Influence of Channel Frequency Selectivity and Modulation Order on the Performance Difference Between DFE-SCCP and OFDM

Cristiano Panazio e Amanda de Paula

***Resumo*— Este artigo apresenta uma comparação de desempenho entre a multiplexação por divisão de frequências ortogonais (OFDM) e a modulação por portadora única implementada com prefixo cíclico e equalizador por decisão realimentada (DFE-SCCP) através de uma análise de capacidade de canal. É proposta uma abordagem semi-analítica ao problema que é verificada por meio de simulações com diferentes modulações, canais e taxas de codificação.**

***Palavras-Chave*— multiplexação por divisão de frequências ortogonais, modulação por portadora única, equalização com decisão realimentada, capacidade de canal, codificação de canal.**

***Abstract*— This article provides a performance comparison between the orthogonal frequency division multiplexing (OFDM) and the single-carrier with cyclic prefix with a decision feedback equalizer (DFE-SCCP) through a channel capacity analysis. A semi-analytical approach to the problem is presented and they are assessed by simulation results for different modulations, channels and coding rates.**

***Keywords*— orthogonal frequency division multiplexing, single-carrier modulation, decision feedback equalization, channel capacity, channel coding.**

I. INTRODUCTION

It is known that orthogonal frequency division multiplexing (OFDM) [1], without channel knowledge at the transmitter side, depends on channel coding to achieve good performance by exploiting channel frequency diversity [2], [3], since it relies on non-faded subcarriers to recover the information carried on attenuated subcarriers. By contrast, the single-carrier is able to exploit such diversity even in the absence of channel coding, since each transmitted symbol spreads throughout the entire used band due to its much smaller symbol duration compared to the OFDM. Furthermore, the cyclic-prefix (CP) and the one-tap equalizer techniques [4], which allow low complexity equalization, are not a privilege of the OFDM and they can also be applied to the single-carrier modulation, giving rise to the so-called single-carrier with cyclic-prefix (SCCP) scheme. Besides linear equalization, implemented by the one-tap equalizer, the SCCP can resort to non-linear equalization techniques that can provide additional performance in frequency-selective channels. One of these techniques is the decision-feedback equalizer (DFE) [5].

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The comparison between both techniques using channel coding is not so immediate, since the characteristics of the received signal differ significatively and may lead to different performances when used in the same channel. In addition to Gaussian noise, the received signal of the SCCP is corrupted by intersymbol interference (ISI), whereas the OFDM signal is just a rotated and scaled version of the transmitted signal plus Gaussian noise. Therefore, the equalizer in the SCCP has to mitigate ISI by using, for example, a minimum mean square error (MMSE) criterion, and the one-tap equalizer in the OFDM has just to provide gain and phase corrections, without any signal-to-noise ratio (SNR) loss at each subcarrier. Furthermore, since each subcarrier is subject to a different gain, in the OFDM the signal that goes to the channel decoder is subject to different SNRs, which is analogous to a time-varying flat fading channel, while the DFE-SCCP averages the signal over all subcarriers. Since the same error correcting code performs differently in both cases, we should expect some performance differences. Therefore, a natural question arises: how do their performances compare?

Some papers have deal with this comparison. For instance, [2], [6] and [7] are restricted to Monte Carlo simulations that naturally hinders the extent of any conclusion, and also do not provide comprehensive scenarios. For instance, [2] and [7] are restricted to just one kind of modulation, such as binary phase-shift keying or quadrature phase-shift keying (QPSK) and finally, they fail to generalize the comparison to any channel configuration. Other comparisons establish an analytical treatment to the problem, but their results are not applied to systems using channel coding [8]–[11]. Concerning the comparison with analytical results under a coded context, [12] provides an interesting result using the cutoff rate [13], but it solely analyzes the effect of the coding rate considering linear equalization for the SCCP and a two-tap block-fading channel configuration scenario. In [3], it is shown that both schemes, when using channel coding, can exploit the frequency diversity on frequency selective block-fading channels, but there are no results on possible coding gains differences.

