them. Henceforth, we will add the index n in some of the previously defined variables in order to indicate the current iteration. More specifically, at a given iteration n, based on the knowledge of the announced prices, $\mathbf{p}^n$, each user i should determine his/her demand, $\mathbf{q}_i^n$. Once all users defined their demand to each provider, $\mathbf{q}^n$, the providers are now able to adjust the previous prices in order to increase their revenues.

In Figure 2 we illustrate the information flow for the proposed distributed algorithm. Firstly, comparing Figures 1 and 2, we can see that in the distributed solution there is no central node. Consequently, the problem to be solved is split in subtasks that should be processed by each node. In this case, the provider and the users iteratively adjust their prices and demands, respectively. The providers adjust their prices based on the knowledge of the users' demands in order to maximize their revenues. On the other hand, the users define their demands to each provider so as to maximize their utility based on the knowledge of the prices.

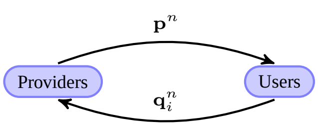

Fig. 2. Information diagram in Distributed Algorithm

In Figure 3 we present a detailed fluxogram of the proposed distributed solution. The first step is the initialization of the prices by the providers. Based on the knowledge of the announced prices at iteration n each user i will independently define their demand q_{ij}^n for each provider j at iteration n. In order to perform this task, each user i will solve the optimization problem of maximization of his/her payoff defined by Equation (4) constrained by $q_{ij}^n \geq 0 \ \forall j$. It is important to highlight here that this optimization problem has not a global maximum as shown in Appendix I. Therefore, suboptimal demands may be defined by the users.

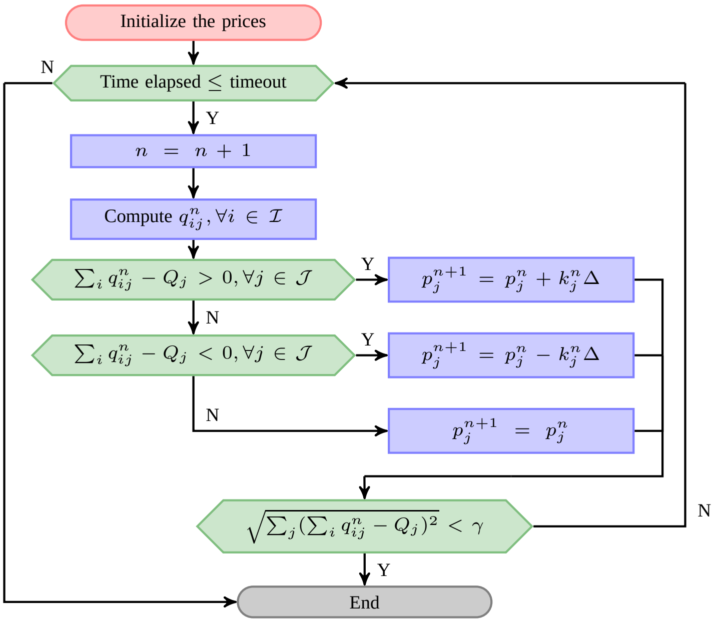

Fig. 3. Flowchart of the distributed algorithm

Once the demands of all users are defined, the providers will adjust their prices so as to increase their revenues. Each provider will verify if the total demand of the users is greater or lower than the total available resources. If greater, this means that the prices should be increased in order to reduce the demand of the users. If lower, the prices should be decreased so as to become more attractive to users and then sell all the available resources. The prices are adjusted by adding or subtracting a term composed of two factors Δ and k_j^n. The former is a positive constant and the latter is an adaptive factor of provider j at iteration n defined as

$$k_j^n = \frac{\sum_i q_{ij} - Q_j}{\sqrt{(\sum_i q_{ij} - Q_j)^2 / J}}, \forall j \in \mathcal{J} \qquad (6)$$

As can be seen in Equation (6), for a given provider j the numerator of k_j^n represents the difference between the total demand and the total available resources while in the denominator we have an average distance of the total demand of all providers and their total available resources. Therefore, when the demand for a provider j at iteration n is lower than the available resources, the factor k_j^n will be negative and the prices will be decreased by a factor proportional to the difference between the demand and available resources. On the other hand, when the demand is higher than the available resources, the prices will be increased.

