Hybrid Local Search Polynomial-Expanded Linear Multiuser Detector for SIMO DS/CDMA Systems

Reinaldo Götz and Taufik Abrão

Abstract— **In this work, low-complexity suboptimum detectors have been analyzed for a single-input multiple-output (SIMO) direct-sequence code division multiple access (DS/CDMA) system. As a contribution, a hybrid multiuser detector based on polynomial expansion (PE-MuD) with alpha-estimation determined by Gerschgorin circles Theorem (GCT), and followed by a low-complexity local search procedure is proposed for synchronous SIMO DS/CDMA systems. The computational complexity of the PE-MuD and the LS-MuDs are expressed in terms of the total number of floating point operations in order to converge. Herein, the spatial diversity is exploited in order to improve the performance of the low-complexity multiuser detectors.**

Keywords— **Near-optimum search algorithms, polynomial-expanded multiuser detection, Gerschgorin circles, SIMO DS/CDMA.**

I. Introduction

In a multipath channel communication environment, spatial diversity can be obtained by employment of multiple antennas at the receiver, provided that the antennas are sufficiently far apart that the channel gains between different antenna pairs can be assumed to be independent [1]. This work considers the antenna-diversity-aided detection in synchronous SIMO systems and investigates the trade-off between performance gain and the complexity increasing provided by the spatial diversity scheme.

Multiuser detection algorithms usually have very high computational complexity, which greatly limits their adoption. The Decorrelator and the MMSE detectors utilize the inverse cross-correlation matrix of signature waveforms of the active users in the system $(\mathbf{R}^{-1})$ to decouple the desired user's signal and have computational complexity of $\mathcal{O}(K^3)$. However a multiple stage detection, presented in [2], approximately implements the inverse cross-correlation matrix through polynomial expansion in $\mathbf{R}$. This scheme is namely polynomial-expanded multiuser detector (PE-MuD) and can be viewed as an iterative approach in order to approximate the linear multiuser detectors with low complexity quadratic order dependence regarding the number of users, $\mathcal{O}(K^2)$.

Other concept widely adopted in this current study is the local search (LS) based on neighborhood with signal detection application. The LS detection method constitutes an optimization mechanism which implement low-complexity local search solutions into a previously established neighborhood [3], [4]. The main advantage of this method lies on its inexpensive reduced complexity.

A structure formed by the PE-based detector as the first stage followed by a local search algorithm has been presented in [5]. This structure is able to offer performance improvements under DSP implementable low-complexity perspective. In a same perspective, [6] and [7] investigate a new hybrid PE-based local search algorithm applicable to single-input single-output (SISO) DS/CDMA systems, which maintains the same convergence shape but with a smaller quantity of operations at the expense of a marginal and acceptable increasing in the bit error rate (BER). The present work considers the hybrid PE-LS-MuD in the multiple antennas environment.

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II. System Model

Herein, a discrete-time baseband system model is adopted, with transmission through a synchronous uplink single-input multiple-output channel, i.e., a single antenna at the mobile terminal (MT) transmitters and N antennas at the base-station (BS) receiver side, subjected to additive white Gaussian noise (AWGN) and flat Rayleigh fading. The same channel is simultaneously shared by K users, which operate under a synchronous DS/CDMA system with binary phase shift keying (BPSK) modulation. This is equivalent to a $K \times N$ MIMO system. In the transmission, the ith information bit generated by the kth user, at a ratio of $R_{\rm b} = 1/T_{\rm b}$ bits per second is denoted by $b_k[i] \in \{\pm 1\}$, $i = 1, 2, \ldots$. At each i bit interval, $b_k[i]$ is modulated by a spreading sequence with pseudo-noise (PN) distribution and length L. The spreading code can be represented by the vector:

$$\mathbf{s}_k[i] = \left[s_{k,1}[i], s_{k,2}[i], \ldots, s_{k,L}[i]\right]^{\rm T}, \tag{1}$$

where $s_{k,\ell}[i] \in \left\{\pm\frac{1}{\sqrt{L}}\right\}$ and herein L denoting the system's processing gain, i.e., the ratio between the bit information period and the chip period, $L = \frac{T_b}{T_c} = \frac{R_c}{R_b}$, with R_b and R_c been the bit and chip rate, respectively. $(\cdot)^{\rm T}$ denotes the matrix transposing operator.

Take into account a scheme of detection that explores the diversity gain, which is given by the utilization of N multiple antennas at the base station, the $L \times 1$ received signal vector at the ith bit interval and the nth receive antenna is:

$$\mathbf{r}_n[i] = \sum_{k=1}^{K} \mathbf{s}_k[i]\, c_{n,k}[i]\, A_k b_k[i] + \mathbf{w}_n[i], \tag{2}$$

where A_k is the amplitude of the signal transmitted by the kth user, admitted constant across the entire message; $\mathbf{w}_n[i]$ is the complex AWGN vector of the nth antenna, with mean zero, variance $\sigma_{\rm w}^2 = N_0$ and bilateral power spectral density given by $N_0/2$ W/Hz.