In this paper, using the Shannon channel capacity [14], we show how both schemes behave for different channels and what is the influence of the modulation cardinality in their capacities. Next, we assess the bit-error rate (BER) performance for some didactic channels and for different modulations and coding schemes through simulations, which corroborate and help further understand the analytical results.

This paper is organized as follows. In Section II, we describe the system model. In Section III, the comparison between the schemes is done through the use of the channel capacity for different modulation cardinalities and different channels. Simulation results are shown in Section IV. Finally, conclusions are stated in Section V.

II. System Model

The use of linear precoding allows us to describe both SCCP and OFDM under a unified framework [8], depicted in Fig. 1.

The linear precoding is performed by the matrix $\mathcal{P}$. For the OFDM, $\mathcal{P} = \mathbf{I}$, *i.e.*, the identity matrix, since the transmitted symbols are obtained from the inverse fast Fourier transform (IFFT) of the data vector $\mathbf{X}$. On the other hand, in the SCCP scheme, the vector of symbols itself is transmitted and, hence, the precoding matrix $\mathcal{P}$ is the Fourier matrix.

After the IFFT, a CP is inserted and that allows us to equalize the received signal in the frequency domain with a simple one-tap equalizer, since it eliminates the interblock interference and, in time-invariant channels, it keeps the orthogonality between the subcarriers. In the OFDM, where there is no ISI, the equalization is reduced to phase and magnitude compensations in each subcarrier by the one-tap equalizer, without introducing any performance penalty for the signal decoding process.

By contrast, the DFE-SCCP, besides the frequency domain one-tap equalizer that implements the feedforward filter, also requires a feedback filter, as seen in Fig. 1, that is implemented in the time-domain. Another important aspect of the DFE-SCCP is that the equalization criterion plays an important role due to a compromise between ISI removal and SNR maximization. A good compromise that will be adopted in this paper is to use the MMSE criterion. An efficient way to calculate the feedforward and feedback filter coefficients is described in [15]. It is worth noting that a gain is applied to the feedforward and feedback coefficients to eliminate the MMSE solution bias when quadrature amplitude modulation (QAM) is used. Also, with regard to the MMSE solution, the number of feedback coefficients that maximizes the performance is equal to the channel length minus one [16].

In order to make analysis more feasible, we do not take into account the error propagation effect, and the DFE-SCCP is said to be ideal. However, in addition, the DFE-SCCP with error propagation is simulated to provide more realistic results.

The DFE-SCCP brings a problem in the initialization of the feedback filter, since it requires the access to the last symbols of the block, which have not yet been processed. An alternative to overcome this issue is to implement the DFE with the unique word (UW) technique [17]. With such approach, we can have the same rate or efficiency than the one obtained with the CP and same signal-to-noise ratio at the equalizer output, but a larger fast Fourier transform (FFT) is required. If the FFT length is the same for both schemes, the rates are different. However, if the block size is large compared to the CP or UW lengths, the difference between both techniques is not significative. Hence, in order to simplify the analysis, we will consider the DFE-SCCP and assume that the last symbols of the block are known and they are used to initialize the feedback filter.

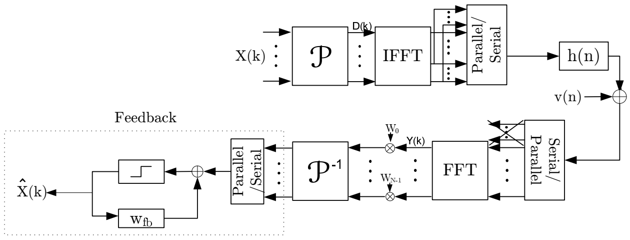

Fig. 1. Unified model, where the feedback filter is appended to implement a decision feedback equalizer for the SCCP.

For all simulations, we have used $N = 512$ subcarriers in the OFDM system, as well considered $N = 512$ symbols in the SCCP block. Random interleaving was used for both schemes and perfect channel estimation was considered. All channels have been normalized to have unitary norm.