Finally, the iterative procedure described previously will be repeated until the average difference between the total demand and available resources is lower than a given threshold or a maximum processing time was achieved. Note that this maximum processing time can be mapped to a maximum number of iterations if desired.

V. Performance Evaluation

In this section we show some results obtained by computer simulations based on the models described on Section II. We performed 100 independent snapshots where in each snapshot a given number of users are randomly disposed in a rectangular area of 2km$\times$2km. Due to the distance based path-loss model in (3) the users also have different channel quality states to the providers in each snapshot. Furthermore, in each snapshot we solve the social welfare maximization problem (5) by using the centralized and the proposed distributed algorithm. The main simulation parameters are described in Table 1.

TABLE 1
Simulation Parameters

Parameter	Value
I	30
J	2 and 3
a_i	exponential distribution with mean 1
W	10 MHz
α	2.4
σ_i	-120dBm
P_j	43dBm
Q_j	20
γ	0.3
Δ	10^{-3}
Timeout	2 hours

In order to better understand the studied problem, we show in Figure 4 the solution of problem (5) for a specific snapshot. In this two-providers case we can see three types of users: the decided users connected to provider 1, the decided users connected to provider 2, the undecided users that are connected to both providers 1 and 2.

Firstly, we can see that users near to a provider and far away from the other provider tend to connect to the nearest one. This comes from the fact that in general the channel quality improves as the distance between the transmitter and receiver decreases. In this case, the channel quality becomes a strong factor in the utility expression in (1). Consequently, in general these users are not affected by price competition. On the other hand, the users that have approximately the same distance to both providers are more sensible to price competition becoming an undecided or eventually a non-connected user.

In Figures 5 and 6 we illustrate the convergence of the distributed algorithm for two- and three-providers scenarios in a given snapshot. As explained in Figure 3, the convergence is achieved when the average difference between the total demand and available resources is lower than a given threshold. In Figure 5 and 6 we show how the total demand of each provider dynamically converges to the total available

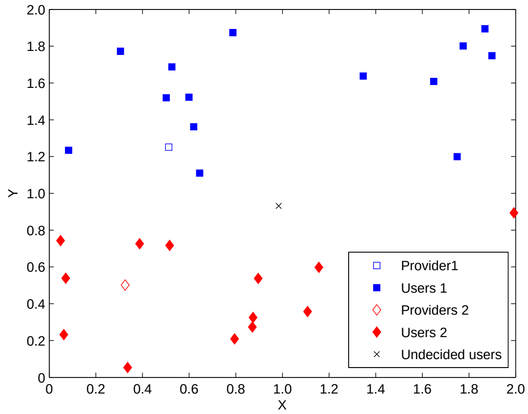

Fig. 4. Association of users and providers in a given snapshot.

resources ($Q_j = 20$) along the iterations. We can see that the price adjustments performed at each iteration are capable of decreasing the difference between the total demand and available resources. For the two-providers case the total demand of the providers converged at iteration 264 while in the three-providers case the convergence was achieved at iteration 275. In general, the convergence speed depends on the number of providers, initial prices defined at the beginning of the algorithm, number of users and adopted criterion for convergence.

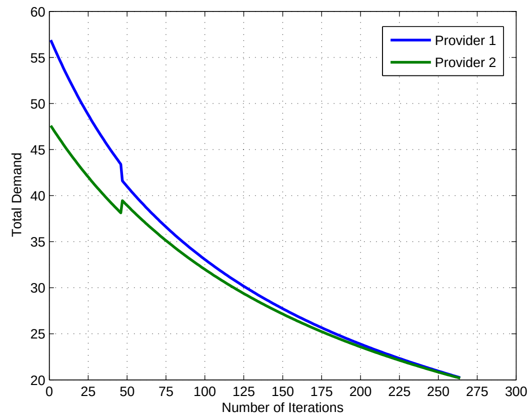

Fig. 5. Convergence of the Distributed Algorithm for 2 providers

The figure of merit used to evaluate our proposed solution is the relative error between the prices found by the distributed and centralized solutions for each provider and snapshot given by

$$e_{j,s} = \left| \frac{p_{j,s}^{dist} - p_{j,s}^{cent}}{p_{j,s}^{cent}} \right|, \quad (7)$$

where $p_{j,s}^{dist}$ and $p_{j,s}^{cent}$ are the final prices of provider j in snapshot s for the distributed and centralized algorithms. In Figure 7 we present the Cumulative Distribution Function (CDF) of this relative error for the two- and three-providers case in all the snapshots. We can see that the maximum error found for the two- and three-providers cases were 1.30% and