The term $c_{n,k}[i]$ denotes the complex coefficient of the channel inherent to the kth user, at the ith bit interval, which corresponds to received signal at the nth antenna and is perfectly known by the receiver, but not at the transmitter side. In statistical terms, $c_{n,k}[i]$ may be represented by a circularly symmetric complex Gaussian random variable, with mean zero and variance $\sigma_{\rm c}^2$, in the form $\mathcal{CN}\left(0, \sigma_{\rm c}^2\right)$. In the polar form, the channel's complex coefficient is described by:

$$c_{n,k}[i] = |c_{n,k}[i]|\, e^{j\theta_{n,k}[i]}, \tag{3}$$

where phase $\theta_{n,k}[i]$ is uniform over the range $[0, 2\pi)$, i.e., omnidirectional BS receive antenna, and independent of the magnitude $|c_{n,k}[i]|$, whose probability density function is given by Rayleigh, $f(r) = \frac{r}{\sigma_c^2} e^{\left(-r^2/2\sigma_c^2\right)}, \quad r \geq 0.$

In the notation of matrices, with bold capital letters representing matrices and bold lower case letters representing vectors, and suppressing the bit interval index i for the sake of convenience, Eq. (2) could be rewritten as follows:

$$\mathbf{r}_n = \mathbf{S}\mathbf{C}_n\mathbf{A}\mathbf{b} + \mathbf{w}_n, \tag{4}$$

with $\mathbf{A} = \mathrm{diag}(A_1, A_2, \ldots, A_K)$ being the diagonal matrix of the amplitudes of the transmitted signals, $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$ being the $L \times K$ spreading code matrix, and $\mathbf{b} = [b_1, b_2, \ldots, b_K]^{\mathrm{T}}$ the bit information vector transmitted by the K users. $\mathbf{C}_n = \mathrm{diag}(\mathbf{c}_n)$ is the channel complex coefficients matrix, where $\mathbf{c}_n = \left[c_{n,1}, c_{n,2}, \ldots, c_{n,K}\right]^{\mathrm{T}}$ in which $c_{n,k} = |c_{n,k}| \cdot e^{j\theta_{n,k}}$. Vector $\mathbf{w}_n = \left[w_{n,1}, w_{n,2}, \ldots, w_{n,L}\right]^{\mathrm{T}}$ represents the complex noise with distribution $\mathcal{N}\left(0, \sigma_{\mathrm{w}}^2\right)$. Conventional matched filters bank (MFB), also called the Correlator filter, is defined by the signature waveform at the first stage of the receiver:

$$\begin{aligned} \mathbf{y}_n^{\text{MFB}} &= \mathbf{S}^{\mathrm{T}}\mathbf{r}_n \\ &= \mathbf{S}^{\mathrm{T}}\mathbf{S}\mathbf{C}_n\mathbf{A}\mathbf{b} + \mathbf{S}^{\mathrm{T}}\mathbf{w}_n \\ &= \mathbf{R}\mathbf{C}_n\mathbf{A}\mathbf{b} + \widetilde{\mathbf{w}}_n, \end{aligned} \tag{5}$$

where $\mathbf{y}_n^{\text{MFB}} = \left[y_{n,1}^{\text{MFB}}, y_{n,2}^{\text{MFB}}, \ldots, y_{n,K}^{\text{MFB}}\right]^{\mathrm{T}}$ is the $K \times 1$ MFB output information vector; the cross-correlation matrix of the signature waveforms is obtained via $\mathbf{R} = \mathbf{S}^{\mathrm{T}}\mathbf{S}$; vector $\widetilde{\mathbf{w}}_n = \mathbf{S}^{\mathrm{T}}\mathbf{w}_n$ corresponds to the filtered noise with variance $\sigma_{\mathrm{w}}^2\mathbf{R}$.

