The α-μ/Generalized Gamma composite distribution: Theory and Experimental Validation

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***Abstract*— This paper presents the experimental validation of the α-μ/Generalized Gamma composite fading distribution. The PDF of such fading model is obtained in closed-form, in which its physical parameters are related with the presence of both long and short term fading. The study is based on field measurements carried out in the frequencies of 700MHz and 1800MHz. The practical data resulting from the measurements show an excellent agreement between theoretical and practical curves.**

***Keywords*— Composite fading, experimental validation, α-μ/Generalized Gamma.**

I. Introduction

Large scale fading of the wireless channel are caused by the phenomenon of shadowing, which is widely modeled by the Lognormal distribution. Besides, in small scale in which multipath situations occur, several distributions are widely accepted for modeling fading, such as Rayleigh, Rice, Nakagami-*m*, Weibull and others. Although the latest distributions consider constant signal power environments, there are environments, such as those who are characterized by slow movement of pedestrians and cars, where the signal strength may not be constant, but rather random. In these cases, the resulting fading is given by the combination of shading and multipath.

For such situations of composite fading, several distributions that well describe the statistics of the mobile radio signal has been suggested in the literature. One of the most popular distributions, the Rayleigh-Lognormal distribution, which is a composition between the Rayleigh slow fading models and Lognormal shadowing, has a drawback that considerably complicates the application of this distribution: the Rayleigh-Lognormal distribution has no closed formula. Trying to resolve this issue, Abdi [1] proposes the use of the Gamma function in place of the Lognormal distributions in order of modeling shading.

In this context, this paper proposes the use of the composite distribution α-μ/Generalized Gamma, in which the distribution α-μ [2], which has two degrees of freedom and therefore is more general and flexible than the Rayleigh distribution is suggested to provide a more realistic analysis of signal and multipath propagation, and the generalized Gamma distribution, a generalization of the form suggested by Liouville's extension to Dirichlet's integral formula [2], is suggested to model the shadowing. Thus, this work introduces field trials studies in the frequencies of 700Mz and 1800Mz and the experimental validation of the data collected in the field with the theoretical models obtained from the composite distribution suggested.

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II. The α-μ /Generalized Gamma distribution

A. The α-μ distribution

The α-μ distribution can be used to better represent the small-scale variation of the fading signal in a non line-of-sight fading condition [3]. As its name implies, it is written in terms of two physical parameters, namely α and μ. The power parameter $\alpha > 0$ is related to the non-linearity of the environment, whereas the parameter $\mu > 0$ is associated to the number of multipath clusters.

For a α-μ fading signal with envelope R, an arbitrary parameter $\alpha > 0$, with $\hat{r} = \sqrt[\alpha]{E(R^\alpha)}$, in which $E(\cdot)$ means the expectation operator, the α-μ envelope PDF, $f_{\mathrm{R}}(r)$, is written as

$$f_R(r) = \frac{\alpha\mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right), \tag{1}$$

for which $\mu > 0$ is the inverse of the normalized variance of R^α. Interestingly, the α-μ distribution includes other traditional distributions as special cases, such as the Nakagami-*m* (α = 2), Weibull (μ = 1) and Rayleigh (α = 2, μ = 1).

B. The Generalized Gamma distribution

The generalized gamma function is a 3-parameter distribution, which means to be a flexible distribution, that contains, as special cases, the exponential, Weibull, lognormal and gamma distributions. For a signal on long-term fading, or shadowing, the PDF of r_s, $f_\Gamma(r_s)$, can be written as [4]

$$f_R(r_s) = \frac{\gamma\beta^{\lambda/\gamma} r_s^{\lambda-1}}{\Gamma(\lambda/\gamma)} \exp(-\beta r_s^\gamma), \tag{2}$$

in which $\gamma = k^{-1/2}/\sigma$ e $\beta = k\exp(-\mu_L k^{-1/2}/\sigma)$. Note that, for $k \to \infty$, it is possible to reduce (2) to the LogNormal distribution with lease parameter μ_L and scale parameter σ, extensively used in environments characterized by shading effects. The distribution α-μ is also a special case of the generalized gamma distribution when $\alpha = \gamma$; $\mu = \lambda/\gamma$ and $\hat{r} = (\frac{\lambda}{\gamma\beta})^{1/\gamma}$.

