Reduced Rank Adaptive Filter with Variable Step Size for Impulsive UWB PLC Systems

Marlon Lucas Gomes Salmento, Eduardo Pestana de Aguiar and Moisés Vidal Ribeiro

***Abstract*— This paper addresses the reception of UWB symbol in an impulsive UWB PLC system. Regarding this topic, it is introduced the use of the Delta-Rule-Delta technique together with the SM concept to decrease the computational complexity of Reduced-Rank Adaptive Filter. Performance analyzes reveal that both proposed techniques together with set-membership concept can result in substantial improvement (computational complexity, bit error rate and convergence rate), when they are compared with previous approaches. The performance results are carried out on indoor PLC channels corrupted by the presence of AWGN noise. Finally, a computational complexity analysis shows that the proposed techniques for adapting the coefficients of the Reduced-Rank Adaptive Filter offer the lowest complexity in term of number of multiplications and additions and highest convergence rate.**

***Keywords*— Power Line Communications, UWB modulation, Reduced-Rank Adaptive Filter, Adaptive Algorithms.**

I. INTRODUCTION

One of major challenges in adaptive filtering area is to perform the signal processing with low computational complexity and high convergence speed. For this, several techniques have been investigated among which stands out the filter Reduced-Rank Adaptive Filter (RRAF) [1]. Filtering with reduced-rank is a technique which has gained considerable attention in recent years due to its ability to converge from a reduced training set and due to its low computational complexity.

Current research indicates that the use of RRAF with set-membership (SM) [2] increases the speed of convergence. Therefore, the investigation of the RRAF technique with SM for receiving symbols in an Ultra Wide-band (UWB) Power-line communication (PLC) systems is of great interest, since increasing the convergence speed results in short training sequence and lower computational complexity.

In this context, this article discusses the use of RRAF with SM and a technique that controls the dynamics of the step size, which contributes to increase the speed convergence and decreasing the RRAF computational complexity even more and, consequently, enable the implementation of an impulsive UWB PLC systems for low cost applications, such as smart appliances, machine-to-machine and vehicular applications.

The paper is organized as follows: Section II presents the problem formulation and the RRAF technique. Section III discusses Delta-Rule-Delta (DRD) technique for dynamic update of the step size and its combination with the SM. Section IV deals with performances and the computational complexity analyses and compares them with previous works. Finally, Section V presents the conclusions and final considerations.

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II. PROBLEM FORMULATION

Considering the block diagram of a baseband and discrete-time system model adopted for modeling the receiver for single-user impulsive UWB modulation scheme for a PLC system, shown in Fig. 1. Note that $x[k]$ is the M-ary pulse

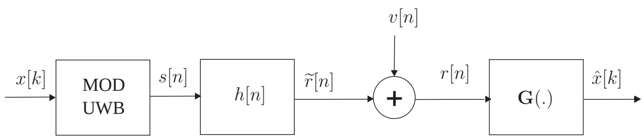

Fig. 1. The block diagram of UWB PLC system.

amplitude modulation (M-PAM) constellation, $s[n]$ is the modulated symbol, $h[n]$ is the impulse response of a linear and time invariant PLC channel, $\widetilde{r}[n]$ denotes the output of channel free of noise and finally, $v[n]$ is the additive noise.

A M-PAM modulated symbol signal can be represented as:

$$s[n] = \sqrt{E} \sum_{k=-\infty}^{\infty} x[k] y_{g1}[n - kN_p], \tag{1}$$

where N_p is a UWB symbol period, E is the pulse energy and $y_{g1}[n]$ is the discrete time domain representation of the UWB pulse.