III. Capacity Analysis

For the OFDM, the SNR for each subcarrier is given by:

$$\mathrm{SNR}(k) = \gamma \left| H_k \right|^2 \tag{1}$$

where γ is the average SNR, $\gamma = \frac{\sigma_X^2}{\sigma_v^2}$, σ_X^2 is the average symbol power, σ_v^2 is the noise variance, H_k is the k^{th}-subcarrier frequency response component of a unitary norm channel for a N-point discrete Fourier transform. The coded message is interleaved and modulated to the symbols $X(k)$ and, hence, the capacity of the OFDM is the average capacity of the subcarriers [18]:

$$C_{\mathrm{OFDM}} = \frac{1}{N} \sum_{k=0}^{N-1} \frac{1}{2} \log_2 \left(1 + \gamma \left| H_k \right|^2 \right) \tag{2}$$

Now, for the DFE-SCCP, let us consider that the residual ISI is Gaussian. Thence, the DFE-SCCP capacity is:

$$C_{\mathrm{DFE}} = \frac{1}{2} \log_2 \left(1 + SNR_{\mathrm{DFE}} \right) \tag{3}$$

where $\mathrm{SNR}_{\mathrm{DFE}}$ is the SNR at the equalizer output and is given by [19]:

$$\mathrm{SNR}_{\mathrm{DFE}} = \exp \left\{ \frac{1}{N} \sum_{k=0}^{N-1} \log \left(1 + \gamma \left| H_k \right|^2 \right) \right\} - 1 \tag{4}$$

which results in:

$$C_{\mathrm{DFE}} = \frac{1}{N} \sum_{k=0}^{N-1} \frac{1}{2} \log_2 \left(1 + \gamma \left| H_k \right|^2 \right) \tag{5}$$

Therefore, the OFDM and the DFE-SCCP systems present the same capacity.

However, such analysis consider that Gaussian symbols are transmitted, which is not the case in practice. For ordinary modulations, such as M-QAM, the capacity can still be numerically evaluated using [14]:

$$C^{\text{M-QAM}} (\mathrm{SNR}) = - \int_{-\infty}^{+\infty} f_Y(y) \log_2 \left(f_Y(y) \right) dy - \log_2 \left(\frac{2\pi e}{\mathrm{SNR}} \right)^{\frac{1}{2}} \tag{6}$$

where $f_Y(y)$ is the probability density function associated to the received symbol.

Equation (6) has a shape similar to the Gaussian capacity and for $M \to \infty$, with $SNR \to \infty$, it presents a performance gap known as the shaping gain that is equal to 1.53 dB [20]. However, when M is finite, it will saturate in $\log_2(M)$. This saturation is the reason that the OFDM will present a capacity degradation when compared to the DFE-SCCP, for certain average SNR values.

In order to show this, let us consider that the capacity of the DFE-SCCP system can be evaluated applying the $\mathrm{SNR_{DFE}}$ in (6):

$$C_{\mathrm{DFE}}^{\text{M-QAM}} = C^{\text{M-QAM}}\left(\mathrm{SNR_{DFE}}\right) \tag{7}$$

On the other hand, the OFDM capacity is the average of the capacities in the different subcarriers:

$$C_{\mathrm{OFDM}}^{\text{M-QAM}} = \frac{1}{N}\sum_{k=0}^{N-1} C^{\text{M-QAM}}\left(\gamma\left|H_k\right|^2\right) \tag{8}$$