Let $\mathbf{p}_n = \left[e^{j\theta_{n,1}}, e^{j\theta_{n,2}}, \ldots, e^{j\theta_{n,K}}\right]^{\mathrm{T}}$ be the vector of phases and $\mathbf{M}_n = \mathrm{diag}(|c_{n,1}|, |c_{n,2}|, \ldots, |c_{n,K}|)$ be the diagonal matrix of magnitudes of the complex channel, so that $\mathbf{c}_n = \mathbf{M}_n\mathbf{p}_n$. The channel phase effect can be mitigated by applying the conjugate of the channel phases' vector after the Correlator filter:

$$\mathbf{z}_n^{\text{MFB}} = \mathbf{R}\mathbf{C}_n\mathbf{A}\mathbf{b} \odot \mathbf{p}_n^* + \widetilde{\mathbf{w}}_n \odot \mathbf{p}_n^*, \tag{6}$$

with $\odot$ denoting the component-wise vector multiplication in a same way as Hadamard matrix product and $(\cdot)^*$ denoting the conjugate operator. Considering the maximum rate combining (MRC) rule applied to the received signals from the N antennas, the decision variable utilized in the estimation of the information bit transmitted by the kth user is given by:

$$\xi_{n,k}^{\text{MFB}} = \sum_{n=1}^{N} z_{n,k}^{\text{MFB}} \cdot |c_{n,k}|, \tag{7}$$

and, finally, the kth user's information bit is given by:

$$\widehat{b}_k^{\text{MFB}} = \mathrm{sgn}\left(\Re\left\{\xi_{n,k}^{\text{MFB}}\right\}\right), \tag{8}$$

where $\mathrm{sgn}(\cdot)$ represents the signum function and $\Re\{\cdot\}$ is the real part operator. As a result, the estimated information bits vector is obtained as $\widehat{\mathbf{b}}^{\text{MFB}} = \left[\widehat{b}_1^{\text{MFB}}, \widehat{b}_2^{\text{MFB}}, \ldots, \widehat{b}_K^{\text{MFB}}\right]^{\mathrm{T}}$.

However, as well known, the performance of the MFB becomes remarkably poor when the system loading $\mathcal{L} = K/L$ increases, i.e., due to the MAI level increasing as a function of the number of active users.

A. Linear Methods of Multiuser Detection

The linear Decorrelator detector operates from multiplication of the discrete signals at the matched filters output by the inverse cross-correlation matrix $\mathbf{R}^{-1}$. Considering the coherent reception model, the information bits vector at the nth antenna, which is estimated after the application of the Decorrelator filter, can be conveniently described as:

$$\begin{aligned} \mathbf{z}_n^{\text{DEC}} &= \mathbf{T}_n^{\text{DEC}}\mathbf{y}_n^{\text{MFB}} \odot \mathbf{p}_n^* \\ &= \mathbf{R}^{-1}\mathbf{R}\mathbf{C}_n\mathbf{A}\mathbf{b} \odot \mathbf{p}_n^* + \mathbf{R}^{-1}\widetilde{\mathbf{w}}_n \odot \mathbf{p}_n^* \\ &= \mathbf{C}_n\mathbf{A}\mathbf{b} \odot \mathbf{p}_n^* + \breve{\mathbf{w}}_n \odot \mathbf{p}_n^*, \end{aligned} \tag{9}$$

where $\mathbf{T}_n^{\text{DEC}} = \mathbf{R}^{-1}$, $\forall n$ is the transformation matrix for the MuD Decorrelator filter, and $\mathbf{z}_n^{\text{DEC}} = [z_{n,1}^{\text{DEC}}, z_{n,2}^{\text{DEC}}, \ldots, z_{n,K}^{\text{DEC}}]^{\mathrm{T}}$ is the Decorrelator output information vector. Note that the cross-correlation inverse matrix $\mathbf{R}^{-1}$ is the common filter for all antennas' received signals. The kth user's information bit is estimated through the MRC of the received signals in phase at all N antennas:

$$\xi_{n,k}^{\text{DEC}} = \sum_{n=1}^{N} z_{n,k}^{\text{DEC}} \cdot |c_{n,k}|, \tag{10}$$

and the result obtained at Decorrelator detector output is given by:

$$\widehat{b}_k^{\text{DEC}} = \mathrm{sgn}\left(\Re\left\{\xi_{n,k}^{\text{DEC}}\right\}\right). \tag{11}$$

The Decorrelator detector presents a gain in the performance regarding the matched filters bank, although the power associated to the noise term $\breve{\mathbf{w}}_n = \mathbf{R}^{-1}\widetilde{\mathbf{w}}_n$, obtained at the Decorrelator output, is always higher or equal to the noise term obtained at the MFB output [8], [9].