The received signal can be denoted as

$$r[n] = \widetilde{r}[n] + v[n] = \sum_{m=-\infty}^{\infty} s[m]h[n-m] + v[n], \tag{2}$$

such that $N_f = N_p + N_g$ is a UWB frame period and N_g is the guard interval. Now assuming that N_c is the impulse response duration of PLC channel. It is important to ensure that $N_g \geq N_c$ in order to avoid the occurrence of Inter-Symbolic Interference (ISI) in UWB impulsive system. According to adopted formulation, we define the vector $\boldsymbol{r}[k] = [r\ [kN_f]\ r[kN_f+1]\ \ldots\ r[(k+1)N_f-1]]^T$, N_f consecutive samples $r[n]$. The k-th estimated symbol $\hat{x}[k]$ obtained by receiver $\mathbf{G}(.)$ is

$$\hat{x}[k] = \mathbf{G}(\boldsymbol{r}[k]). \tag{3}$$

Among several proposals for $\mathbf{G}(.)$ found in the literature, this paper analyzes the use of RRAF introduced in [1] to represent the operator $\mathbf{G}(.)$ and investigates the use of variable step size.

The block diagram of RRAF is shown in Fig. 2, which is constituted by an interpolator filter $\mathbf{v}[k]$, a decimator $\mathbf{D}$ and a reduced-rank filter $\mathbf{w}[k]$.

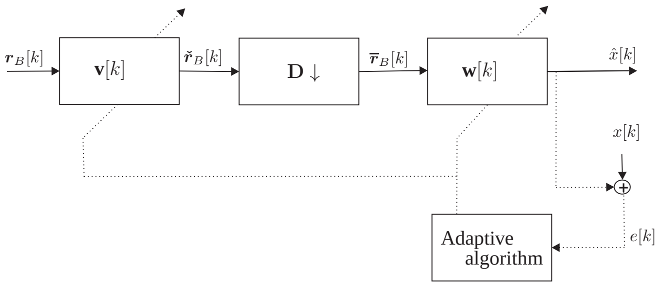

Fig. 2. The block diagram of RRAF.

The input vector $\boldsymbol{r}_B[k]$ is filtered by interpolator filter $\mathbf{v}[k] = [v_0[k]...v_{N_I-1}[k]]^T$, where N_I is the length of the interpolator filter. Finally, the interpolated vector $\boldsymbol{r}_B[k] \in \mathbb{R}^{N_f \times 1}$ is expressed by

$$\check{\boldsymbol{r}}_B[k] = \mathbf{V}^T[k]\boldsymbol{r}_B[k], \quad (4)$$

where $\mathbf{V}[k] \in \mathbb{R}^{N_f \times N_f}$ is a Toeplitz convolution matrix that is expressed by

$$\mathbf{V}[k] = \begin{bmatrix} v_0[k] & 0 & \dots & 0 \\ \vdots & v_0[k] & \dots & 0 \\ v_{N_I-1}[k] & \vdots & \dots & 0 \\ 0 & v_{N_I-1}[k] & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_0[k] \end{bmatrix}. \quad (5)$$

To facilitate the mathematical description of RRAF,

$$\begin{aligned} \check{\boldsymbol{r}}_B[k] &= \mathbf{V}^T[k]\mathbf{r}_B[k] \\ &= \Re_o[k]\mathbf{v}[k], \end{aligned} \quad (6)$$

where $\Re_o \in \mathbb{R}^{N_f \times N_I}$ is denoted by

$$\Re_{\mathbf{o}}[k] = \begin{bmatrix} r_0[k] & r_1[k] & \dots & r_{N_I-1}[k] \\ r_1[k] & r_2[k] & \dots & r_{N_I}[k] \\ \vdots & \vdots & \ddots & \vdots \\ r_{N_f-1}[k] & r_{N_f}[k] & \dots & r_{N_f+N_I-2}[k] \end{bmatrix}. \quad (7)$$