Consider first that we have $\gamma \to \infty$, then (7) and (8) converge to $\log_2(M)$. If γ is small enough to have $C^{\text{M-QAM}}(\gamma\left|H_k\right|^2) \approx C^{\text{M'-QAM}}(\gamma\left|H_k\right|^2)$, with $M' \gg M$, $\forall k$ then the capacities are also equal. However, if $C^{\text{M-QAM}}(\gamma\left|H_k\right|^2)$ falls close to the saturation region of (6) for certain values of k, then the capacity of the OFDM will be inferior to that of the DFE-SCCP. As an example, we will calculate the OFDM and DFE-SCCP capacities for the unitary norm channel with zeros in $0.95\exp\left(\pm j0.9\pi\right)$, considering 16-QAM modulation and $N = 8$ subcarriers or symbols for γ equal to 6 dB and 11 dB. The results are depicted in Figs. 2 and 3. Also, in addition to the 16-QAM, the 64-QAM capacity curve is also shown. As we can see in Fig. 2, the capacities are almost the same for both schemes, even for the 16-QAM. However, when using the same modulation for a higher value of γ, as shown in Fig. 3, certain values of $\gamma\left|H_k\right|^2$ fall close to the saturation region generated by (6) which clearly leads to a capacity difference between the OFDM and the DFE-SCCP. Additionally, we also show in Fig. 3 that for 64-QAM and the same γ that both schemes have the same capacity, since for such modulation, the values of $\gamma\left|H_k\right|^2$ are off the saturation region again.

From the results discussed in the previous paragraph, we can predict that larger deviations of $\left|H_k\right|^2$ will be more prone to create differences between the capacities of the OFDM and the DFE-SCCP, which will evidently reflect on the BER of both schemes. One way to prevent such differences is to choose wisely the modulation and coding schemes. In the next section, we show through simulations some of cases to illustrate these characteristics.

IV. Simulation Results

In this section, we run some simulations that show the behavior predicted by the analysis in the previous section. First, we use 16-QAM modulation and a channel with zeros at $\exp(\pm j0.22\pi)$. This channel has zeros over the unitary circle generating deep fades and large gains that will spread $\gamma\left|H_k\right|^2$ values and will create large performance difference between the OFDM and the DFE-SCCP for mild to high SNR values. We then use two convolutional coding schemes: a rate 1/2 with polynomial (133,171) and a rate 1/4 with polynomial (135,135,147,163). The former code can only provide low BER for higher values of average SNR, in which certain subcarriers of the OFDM will operate closer to or in the saturation region of (6), and the latter makes both schemes have similar performances for lower average SNRs, since (7) and (8) are approximately the same. The results are depicted in Fig. 4.

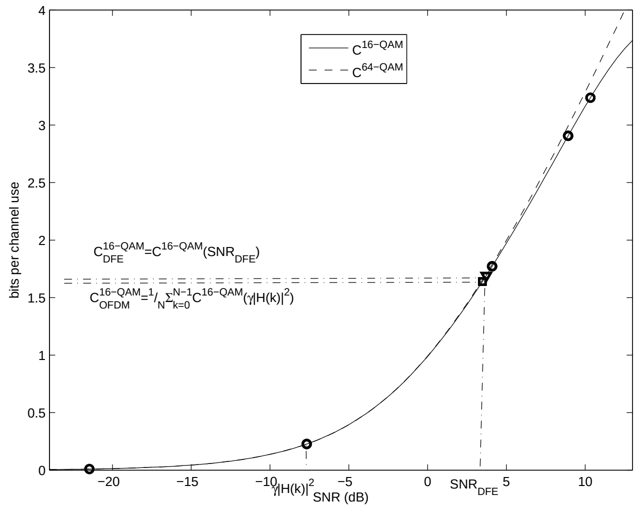

Fig. 2. Capacity for the OFDM and DFE-SCCP systems considering 16-QAM and 64-QAM for a low SNR value, equal to 6 dB.

