On the Error Performance of Precoded Filterbank Multicarrier Systems Transmitting Through Highly Frequency Selective Channels

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***Abstract*—When linear precoding is applied to filterbank multicarrier systems, a combination of all subcarriers is transmitted, minimizing the effect of a deep fade in a subcarrier while also reducing frequency offset sensibility and the peak-to-average power ratio. However, their equalization becomes even more challenging than usual. In this paper, we present an analysis on the effect of multitap subchannel equalizers in precoded filterbank multicarrier systems transmitting through highly frequency selective channels. It is possible to see that subchannel equalizers employing more than three taps allow for a negligible increase in the error performance.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) systems were proposed to simplify the equalization of channels with long impulse responses. However, its subchannels are separated by a rectangular window, whose spectral characteristics are poor. They also require a cyclic prefix to eliminate the intersymbol interference (ISI) and post-filtering to conform to a spectral mask. Filterbank Multicarrier (FBMC) systems replace the rectangular window by a filter well conformed in time and frequency, eliminating the cyclic prefix and gaining spectral efficiency. This cyclic prefix elimination, even with the optimization of the subchannel pulse, means that residual ISI is often present in FBMC systems. To completely eliminate it, subchannel equalizers with more than one tap may be necessary. Examples of the effect of the multitap subchannel equalizers in regular FBMC systems can be found in [1], [2].

In these multicarrier systems, linear precoding is applied to overcome the effect that a deep fade in one or more subchannels can cause in the error performance, through the transmission of a combination of all subcarriers. They also have a higher resistance to carrier frequency offset (CFO) and lower peak to average power ratio (PAPR) when compared to regular OFDM systems [3].

Due to the demodulation of a combination of the signal transmitted in all subcarriers, equalization in precoded FBMC systems can be even more challenging than usual. This paper presents a study on the effect of multitap subchannel equalizers in precoded filterbank multicarrier systems transmitting through highly frequency selective channels. It is divided as follows. Section II shows a brief introduction to filterbank multicarrier systems. Precoded FBMC systems are presented in Section III, and the subchannel equalizers based on the geometric interpolation and the IFFT in Section IV. Simulation results are presented in Section V and the concluding remarks in Section VI.

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II. Filterbank Multicarrier Systems

A computationally efficient way to implement a large number of well conformed in time and frequency filters to separate the subchannels is the usage of a filterbank, which can be implemented through the filters' polyphasic decomposition associated with the fast Fourier transform (FFT). This way, the synthesis (SFB) and the analysis (AFB) filterbanks implement the systems' modulator and the demodulator, respectively. The synthesis filterbank associates the polyphase network (PPN), which is the set of the so-called prototype filters, with the inverse fast Fourier transform (IFFT) to divide the bandwidth in M subchannels for transmission. The inverse operation in reversed order is realized in the receiver by the analysis filterbank to recover the transmitted data. However, the usage of these efficient filters to separate the subchannels instead of the rectangular window imposes the use of a real modulation to maintain the orthogonality between subcarriers, since the transmultiplexer's impulse response is non-unitary due to interference from the neighboring subchannels and time instants. Thus, the FBMC system has to transmit a real symbol every half OFDM symbol duration, yielding to the so-called FBMC/OQAM system.

The baseband discrete signal at a time instant k at the output of a synthesis filterbank in a FBMC/OQAM (Offset Quadrature Amplitude Modulation) system is expressed by

$$s[k] = \sum_{m=0}^{M-1} \sum_{n} a_{m,n} q_{m,n}[k], \tag{1}$$

where $a_{m,n}$ are real OQAM symbols with energy E_s and

$$q_{m,n}[k] = q\left[k - n\frac{M}{2}\right] e^{-j\frac{2\pi}{M}n(k-\frac{L-1}{2})} e^{-j\phi_{m,n}}, \tag{2}$$

with $q[k]$ being the impulse response of a real and symmetric prototype filter (with unit energy and length L), M the number of subchannels, m the subchannel index, n the time index for the OQAM symbol and $\phi_{m,n} = \phi_0 + \frac{\pi}{2}(m+n) \pmod{\pi}$, with