The decimation matrix $\mathbf{D} \in \mathbb{R}^{D \times N_f}$ is expressed by

$$\mathbf{D} = \begin{bmatrix} 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & 0 \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \end{bmatrix}, \quad (8)$$

where $D = \lfloor N_f/L \rfloor$, $\lfloor x \rfloor = \max\{m \in \mathbb{Z} \mid m \leq x\}$, L is the decimation factor. In the matrix $\mathbf{D}$ there is only one component equal 1 per line and its position follows the rule $\left[\underbrace{0 \ \dots \ 0}_{(p-1)L \ zeros} \ 1 \ \underbrace{0 \ \dots \ 0}_{N_f-(p-1)L-1 \ zeros} \right]$, where $p = 1, 2, ..., D$ is p-th line of matrix $\mathbf{D}$. The vector projection $\check{\boldsymbol{r}}_B[k]$ in the matrix space $\mathbf{D}$ is expressed by

$$\begin{aligned} \overline{\boldsymbol{r}}_B[k] &= \mathbf{D}\check{\boldsymbol{r}}_B[k] \\ &= \mathbf{D}\Re_o[k]\mathbf{v}[k]. \end{aligned} \quad (9)$$

The error at the output of the the RRAF is given by

$$e[k] = x[k] - \mathbf{w}^T[k]\overline{\boldsymbol{r}}_B[k], \quad (10)$$

where $\mathbf{w}[k] = [w_0[k]...w_{D-1}[k]]^T$ is the reduced-rank filter and $x[k]$ the transmitted symbol.

Finally, an the estimate of *k*-th transmitted symbol is given by

$$\hat{x}[k] = \mathbf{w}^T[k]\mathbf{D}\Re_o[k]\mathbf{v}[k], \quad (11)$$

or

$$\hat{x}[k] = \mathbf{v}^T[k]\Re_o^T[k]\mathbf{D}^T\mathbf{w}[k]. \quad (12)$$

Next section we present a RRAF with SM that makes use of a variable step size technique to update its coefficients.

III. Proposed Technique

In this section we introduce a technique for training RRAF, by updating step size, which contributes to increase the convergence speed of RRAF and decrease the computational complexity.

A. *RRAF with* Set-membership *and DRD*

The use of adaptive filters that have improved convergence speed receiver. Currently, there are several techniques to dynamically adjust the step size of a typical Steepest Descent (SD) kind of algorithm in the literature. Currently, there are several techniques to dynamically adjust the step size of a typical Steepest Descent (SD) kind of algorithm in the literature, where is cited by [2]. Among these techniques, deserve special attention Delta-Rule-Delta (DRD) [3].

The DRD technique is a procedure in which for each iteration the coefficients individually contribute to reach the minimum of the cost function. If it does not, the coefficients are individually subjected to a procedure which ensures that. Therefore, the DRD technique seeks to correct the presented problem in SD, which significantly influences the convergence speed of it.

Applying DRD technique [3] in RRAF, the update equation of $\mathbf{w}[k]$ is now expressed by:

$$\mathbf{w}[k+1] = \mathbf{w}[k] - (1-\alpha_{\mathbf{w}})\mathbf{A}_{\mathbf{w}}[k]\nabla_{\mathbf{w}} J(\mathbf{v}[k], \mathbf{w}[k]) + \alpha_{\mathbf{w}}\Delta\mathbf{w}[k], \tag{13}$$

where $J(\mathbf{v}[k], \mathbf{w}[k])$ is the cost function and $\mathbf{A}_{\mathbf{w}}[k] \in \mathbb{R}^{D\times D}$ is a diagonal matrix responsible for updating the coefficient of $\mathbf{w}[k]$, and is expressed by

$$\mathbf{A}_{\mathbf{w}}[k] = \begin{bmatrix} \mu_0[k] & 0 & \dots & 0 \\ 0 & \mu_1[k] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mu_{D-1}[k] \end{bmatrix}, \tag{14}$$

where $\mu_j[k]$ is the step size associated with the *j*-th filter coefficient $w_j[k]$. The updating rule, in accordance with the DRD technique, is expressed by

$$\mu_j[k+1] = \begin{cases} \mu_j[k] + K_{\mathbf{w}}, & \text{if } \Theta[k+1]J(\mathbf{v}[k], \mathbf{w}[k]) > 0 \\ \mu_j[k] - K_{\mathbf{w}}\mu_j[k], & \text{if } \Theta[k+1]J(\mathbf{v}[k], \mathbf{w}[k]) < 0 \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