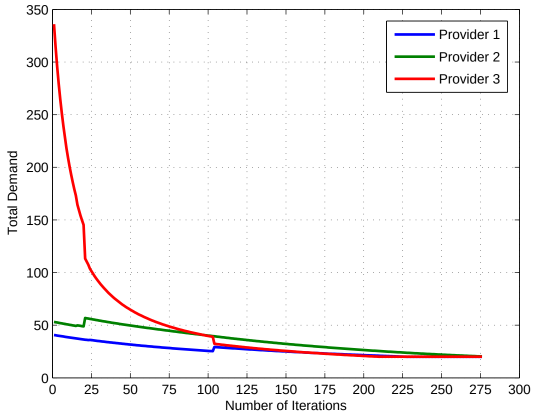

Fig. 6. Convergence of the Distributed Algorithm for 3 providers

8.3%. Looking at the $97th$ percentile we can observe that 97% of the samples experienced an error lower than 1.25% and 2.25% for the two- and three- providers, respectively. Based on these results, we highlight that the distributed solution is capable of providing suboptimal but good solutions with small errors compared to the centralized solution without the need of a central node.

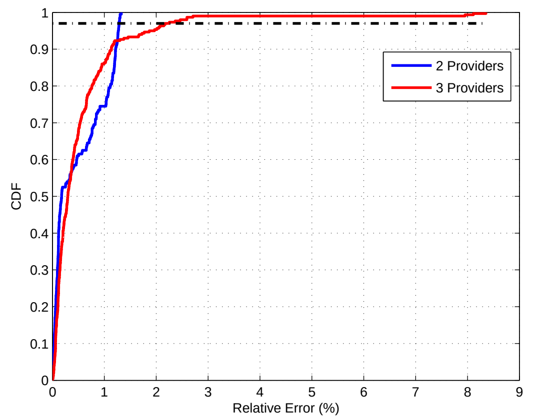

Fig. 7. CDF of the relative error for the two- and three-providers case in all snapshots.

VI. Conclusions and Perspectives

In this work we study the problem of competition between wireless data providers for users. We reviewed the article [8] that provided a centralized solution to the social welfare maximization problem. Based on this we proposed a distributed solution to this problem without the shortcomings of centralized solutions.

The performance evaluation presented in this article shows that our proposed solution is capable of finding small error solutions compared to the centralized algorithm. Therefore, the distributed algorithm achieves a good trade-off between performance and complexity. As perspective, we intend to study the impact of the parameters in the model.

Appendix I
Proof of no critical point of payoff function

Our objective is to show that the payoff function has not a global maximum. First, we have to analyse where the function defined in Equation (4) has critical points. The first derivative of v_i $\forall j \in \mathcal{J}$ is

$$\frac{\partial v_i}{\partial q_{i1}} = \frac{a_i c_{i1}}{1+\sum_{j=1}^{J} c_{ij} q_{ij}} - p_1 \tag{8}$$

$$\vdots$$

$$\frac{\partial v_i}{\partial q_{iJ}} = \frac{a_i c_{iJ}}{1+\sum_{j=1}^{J} c_{ij} q_{ij}} - p_J \tag{9}$$

Critical points can be found at the points where the first derivative is zero that give us

$$\sum_{j=1}^{J} c_{ij} q_{ij} = \frac{a_i c_{i1}}{p_1} - 1 \tag{10}$$

$$\vdots$$

$$\sum_{j=1}^{J} c_{ij} q_{ij} = \frac{a_i c_{iJ}}{p_J} - 1 \tag{11}$$

In order to allow critical points we have to guarantee the equality of all right-hand terms of Equations (10)-(11), i.e,

$$\frac{c_{i1}}{p_1} = \ldots = \frac{c_{iJ}}{p_J}. \tag{12}$$

However, the condition in (12) is unlikely according to the assumed model. The Fermat Theorem says that every local maximum or minimum point has the first derivative equals to zero in all coordinates, if it exists. Therefore, the function in (4) has neither critical points nor local maximum points.

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