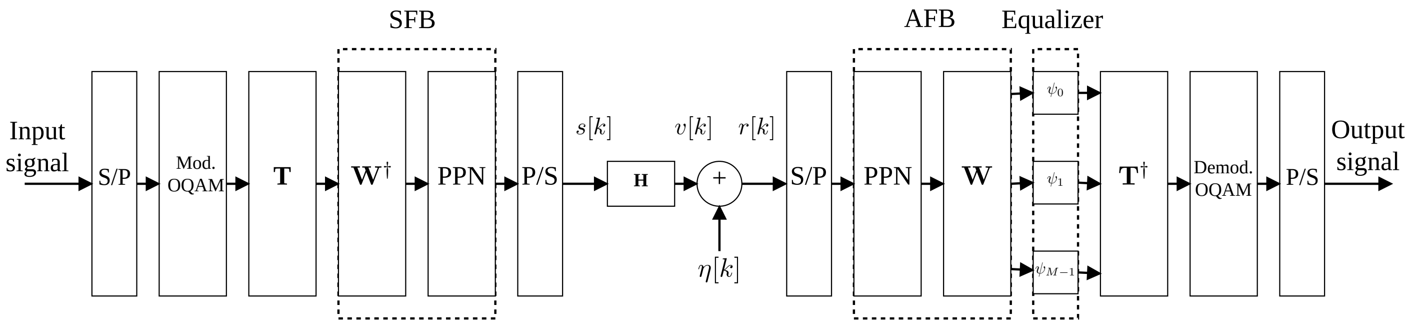

Fig. 1: A precoded FBMC/OQAM system

a arbitrary ϕ_0. The filter length L can be expressed by KM, where K is the overlapping factor [4].

The filters $q_{m,n}$ are only orthogonal in the real field, that is

$$\Re\left\{\sum_k q_{m,n}[k]q^*_{m',n'}[k]\right\} = \delta_{m,m'}\delta_{n,n'}, \tag{3}$$

where $\delta_{i,j}$ is the Kronecker delta (1 if $i = j$ and 0 otherwise). This condition implies that, even in a transmission with a perfectly equalized channel and no channel noise, there will be residual imaginary intercarrier/intersymbol intereference (ICI/ISI).

In OFDM/QAM systems, a cyclic prefix with length greater than the channel impulse response guarantees that a one-tap per subchannel equalizer will completely eliminate the ISI. Since the FBMC/OQAM systems do not use the cyclic prefix, if the transmission channel is highly selective in frequency equalizers with multiple taps per subchannel may be necessary. These equalizers are fractionally spaced (because the system operates at twice the symbol rate) and can deal with residual misalignments and severe channel impairments [5].

III. Precoded Filterbank Multicarrier Systems

If the transmitter has knowledge of the channel state information in a multicarrier system, adaptive coding and modulation can be employed to transmit more data on the subcarriers with the highest SNR, discarding the ones who are not fit to transmit data with a low error probability. However, this knowledge can be hard to acquire precisely, since the channel in a wireless environment changes rapidly. For the situations when the transmitter does not have the channel state information (CSI), linear precoding [6], [7] can be employed to minimize the effect that a deep fade in one of the subchannels can have on the overall bit error ratio. Precoded systems also have lower peak-to-average power ratio (PAPR) and lower sensibility for (sub)carrier frequency offset [8].

Figure 1 presents a precoded FBMC system. In this system, symbols being drawn from an OQAM alphabet are precoded by a matrix $\mathbf{T}$ that satisfies the following condition:

$$|t_{n,l}| = \frac{1}{\sqrt{N}}, 0 \leq n, l \leq N-1, \tag{4}$$

where $t_{n,l}$ is the n,l-th element of the matrix $\mathbf{T}$. The Hadamard matrix and the Discrete Fourier Transform one, among other unitary matrixes, comply with this condition. After the precoding operation, the symbols are modulated by a synthesis filterbank. The transmitted symbols are corrupted by a fading channel with an impulse response $\mathbf{h}[k]$ and the corresponding channel frequency response matrix $\mathbf{H}$, with H_0, H_1, …, H_{M-1} (the channel frequency response in the center of each subchannel) being the diagonal elements of this matrix. White gaussian noise $\eta[k]$ with variance σ^2_η is also added to the transmitted signal. The signal at the input of the receiver is demodulated by the analysis filterbank. At this stage, a subchannel equalizer (which can have one or more taps) compensates the channel effect on the two times oversampled signal. Finally, after downsampling and deprecoding, the decision on the symbol is realized.