C. The α-μ /Generalized Gamma distribution

A composite probability distribution can be created from the superposition of two or more statistical distributions. In this way, the α-μ distribution, combined with the 3 parameters γ has the following probability density function of r_c, $f_{\mathrm{P}}(r_c)$

$$f_{\mathrm{P}}(r_c) = \frac{\alpha\mu^\mu r_c^{\alpha\mu-1}\gamma\beta^{\lambda/\gamma}}{\Gamma(\mu)\Gamma(\lambda/\gamma)} \int_0^\infty x^{\lambda-\mu-1}\exp(-\mu\frac{r^\alpha}{x} - \beta x^\gamma)dx. \tag{3}$$

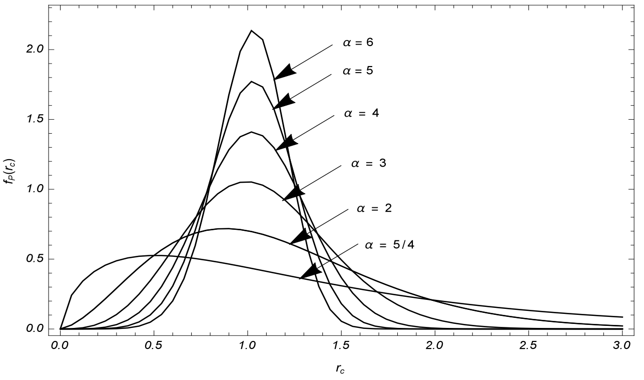

Fig. 1. PDFs of α-μ/Generalized Gamma distribution for μ = 2 and k = 1

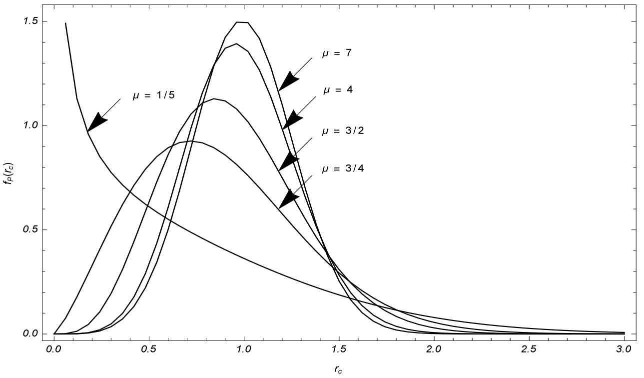

Fig. 2. PDFs of α-μ/Generalized Gamma distribution for α = 3 and k = 1

Using standard statistical procedures, the PDF of α-μ /Generalized Gamma distribution is obtained in closed-form as

$$f_{\mathrm{P}}(r_c) = \frac{\alpha\mu^{\mu} r_c^{\alpha\mu-1}\beta^{\mu/\gamma}}{\Gamma(\mu)\Gamma(\lambda/\gamma)} H_{0,2}^{2,0}\left[r_c^{\alpha}\mu\beta^{1/\gamma} \,\middle|\, {}^{\;-}_{(0,1);(\frac{\lambda-\mu}{\gamma},\frac{1}{\gamma})} \right], \tag{4}$$

in which H is the Fox H-Function. It is interesting to note that using the parameters $\alpha = 2$ e $\mu = 1$, the distribution α-μ/Generalized Gamma of 3 parameters reduces to the composite Rayleigh/Generalized Gamma distribution. Similarly, for α=2 the composite Nakagami-*m*case is obtained and for $\mu = 1$ the composite Weibull case is also obtained.