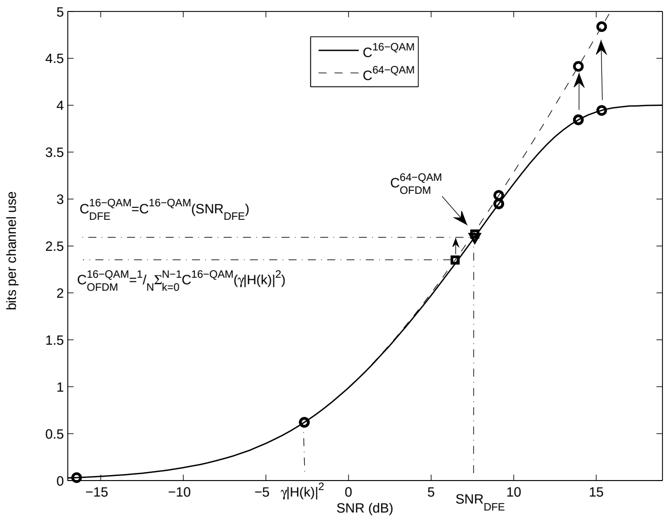

Fig. 3. Capacity for the OFDM and DFE-SCCP systems considering 16-QAM and 64-QAM for a high SNR value, equal to 11 dB.

In the following, we use QPSK modulation and the convolutional code (15,17) for three different channels. The first channel is the channel with zeros at $0.5\exp(\pm j0.5\pi)$. The second one presents two complex conjugates zeros in the unitary circle: $\exp(\pm j0.22\pi)$. The third channel presents three zeros at $\exp(j0.33\pi)$ and other three zeros at $\exp(-j0.33\pi)$. The first channel produces a small dispersion of $\left|H_k\right|^2$ and the capacity calculation for both schemes results in almost the same values for a large range of SNRs. The second and

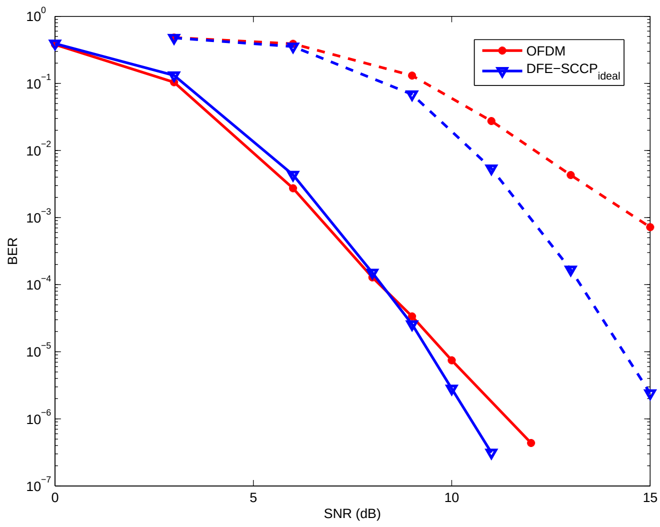

Fig. 4. BER performance versus average SNR for 16-QAM. Solid lines (–) are for the convolutional code with polynomial (135,135,147,163) and the dashed lines (- -) are for the convolutional code with polynomial (133,171).

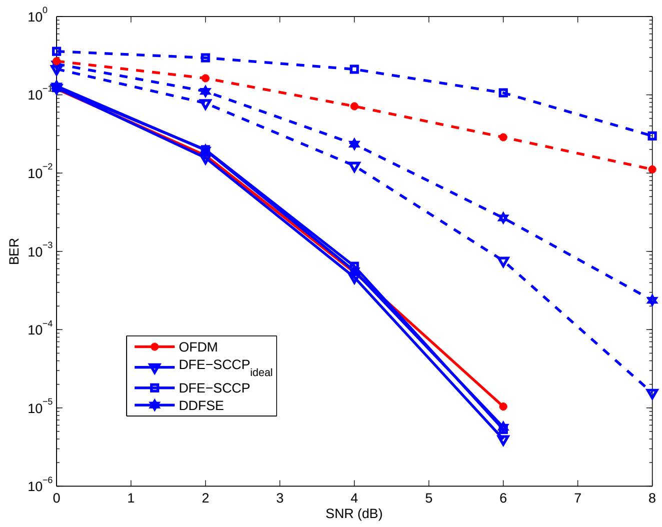

Fig. 5. BER performance versus average SNR for QPSK. Solid lines (–) are for the channel with zeros at $0.5\exp(\pm j0.5\pi)$ and the dashed lines (- -) are for the channel with zeros at $\exp(\pm j0.22\pi)$. The convolutional code for both channels has the polynomial (15,17)