In precoded filterbank multicarrier systems, which transmit a combination of all subcarriers, the use of minimum mean square equalization (MMSE) is preferred to the zero-forcing (ZF) based one due to the fact that, unlike ZF equalization, the MMSE-based one does not amplify the noise combination. With MMSE equalization, these systems can also have a higher diversity order; in contrast, systems employing ZF equalization can only have a maximal diversity order of one [9]. In this paper, we focus on designing the MMSE subchannel equalizer based on geometric interpolation.

IV. Geometric Interpolation-based Subchannel Equalizers

In the following, we assume that a noisy estimate $H(i)$ of the channel at the frequencies i is known. Then, for the one-tap equalizer case, the equalizer coefficient can be computed through the minimum mean-square error (MMSE) criterion as

$$EQ(i) = \frac{H^*(i)}{|H(i)|^2 + \sigma^2_n}, \tag{5}$$

where i is the sub-channel index.

For the 3-tap equalization of the i-th sub-channel we need three points of the inverse sub-channel frequency response: EQ(i) and two intermediate points EQ1 and EQ2, as shown in Figure 2a. The same procedure is realized for the 5-tap and 7-tap equalization, dividing equally the frequency interval between EQ(i-1) and EQ(i+1), as shown in Figures 2b and 2c.

A. Geometric Interpolation

The objective of the equalizer is to compensate channel distortions and timing offset, which introduce a possibly large linear component in the phase characteristic. Therefore, it is

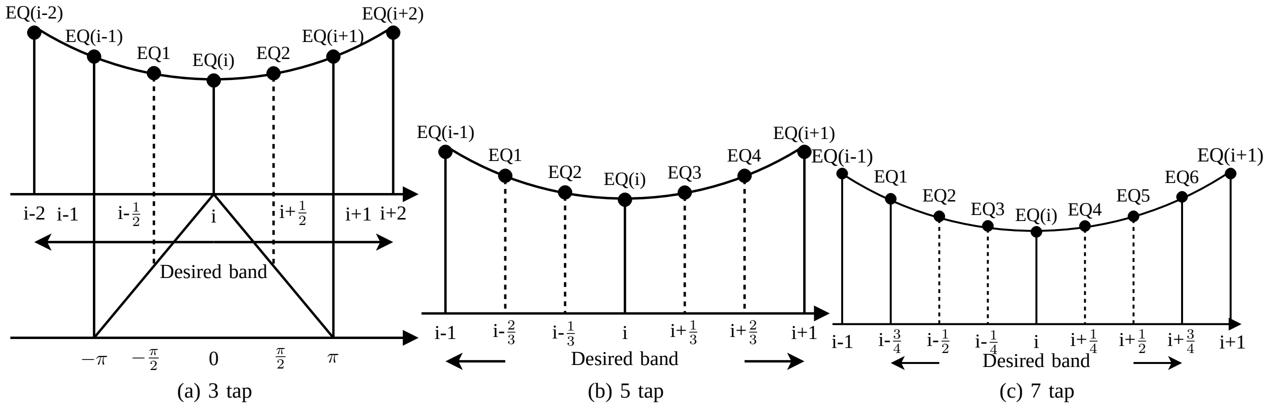

Fig. 2: Inverse subchannel frequency response values used to design the equalizers

appropriate to carry out the phase and magnitude interpolation separately and linear techniques can be applied. However, a simple approach to carry out both interpolations jointly is to use geometric interpolation, which consists of computing the geometric mean of two known values. It is a linear interpolation of the phase and it is close to linear for amplitude, if the known amplitude values are close.