where $j = 0,\ 1,\ \dots,\ D-1$, $\alpha_{\mathbf{w}} \in \mathbb{R}\,|0 < \alpha_{\mathbf{w}} < 1$, $K_{\mathbf{w}} \in \mathbb{R}$ a constant responsible for increasing and decreasing the value of step size and

$$\Theta[k+1] = (1-\rho_{\mathbf{w}})\partial_j J(\mathbf{v}[k+1], \mathbf{w}[k+1]) + \rho_{\mathbf{w}}\Theta[k], \tag{16}$$

where $\rho_{\mathbf{w}} \in \mathbb{R}\,|0 < \rho_{\mathbf{w}} < 1$ is a constant and ∂_j is the partial derivative of the cost function $J(\mathbf{v}[k], \mathbf{w}[k])$ against the *j*-th coefficient.

Analyzing $\mu_j[k+1]$, it is possible to conclude that it is incremented only when the partial derivative for the next iteration coefficient has the same sign of the current partial derivative coefficient, i.e., if for some reason the coefficient moves in the opposite direction to that resulting at the minimum value of the cost function, so it is redirected to converge for the right direction.

The combined use of the DRD technique and SM in RRAF, results in the following update equation:

$$\mathbf{w}[k+1] = \mathbf{w}[k] - \Lambda[k], \tag{17}$$

where

$$\Lambda[k] = (1-\alpha_{\mathbf{w}})\mu_{SMF}[k]\mathbf{A}_{\mathbf{w}}[k]\nabla_{\mathbf{w}} J(\mathbf{v}[k], \mathbf{w}[k]) + \beta[k], \tag{18}$$

$$\beta[k] = \alpha_{\mathbf{w}}\mu_{SMF}[k]\Delta\mathbf{w}[k] \tag{19}$$

and

$$\mu_{SMF}[k] = \begin{cases} 1 - \frac{\gamma_{\mathbf{w}}}{|e[k]|}, & \text{if } |e[k]| \geq \gamma_{\mathbf{w}} \\ 0, & \text{otherwise} \end{cases}. \tag{20}$$

For the filter $\mathbf{v}[k]$ will not apply the use of variable step size by DRD and SM, since the filter $\mathbf{v}[k]$ showed unstable behavior as verified in our simulations. Thus, we update $\mathbf{v}[k]$ by using

$$\mathbf{v}[k+1] = \mathbf{v}[k] + \eta[k]\nabla_{\mathbf{v}} J(\mathbf{v}[k], \mathbf{w}[k]). \tag{21}$$

In this case, the Normalized Least Mean Square (NLMS) is used to update $\mathbf{v}[k]$ filter. The combination of (13) and (21) results in the RRAF-DRD technique. On the other hand, the combination of (17) and (21) results in RRAF-DRD-SM technique.

IV. Numerical Results

In all simulations, we considered that the transmission bandwidth is at the baseband in the frequency band from 0 up to 50 MHz (frequency bandwith is $B = 50$ MHz) and sampling frequency equal to 100 MHz. Also, we used a typical indoor PLC channel, which was measure in one house in Juiz de Fora city Brazil. Fig. 3 shows its magnitude response.

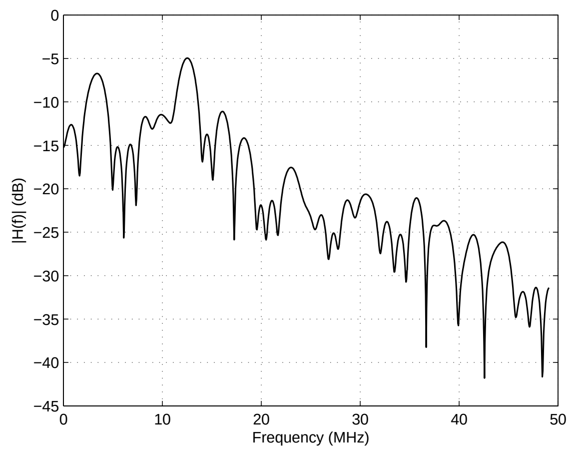

Fig. 3. Magnitude response of the chosen indoor PLC channel.