Another classical linear detection method known in literature is the MMSE detector, proposed for CDMA systems in [10]. This method is based on the appropriate choice of a linear transformation vector, $\mathbf{t}_{n,k} = \left[t_{n,k,1}, t_{n,k,2}, \ldots, t_{n,k,K}\right]^{\mathrm{T}}$, that minimizes the mean square error (MSE) between the kth user's information bit and the kth linear transformation output, $(\mathbf{t}_{n,k})^{\mathrm{T}}\,\mathbf{y}_n^{\text{MFB}}$, resulting in:

$$\mathbf{t}_{n,k}^{\text{MMSE}} = \min_{\mathbf{t}_{n,k}} \mathbb{E}\left\{\left[b_k - (\mathbf{t}_{n,k})^{\mathrm{T}}\,\mathbf{y}_n^{\text{MFB}}\right]^2\right\}. \tag{12}$$

The vector that minimizes (12) involves the covariance of colored noise $\widetilde{\mathbf{w}}_n$ and the estimated amplitudes of the received users' signals at the nth BS receive antenna, expressed by the $K \times K$ diagonal matrix:

$$\mathbf{G}_n = \mathbf{M}_n\mathbf{A}. \tag{13}$$

By applying this MMSE solution to the joint detection of the K users, the transformation matrix $\mathbf{T}_n^{\text{MMSE}} = \left[\mathbf{t}_{n,1}^{\text{MMSE}}, \mathbf{t}_{n,2}^{\text{MMSE}}, \ldots, \mathbf{t}_{n,K}^{\text{MMSE}}\right]$ with dimensions $K \times K$ is given by:

$$\mathbf{T}_n^{\text{MMSE}} = \left(\mathbf{R} + \sigma_{\mathrm{w}}^2\mathbf{G}_n^{-2}\right)^{-1}. \tag{14}$$

Hence, the decision information vector at the nth antenna, which is obtained after the application of the MMSE multiuser filter, is described by $\mathbf{z}_n^{\text{MMSE}} = \mathbf{T}_n^{\text{MMSE}}\mathbf{y}_n^{\text{MFB}} \odot \mathbf{p}_n^*$, where $z_{n,k}^{\text{MMSE}} = (\mathbf{t}_{n,k}^{\text{MMSE}})^{\mathrm{T}}\mathbf{y}_n^{\text{MFB}} \cdot e^{-j\theta_{n,k}}$.

And applying the MRC to the received signals in phase at all N antennas:

$$\xi_{n,k}^{\text{MMSE}} = \sum_{n=1}^{N} z_{n,k}^{\text{MMSE}} \cdot |c_{n,k}|. \tag{15}$$

Therefore, the estimated information bit for the kth user at the output of the linear SIMO BPSK MMSE detector is given by:

$$\widehat{b}_k^{\text{MMSE}} = \text{sgn}\left(\Re\left\{\xi_{n,k}^{\text{MMSE}}\right\}\right). \quad (16)$$

III. Polynomial-Expanded Multiuser Detectors

The computational complexity of the linear MuDs, which originates in the operations associated to the cross-correlation matrix inversion, grows with the third order of the matrix size, i.e., $\mathcal{O}\left((m\mathcal{P}K)^3\right)$, where $\mathcal{P}$ is the transmitted message length and m is the modulation order. However, any linear transformation matrix, represented by $\mathbf{T}_n$ in the SIMO model context, can be approximated through the iterative polynomial expansion method with complexity of $\mathcal{O}\left((m\mathcal{P}K)^2\right)$. In general, PE methods approach approximates the cross-correlation matrix inversion via Neumann iterative series expansion.

A. General Result for PE Matrix Approximation

The general result for the $K \times K$ PE-transformation matrix $\mathbf{T}_{\text{PE}}$, which is able to implement a PE multiuser detector (by approximating a matrix inversion), is given by [8]:

$$\mathbf{T}_{\text{PE}} = \sum_{i=0}^{N_{\text{t}}} w_i \mathbf{Q}^i, \quad (17)$$

where N_{t} denotes the number of terms of the polynomial expansion. The matrix $\mathbf{Q}$ and the weights w_i, interpreted as the coefficients for the series convergence rate, have to be chosen such that they suitably approximate the desired multiuser detector.

As a result, the polynomial expansion transformation matrix $\mathbf{T}_{\text{PE}} \to \mathbf{Q}^{-1}$ when the number of expansion terms $N_{\text{t}} \to \infty$.

B. Polynomial Expansion via Neumann Series

By using the Neumann series expansion method [11], the inverse cross-correlation matrix $\mathbf{R}^{-1}$, for the case of Decorrelator filter, may be approximated as:

$$\mathbf{R}^{-1} \approx \mathbf{T}_{\text{PE}}^{\text{DEC}} = \alpha \sum_{i=0}^{N_{\text{t}}} \left(\mathbf{I}_K - \alpha\mathbf{R}\right)^i, \quad \|\mathbf{I}_K - \alpha\mathbf{R}\| < 1 \quad (18)$$

where $\mathbf{I}_K$ is an identity matrix of size K, and the associated residual error matrix is given by:

$$\boldsymbol{\varepsilon}_{\text{PE}}^{\text{DEC}} = \alpha \sum_{i=N_{\text{t}}+1}^{\infty} \left(\mathbf{I}_K - \alpha\mathbf{R}\right)^i, \quad (19)$$

such that the equality $\mathbf{R}^{-1} = \mathbf{T}_{\text{PE}}^{\text{DEC}} + \boldsymbol{\varepsilon}_{\text{PE}}^{\text{DEC}}$ holds.