third channels generate large dispersions of $|H_k|^2$ and should lead to larger performance differences. Particularly, the third channel, due to broader deep fades, will generate the largest performance differences. We also show the performance of the DFE-SCCP considering error propagation and a technique that can mitigate error propagation based on the delayed decision feedback sequence estimator (DDFSE) [21]. The results are shown in Fig. 5 for the first two channels and in Fig. 6 for the third channel. In Fig. 5, the less frequency-selective channel results in the same performance for all techniques, except that the OFDM starts to show a small performance loss for a SNR equal to 6 dB. For the second channel, the ideal DFE-SCCP is much better than the OFDM, since this channel provides a large deviation of $|H_k|^2$. Due to the large coefficients present in the feedback filter, the DFE-SCCP suffers a large performance degradation from error propagation. It is worth noting that the error burst generated by the error propagation, even if an interleaver is present, generates a large impact on coded systems [22]. The DDFSE technique provides a performance better than the OFDM and closer to the ideal DFE-SCCP, but with additional complexity. Finally, the third channel provides an extreme case with a very large spread of $|H_k|^2$. In this case, the OFDM fails to provide acceptable performance. The DFE-SCCP is similar to the OFDM, but will outperform it for higher SNR values. The ideal DFE-SCCP largely surpasses the OFDM and the DDFSE follows closely.

V. CONCLUSIONS

We have shown what kind of channels will or will not generate performance differences between the OFDM and the ideal DFE-SCCP when considering channel coding for both schemes. It is not surprising that channels with deep frequency fades are detrimental to OFDM, but not all frequency-selective channels are equal. Channels with broad deep fades will also generate large gains that will create a performance difference between the OFDM and the ideal DFE-SCCP. On the other

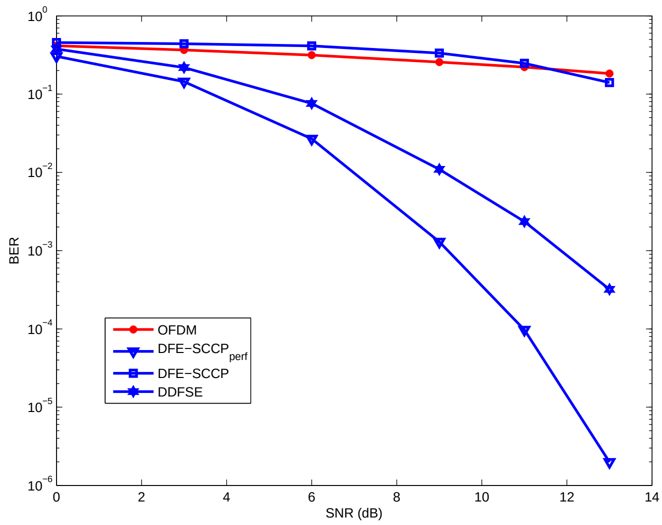

Fig. 6. BER performance versus average SNR for QPSK, a channel with three zeros at $\exp(j0.33\pi)$ and three zeros at $\exp(-j0.33\pi)$ and a convolutional code with polynomial (15,17).

hand, channels that are less frequency selective will result in small differences if any at all. From the results, we can also identify that robust OFDM implementations will use larger modulation cardinalities and low coding rates in order to make it have BER performances close to the ideal DFE-SCCP. It is worth noting that the ideal DFE-SCCP is not realizable and the DFE-SCCP suffers a large performance hit due to error propagation, specially in channels with deep fades that usually results in large feedback coefficients. Furthermore, there are techniques that can mitigate the DFE error propagation, but they will result in higher complexity and/or latency. In this sense, the OFDM seems to be a more reasonable solution, if the system parameter, *i.e.*, modulation and coding schemes are wisely chosen and there are no peak-to-average power ratio limitations.

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