The additive noise, was Additive White Gaussian Noise (AWGN) as described in [4]. Also, we assumed that E_b/N_0 refers to the relationship between the normalized bit energy and power spectral density of the background noise. The basis function of the UWB pulse is the first derivative of Gaussian pulse, which is described in [4]. The modulation used was 2-PAM. The number of samples that UWB pulse was $N_p = 20$ samples. The adopted frame period was $N_f = 119$ samples. The length of the impulse response in the discrete time domain was set as 99% of the total energy of the channel, ie, $N_c = 100$ samples.

The techniques discussed in [5], NLMS and NLMS combined with SM (NLMS-SM), were applied with RRAF. They were compared with RRAF-LAMARE [2] and with the techniques introduced in this paper (RRAF-DRD-SM and RRAF-DRD). The threshold values for SM were defined as $\gamma_{\mathbf{v}} = \gamma_{\mathbf{w}} = \sqrt{20\sigma_v^2}$, where σ_v^2 is the noise variance. The parameters adopted in the DRD technique were $\alpha_{\mathbf{w}} = 0.3$, $K_{\mathbf{w}} = 0.001$ and $\rho_{\mathbf{w}} = 0.5$. The convergence speed is evaluated through the Mean Square Error (MSE) curves and the performance as a function of BER $\times$ E_b/N_0. Finally, the MSE is given by:

$$MSE = E\{|e[k]|^2\}, \tag{22}$$

where E{.} and |.| denotes the expectation operator and absolute value, respectively. All performance curves are the average results of 30 Monte Carlo simulations.

1) Convergency and Performance Analysis: For the analysis of convergence speed we adopted $E_b/N_0 = 20$ dB. For the RRAF we considered $D = 60$, and for interpolating filter $N_I = 56$. Fig. 4 shows convergency curves for proposed and previous techniques. We see that RRAF-DRD and RRAF-DRD-SM provide improved convergence rate and RRAF-DRD-SM techniques result in higher rates of convergence speed.

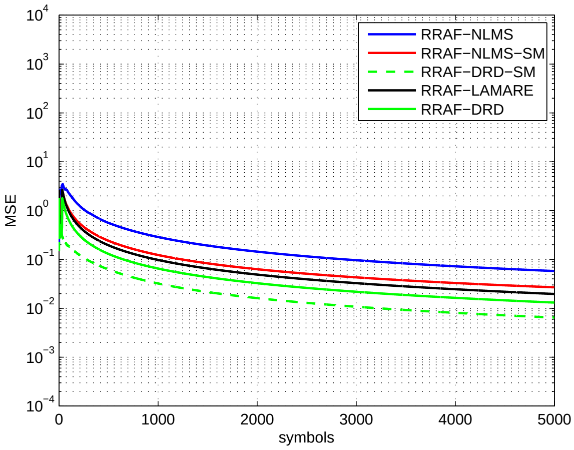

Fig. 4. Convergency: indoor PLC channel.

The performance analysis of RRAF during training is carried out with the coefficients found for $\mathbf{v}[k]$ and $\mathbf{w}[k]$ after a training of $N_T = 5000$ symbols. The choice of $N_T = 5000$ is to ensure that the MSE has achieved a steady state condition. Fig. 5 shows the BER curves $\times$ E_b/N_0 for all techniques. When we compare the proposed technique with previous ones, an improvement higher than 4 dB can be observed at BER= 10^{-3}.