In the same way, the PE matrix transformation for the SIMO linear MMSE detector at the nth receive antenna can be approximated in:

$$\mathbf{T}_{\text{PE},n}^{\text{MMSE}} \approx \left(\mathbf{R} + \sigma_{\text{w}}^2\mathbf{G}_n^{-2}\right)^{-1} = \alpha_n \sum_{i=0}^{N_{\text{t}}} \left[\mathbf{I}_K - \alpha_n\left(\mathbf{R} + \sigma_{\text{w}}^2\mathbf{G}_n^{-2}\right)\right]^i. \quad (20)$$

The decision variable utilized by the polynomial-expanded multiuser detector in the estimation of the information bit transmitted by the kth user is given by using the MRC:

$$\xi_{n,k}^{\text{PE}} = \sum_{n=1}^{N} z_{n,k}^{\text{PE}} \cdot |c_{n,k}|, \quad (21)$$

with $\mathbf{z}_n^{\text{PE}} = \left[z_{n,1}^{\text{PE}}, z_{n,2}^{\text{PE}}, \ldots, z_{n,K}^{\text{PE}}\right]^{\text{T}} = \mathbf{T}_{\text{PE},n}^{\text{DEC}}\,\mathbf{y}_n^{\text{MFB}} \odot \mathbf{p}_n^*$ for the Decorrelator approximation or $\mathbf{z}_n^{\text{PE}} = \mathbf{T}_{\text{PE},n}^{\text{MMSE}}\,\mathbf{y}_n^{\text{MFB}} \odot \mathbf{p}_n^*$ for the MMSE approximation.

Finally, the hard decisions for the PE-MuD SIMO DS/CDMA with BPSK modulation are obtained as:

$$\widehat{b}_k^{\text{PE}} = \text{sgn}\left(\Re\left\{\xi_{n,k}^{\text{PE}}\right\}\right). \quad (22)$$

C. Optimum Value of the Parameter α

Since the convergence factor of an iterative method can be associated with the radius of the matricial operator, the convergence ratio is related to the dimension of this radius. The optimum parameter for linear Decorrelator detector in the PE approximation is given by:

$$\alpha_{\text{opt}}^{\text{DEC}} = 2(\lambda_{\min} + \lambda_{\max})^{-1}. \quad (23)$$

In turn, for the linear MMSE detector approximation, the optimum value of α is given by:

$$\alpha_{\text{opt}}^{\text{MMSE}} = 2(\lambda_{\min} + \lambda_{\max} + 2\sigma_{\text{w}}^2)^{-1}. \quad (24)$$

Important to point out that the deterministic choice of α_{opt} through cross-correlation matrix eigenvalues calculation is prohibitively complex for the implementation of the polynomial expansion method using practical digital signal processing hardware platforms. As a result, the complexity of only one eigenvalue computation, as well as of all eigenvalues calculation from a K squared-dimension matrix results in $\mathcal{O}\left(K^3\right)$. Thus, it is necessary to estimate the optimum value of the parameter α. Next, the estimation of α_{opt} is suggested by using the Gerschgorin circles Theorem [11].

D. Gerschgorin Circles Theorem (GCT)

According to Gerschgorin Theorem, any eigenvalue λ_i of a matrix $\mathbf{R}$, which has elements $r_{i,j}$, $\forall i, j$, is situated in one of the circles of the complex plane that are centered in $r_{i,i}$:

$$|\lambda_i - r_{i,i}| \leq \sum_{i,j\neq i} |r_{i,j}|. \quad (25)$$

Thus, through a simple calculation, by using the elements of $\mathbf{R}$, the approximated values of $\lambda_{\min}$ and $\lambda_{\max}$, which are denoted by $\widehat{\lambda}_{\min}$ and $\widehat{\lambda}_{\max}$, respectively, can be achieved by:

$$\widehat{\lambda}_{\min} \approx \min\left\{r_{i,i} + \textstyle\sum_{i,j\neq i} |r_{i,j}|\right\}, \quad \forall i, \quad (26)$$

$$\widehat{\lambda}_{\max} \approx \max\left\{r_{i,i} + \textstyle\sum_{i,j\neq i} |r_{i,j}|\right\}, \quad \forall i. \quad (27)$$

The GCT guarantees a considerable reduction in the complexity of the calculation of the minimum and maximum eigenvalues; therefore, the GCT results are applied directly to Eq. (23) and (24) in order to estimate the parameter α.