In the present case, an intermediate sample of the equalizer frequency response is given by

$$EQ_i = X_i \left(X_{i\pm1}/X_i\right)^{\rho}, \tag{6}$$

where X_i is a known point of the inverse sub-channel frequency response, for example, EQ(i-1), EQ(i) or EQ(i+1) in the Figure 2, $X_{i\pm1}$ are the adjacent known sub-channels and ρ is the fraction of interpolation [10]. For 3-tap and 5-tap equalization, ρ is always 1/2 and 1/3, respectively. For 7-tap equalization, ρ is 1/2 for EQ2 and EQ5 and 1/4 for the others intermediate points. The intermediate points for 3-tap equalization are calculated by

$$EQ1 = EQ(i-1)(EQ(i)/EQ(i-1))^{1/2} \tag{7}$$

and

$$EQ2 = EQ(i)(EQ(i+1)/EQ(i))^{1/2}. \tag{8}$$

For the 5-tap equalization, we find

$$EQ1 = EQ(i-1)(EQ(i)/EQ(i-1))^{1/3}, \tag{9}$$

$$EQ2 = EQ(i)(EQ(i-1)/EQ(i))^{1/3}, \tag{10}$$

$$EQ3 = EQ(i)(EQ(i+1)/EQ(i))^{1/3}, \tag{11}$$

$$EQ4 = EQ(i+1)(EQ(i)/EQ(i+1))^{1/3}. \tag{12}$$

Similar equations can be found for the 7-tap equalizer. These equations are valid for any absolute phase value, and phase differences of $\pm\pi$. For the quasi-flat sub-channels, the values of EQ(i-1), EQ(i) and EQ(i+1) are very similar. In these cases, the magnitude of EQ2 in the equations (8) and (10), e. g., can be approximated by

$$|EQ2| \approx (|EQ(i)| + |EQ(i+1)|)/2 \tag{13}$$

and

$$|EQ2| \approx (2|EQ(i)| + |EQ(i+1)|)/2, \tag{14}$$

respectively. Therefore, the magnitude is quasi-linearly interpolated. Evidently, for a flat channel both magnitude and phase must be linearly interpolated.

B. Coefficient Computation using IFFT

The use of FFT and IFFT for frequency sampling is a classical technique applied in several applications in digital signal processing. Here, we will use the IFFT to calculate the equalizer coefficients of each sub-channel. In the following, we will discuss only the design for 3-tap FIR equalizer. The design for 5-tap and 7-tap FIR equalizers can be done following the same procedure.

The frequency points $EQ(i \pm n), n = 0, 1, 2...$ are obtained using a frequency step δf, equal to the sub-channel spacing. For 3-tap FIR equalization, we use one frequency point which is located at the center of i-th sub-channel band (ω = 0), and two intermediate frequencies located at $\omega = \pm\frac{\pi}{2}$, as shown in Figure 2a. The equalizer coefficients are calculated so that its frequency response passes by the values EQ1, EQ(i) and EQ2. At the analysis filter bank, the sampling frequency for each sub-channel is $2\Delta f$. As the frequency spacing between EQ(i) and the intermediates points EQ1 and EQ2 is $\frac{\Delta f}{2}$, we will have $2/\frac{\Delta f}{2} = 4$ frequency points per sub-channel. So, one value is unknown. Let denote by A the unknown value. A can be found by a constraint over the equalizer coefficients calculated using the 4th-order inverse Fourier transform (IFFT) of the ensemble A, EQ1, EQ(i), EQ2. Considering the ith sub-channel, the equalizer coefficients obtained after the IFFT is $q_{0i}, q_{1i}, q_{2i}, q_{3i}$. The central coefficient q_{0i} is surrounded by the coefficients $q_{-1i} = q_{3i}$ and q_{1i}, due to the IFFT periodicity property. Considering we want to design a 3-tap equalizer, it is necessary to null the coefficient q_{2i}, thus we can determine the unknown A by the relation