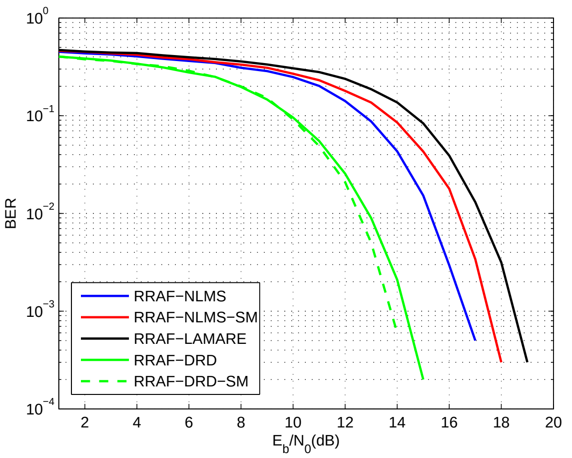

Fig. 5. BER curves $\times$ E_b/N_0 in the presence of AWGN noise.

2) Computational Complexity Analysis: Table I shows the computational complexity per iteration in terms of number of multiplications and additions for each technique.

TABLE I
COMPUTATIONAL COMPLEXITY PER ITERATION FOR RECEIVING TECHNIQUES.

Technique	Multiplications	Additions
RRAF-NLMS	$D(4N_I + 3) + 2$	$D(4N_I + 1) - N_I$
RRAF-NLMS-SM	$D(4N_I + 3) + N_f + 4$	$D(4N_I + 1) - N_I + 2N_f$
RRAF-LAMARE	$3N_f D + 2N_f + D + 12$	$3N_f D + 2N_f - 1$
RRAF-DRD	$N_I(3D + 1) + 4.5D + 5$	$N_I(3D - 2) + 5D + 3$
RRAF-DRD-SM	$N_I(3D + 1) + 4.5D + N_f + 8$	$N_I(3D - 2) + 5D + 2N_f + 2$

Based on Fig. 6 shows computational complexity $\times$ D. The computational complexity is given by the sum of the number of multiplications with additions. To obtain these curves, we considered $N_f = 119$ and $N_I = 56$. Note that RRAF-DRD and RRAF-DRD-SM attained the smallest computational complexity.

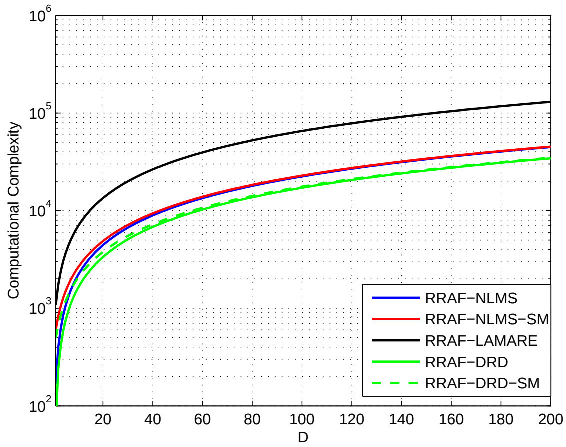

Fig. 6. Computational Complexity $\times$ D.

Carefully analyzing Fig. 6, we can state that RRAF-NLMS-SM and RRAF-NLMS showed similar computational complexity. Also, RRAF-DRD and RRAF-DRD-SM presented similar complexity (see red curves are over the blue one in Fig. 6).

V. CONCLUSION

This paper discussed the application of PLC system based on impulse UWB modulation when indoor PLC channel is corrupted by the presence of AWGN noise. We presented the impulsive UWB PLC system receiver that uses the RRAF, resulting in techniques RRAF-DRD and RRAF-DRD-SM. Between the proposed techniques RRAF-DRD-SM, achieves the best performance in terms of BER $\times$ E_b/N_0 curves obtaining gains of 4 dB at BER $= 10^{-3}$ with respect to RRAF-LAMARE and achieving the lowest computational complexity and highest convergence speed. Future works will consider an analysis of more noises that corrupt the PLC signal, like narrowband, impulsive periodic synchronous to the main frequency, impulsive periodic asynchronous and impulsive non-periodic.

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