IV. Local Search Methods Applied to MuD

The well known local search methods propitiate the attainment of near-optimum solutions from searches guided in subspaces of the dimension of the optimization problem. The 1opt-LS-MuD algorithm performs guided searches for the $\underline{\mathbf{b}}$ vector that maximizes the linearly combining cost function:

$$f\left(\underline{\mathbf{b}}\right) = \frac{1}{N}\sum_{n=1}^{N} \Omega_n\left(\underline{\mathbf{b}}\right), \quad (28)$$

where Ω_n is the Euclidean distance function between the received signal at nth antenna and the reconstructed signal at the receiver from the information candidate-vector, $\underline{\mathbf{b}}$:

$$\Omega_n\left(\underline{\mathbf{b}}\right) = 2\Re\left\{\underline{\mathbf{b}}^{\text{T}}\mathbf{C}_n^{\text{H}}\mathbf{A}\mathbf{y}_n^{\text{MFB}}\right\} - \underline{\mathbf{b}}^{\text{T}}\mathbf{C}_n\mathbf{A}\mathbf{R}\mathbf{A}\mathbf{C}_n^{\text{H}}\underline{\mathbf{b}}. \quad (29)$$

The 1opt-LS-MuD algorithm selects candidate-vectors with unitary Hamming distance[1] from the MFB output vector. Details related to this LS algorithm can be found in [4], [12]. Herein, the users' power profile is considered for classification purpose at the beginning of the guided search process. From the estimated users' received amplitudes diagonal matrix $\mathbf{G}_n = \mathbf{M}_n\mathbf{A} = \mathrm{diag}\left(g_{n,1},\, g_{n,2}, \ldots,\, g_{n,K}\right)$, the average received amplitude of the kth user can be estimated as a linear combination of the corresponding N antenna branches:

$$\mathfrak{g}_k = \frac{1}{N}\sum_{n=1}^{N} g_{n,k}, \tag{30}$$

where vector $\mathbf{g} = \left[\mathfrak{g}_1,\, \mathfrak{g}_2, \ldots,\, \mathfrak{g}_K\right]^{\mathrm{T}}$ is formed to assist the users' power classification. In the following, an adaptation for the 1-opt LS algorithm is proposed and a new algorithm is formed.

A. Local Search Algorithm 1-adapt LS

The quantity of calculations of the cost functions during the search for the best candidate vector can be limited by using a given threshold. Chase establishes a threshold criterion based on channel measurement informations, by selecting a fixed number of the lowest confidence bits to be changed [13]. Differently of Chase search stop criterion, herein for the proposed 1-opt LS algorithm, a dynamic threshold is used in order to create adaptation and reduce complexity. The algorithm one-adaptive local search (1-adapt LS) classifies the received signals in order of increasing amplitude; then, candidate vectors with unitary Hamming distance are generated, following the ordering of the signals (from the weakest to the strongest), and their respective cost functions are evaluated; in case of the linearly combining cost function value does not increase inside of a pre-established quantity of consecutive evaluations, denoted here by the parameter κ, the search process is interrupted and a new search is initiated. In general, κ is taken as a fraction of the number of active users in the system. The pseudo-code for the algorithm 1-adapt LS is described in the Algorithm 1.

B. Hybrid PE-MMSE 1opt-LS-MuD and 1adapt-LS-MuD

A detection structure formed by a suboptimal local search algorithm in conjunction with a primary stage of polynomial-expanded linear multiuser detector was presented in [5]. This structure has been reproduced herein, by deploying in the first stage, the SIMO polynomial-expanded MMSE detector with α estimated via Gerschgorin circles method, and in the second stage, the 1-opt LS algorithm. In this work, the low-complexity 1-adapt LS algorithm is deployed in the second stage of the hybrid detection structure to form the hybrid 1adapt-LS-MuD with spatial diversity exploration. In Section VI, a performance comparison including both hybrid PE-LS-based suboptimal MuDs have been carried out.

V. Computational Complexity

The metric of computational complexity is defined by the total number of floating points operations (FLOP) needed for each detector to achieve convergence. The considered operations are: multiplication, comparison, random number generation and selections. The complexity is expressed as a function of the number of users (K), antennas (N), iterations needed for convergence ($n_{\mathrm{it}} \leq N_{\mathrm{it}}$) and the average quantity of cost function calculations by iteration, which is denoted by ζ_{avg}. The cost function calculation in (29) is the most significant factor in determining the complexity of the detectors. The terms $2\mathbf{C}_n^{\mathrm{H}}\mathbf{A}\mathbf{y}_n^{\mathrm{MFB}}$ and $\mathbf{C}_n\mathbf{A}\mathbf{R}\mathbf{A}\mathbf{C}_n^{\mathrm{H}}$ are evaluated outside the iterations loop and adopted constant during the detector search. The resulting number of operations needed for these two terms is $4K^3 + 6K^2 + 2K$, and this calculation is done N times (one for each antenna). Inside the iterations loop, the number of operations needed for each candidate vector evaluation through cost function becomes $N(K^2 + 3K)$. The Table I shows the complexity for the PE-MMSE-MuD with N_{t} terms and estimated α, the linear MMSE-MuD and the local search algorithms 1-opt LS and 1-adapt LS.