$$q_{2i} = (A - EQ1 + EQ(i) - EQ2)/4. \tag{15}$$

Solving for the other coefficients, we get

$$q_{-1i} = \pm((EQ1 - 2EQ(i) + EQ2) + j(EQ2 - EQ1))/4, \tag{16}$$

$$q_{0i} = (EQ1 + EQ2)/2, \tag{17}$$

$$q_{1i} = \pm((EQ1 - 2EQ(i) + EQ2) + j(EQ2 - EQ1))/4. \tag{18}$$

TABLE I: Simulation parameters

Constellation	OQPSK
Sampling Frequency	10 MHz
Carrier Frequency	2,5 GHz
Number of channel realizations	20000
Block size	53 FBMC symbols
Precoding Matrix	DFT

Here the sign - and + correspond to the even and odd subchannels, respectively. Note that these equations are equivalent to those presented in [11].

V. Simulation Results

Simulation results showing the effect of the geometric interpolation-based equalizers in precoded filterbank multicarrier systems are presented in this section. The parameters used in the following simulations are presented in Table I. Channel estimation at the receiver is considered to be perfect and channel fading is considered to be quasistatic (time-invariant during each transmitted frame). For the filter banks, the prototype filter presented in [4] is used. Figure 3 presents the simulation results for 64 subcarriers and the ITU-T Vehicular A channel model. For this scenario, the channel is highly frequency selective (for example, if regular OFDM systems were used, the appropriate cyclic prefix size to completely eliminate the ISI would be 1/2). It is possible to see that even subchannel equalizers with a higher number of taps are not able to compensate effectively the channel selectivity, leading to residual ISI and an error floor at a bit error rate (BER) of about 10^{-3}.

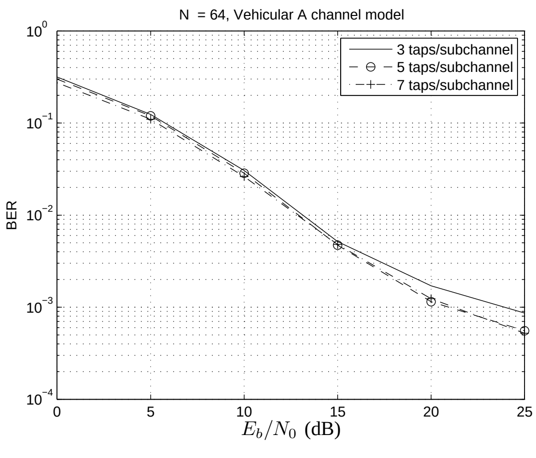

Fig. 3: Error performance for N = 64 and the Vehicular A channel model

For 512 subcarriers and the Vehicular B channel model, Figure 4 presents the simulation results. In this scenario, the channel selectivity is similar to the one presented in Figure 3 (a cyclic prefix size of 1/2 would be needed too), and the subchannel equalizers cannot eliminate all the ISI.

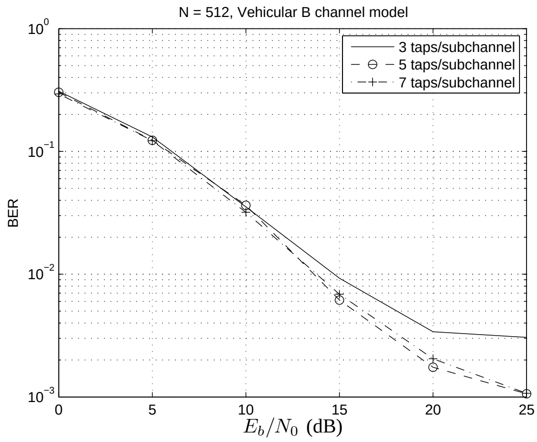

Fig. 4: Error performance for N = 512 and the Vehicular B channel model

VI. Concluding Remarks

We have presented in this paper a study on the effect of the number of taps of the subchannel equalizer in the error performance of precoded FBMC systems transmitting through channels with severely fading characteristics. It is possible to see that subchannel equalizers with more than three taps do not bring a performance improvement in this case. Therefore, new equalizer structures should be proposed to overcome the channel effect and compensate the residual ISI.

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