The family of curves for PDF $f(r_c)$, with $\mu = 2$, $k = 1$ and α as variable is presented in Figure 1. As expected, while increasing the power parameter α, the PDF becomes more concentrated. The same behavior is observed when increasing the number of multipath cluster, μ. Note that for α less than 2 the fading is extremely severe, as expected.

Figure 2 presents the $f(r_c)$ PDF varying the μ parameter and keeping the others parameters fixed. As expected, for $\alpha\mu < 1$ (i.e., $\mu < 1/\alpha$), $f(r_c)$ tends to infinity as r approaches zero and $f(r_c)$ decreases monotonically with the increase of r. For $\alpha\mu > 1$ (i.e., $\mu > 1/\alpha$), $f(r_c)$ is nil at the origin and it increases with the increase of r to reach a maximum and then decrease toward zero as r increases.

III. Field Trials and Experimental Validation

A series of field trials was conducted at the the University of Brasilia (UnB) and the University of Campinas (Unicamp), Brazil, in outdoor and indoor environments, in order to investigate and validate in practice the α-μ/Generalized Gamma distribution. To this end, the transmitter was placed on the rooftop of one of the buildings and the receiver travelled through each campus as well as within the buildings. The mobile reception equipment was especially assembled for this purpose. Basically, the setup consisted of a vertically polarized omnidirectional receiving antenna, a low noise amplifier, a spectrum analyzer, data acquisition apparatus, a notebook computer, and a distance transducer for carrying out the signal sampling. The transmission consisted of individual CW tones at 700 MHz and 1800 MHz.

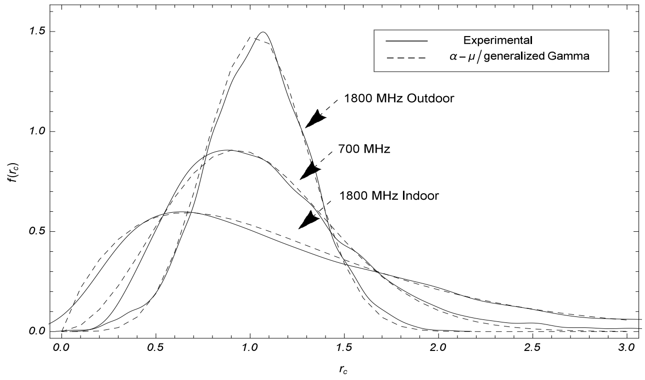

Fig. 3. Practical and theoretical shapes for PDFs in 700MHz and 1800MHz.

Figure 3 presents the PDF $f_P(r_c)$ for 700 MHz and 1800 MHz. It is possible to observe an excellent fit between the curves of the practical and theoretical PDFs for each one of the frequencies, being observed a mean deviation error, ϵ[1], of 2.94% for PDF of 1800MHz in outdoor environment, 1.36% for PDF of 1800MHz in indoor environment and 2.27% for the PDF of 700MHz. For 1800MHz, when analyzing the PDFs, it was observed that the most commom parameters values for this frequency was α = 2.3, μ = 3.3, μ_1 = 1.5, σ = 1 and k = 1. Similarly, the most commom parameters values for 700Mhz was α = 3.4, μ = 4.3, μ_1 = 1.3, σ = 1 and k = 1.

IV. Conclusion

In this paper, we reported the results of field trials aimed at investigating the first-order statistics of the α-μ/Generalized Gamma distribution. An *excellent* visual agreement between the experimental and the theoretical data was found. The measurements validate the PDF derived in closed-form for the composite α-μ/Generalized Gamma fading signal.

References

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[1] The mean error deviation between the measured data x_i and the theoretical value y_i is defined as $\epsilon = \frac{1}{N}\sum_{i=1}^{N}\frac{|y_i - x_i|}{x_i}$, where N is the number of points.