Algorithm 1 One-adaptive LS

Input: $\widehat{\mathbf{b}}^{\mathrm{MFB}}$; N_{it}; $\mathbf{g}$; κ; **Output**: $\widehat{\mathbf{b}}$;
begin
$t = 0$;
1. Classify signals: $\mathbf{g}$ (increasing amplitude order), given $\mathfrak{g}_k\,[t]$, $k = 1, 2, \ldots, K$, with $\mathfrak{g}_k\,[t] \leq \mathfrak{g}_{k+1}\,[t]$;
2. Initialize the local search: $t = 1$; $\ell = 0$;
  a. Let $\mathbf{b}_{\mathrm{best}}\,[1] = \widehat{\mathbf{b}}^{\mathrm{MFB}}$;
  b. Calculate the cost function $f_{\mathrm{best}}\,[1] = f\left(\mathbf{b}_{\mathrm{best}}\,[1]\right)$;
3. **for** $t = 1, 2, \ldots, N_{\mathrm{it}}$,
    while $\ell < \kappa$,
      a. Generate candidate vectors with unitary Hamming distance denoted by $\underline{\mathbf{b}}_i\,[t]$, $i = 1, 2, \ldots, K$;
      b. Calculate the cost function $f_i\,[t] = f\left(\underline{\mathbf{b}}_i\,[t]\right)$;
      if $f_i\,[t] > f_{\mathrm{best}}\,[t]$,
        $f_{\mathrm{best}}\,[t+1] \leftarrow f_i\,[t]$;
        $\mathbf{b}_{\mathrm{best}}\,[t+1] \leftarrow \underline{\mathbf{b}}_i\,[t]$;
        $\ell = 0$;
      else
        $\ell = \ell + 1$;
      end if
    end while
    if $f_{\mathrm{best}}\,[t+1] = f_{\mathrm{best}}\,[t]$,
      go to 4;
    end if
  end for
4. $\widehat{\mathbf{b}} = \mathbf{b}_{\mathrm{best}}$;
end

TABLE I: Computational Complexity in [FLOPs]

MuD	Operations
Linear MMSE	$N\left[2K^3/3 + 3K + 2\left(K-1\right)\right]$
PE-MMSE	$N\left[3K^2 + 3K\left(N_{\mathrm{t}}(N_{\mathrm{t}}-1)\right) + K\right]$
1-opt LS ($n_{\mathrm{it}} \leq N_{\mathrm{it}}$)	$n_{\mathrm{it}}\left[KN\left(K^2+3K\right) + 2K + 2\right] + N\left(4K^3 + 6K^2 + 4K\right) + 1$
1-adapt LS ($n_{\mathrm{it}} \leq N_{\mathrm{it}}$)	$n_{\mathrm{it}}\left[\zeta_{\mathrm{avg}}N\left(K^2+3K\right) + 3K + 3\right] + N\left(4K^3 + 6K^2 + 4K\right) + 1$

VI. Performance-Complexity Analysis

The suboptimal MuD performances are evaluated by means of Monte Carlo simulation (MCS) method. The flat Rayleigh fading channels with magnitude and phase coefficients perfectly estimated at the receiver side have been assumed, while the number of antennas at receiver ranges from $N = 2$ to

[1]Hamming distance between $\mathbf{b}_1$ and $\mathbf{b}_2$ vectors is defined by $\|\mathbf{b}_1 - \mathbf{b}_2\|$, which corresponds to the amount of elements that differ between the vectors.

$N = 4$ antennas. The average SNR, denoted by SNR_{avg}, is deployed in the context of the near-far effect, i.e., there are two interfering group of users with near-far ratio $\text{NFR}_{+} = P_{\text{interf}}\ (\text{dB}) - P_{\text{interest}}\ (\text{dB}) = +5$ dB ($K/3$ users), and $K/3$ users with $\text{NFR}_{-} = -5$ dB, where $P_{\text{interf}} = \mathfrak{g}^2_{i,\text{interf}}$ and $P_{\text{interest}} = \mathfrak{g}^2_{j,\text{interest}}$ is the received power related to the linear combination of received amplitudes, Eq. (30), for the ith interfering signal and jth interest signal, respectively.

The SNR_{avg} considered in simulation ranges from 0 to 18 dB. Random spreading codes with processing gain length of $L = 31$ have been adopted in a single-rate DS/CDMA system. Furthermore, the number of terms in polynomial expansion is limited to $N_{\text{t}} \in \{1, 3\}$ terms and the number of local search iterations is limited to $N_{\text{it}} = [1; 5]$ iterations. Besides, in the 1-adapt LS algorithm, a good performance-complexity trade-off is attainable with $\kappa = \lfloor 0.6 \cdot K \rceil$.

For the 1adapt-LS-MuD, Fig. 1 depicts the average quantity of cost function calculations by iteration and antenna, ζ_{avg}, as a function of the number of users, considering (a) $N = 2$ and (b) $N = 4$ antennas, with $\text{SNR}_{\text{avg}} = 10$ dB. The quantity of active users in the system ranges from $K = [21; 30]$ users. As the number of antennas at receiver grows, decreases the influence that the number of iterations has in the local search algorithm.

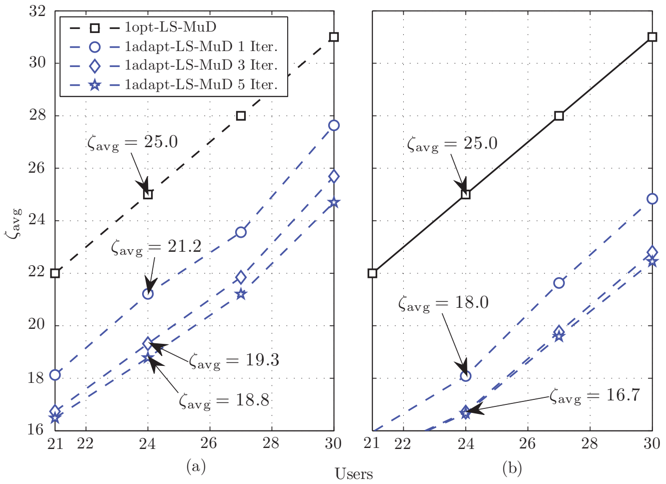

Fig. 1: Average quantity of cost function calculations necessary in the 1opt-LS-MuD and 1adapt-LS-MuD with (a) $N = 2$ and (b) $N = 4$ antennas; $\text{SNR}_{\text{avg}} = 10$ dB.

Fig. 2 depicts the BER performance of the MuDs under SIMO BPSK environment for low system loading $\mathcal{L} \approx 29\%$ ($K = 9$ users) configuration. In this scenario, the hybrid PE-MMSE 1opt-LS-MuD with $N_{\text{t}} = 3$ terms keeps the same diversity gain achieved by the linear MMSE-MuD but with a BER performance improvement, i.e., a SNR gap of ≈ 2 dB in high SNR region. On the other hand, the proposed hybrid PE-MMSE 1adapt-LS-MuD follows the performance of the PE-MMSE 1opt-LS-MuD in the low and medium SNR region; however its achieves BER floor at $\text{SNR}_{\text{avg}} > 20$ dB. A reduction in BER floor effect can be obtained by adding more terms (5 or 7 terms), with marginal complexity increasing. Hence, the proposed hybrid PE-MMSE 1adapt-LS-MuD represents a suitable solution in terms of performance-complexity trade-off when compared to the PE-MMSE-MuD.

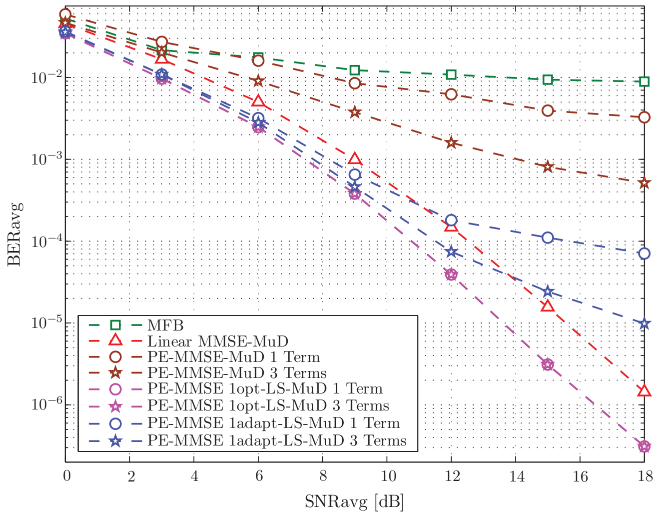

Fig. 2: Hybrid 1adapt-LS-MuD performance, in the flat Rayleigh channel with $N = 4$ antennas and $K = 9$ users; Algorithm 1 with $N_{\text{it}} = 3$ iterations.

VII. Conclusions

The proposed 1-adapt LS algorithm promotes a remarkable gain in the SIMO DS/CDMA system performance equipped with polynomial expansion-based hybrid multiuser detectors. When associated to low-complexity PE-MuDs, it provides reliability to the detection process, without an excessive increasing in its implementation cost, been able to offer a promising performance-complexity trade-off.

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