Stochastic Analysis of the Orthogonal Frequency Division Multiplexing Scheme

Marcelo S. Alencar, Francisco Madeiro, Wamberto J. L. Queiroz and Waslon T. A. Lopes

***Resumo*— A Multiplexação por Divisão em Frequências Ortogonais tem sido escolhida como esquema de transmissão para a maioria dos padrões de TV digital e de comunicações sem fio. Nesse esquema, as subportadoras formam um conjunto de funções que são ortogonais entre si, ou seja, o produto interno dessas funções é nulo em um intervalo de símbolo. A ortogonalidade assegura que a interferência intersimbólica nas frequências das subportadoras seja nula. Esse artigo apresenta uma análise estocástica do sinal OFDM para que se possa obter expressões gerais para a autocorrelação e para a densidade espectral de potência. A densidade espectral obtida é exatamente a que é produzida pelos analisadores de espectro.**

***Palavras-Chaves*— OFDM, análise estocástica.**

***Abstract*— The Orthogonal Frequency Division Multiplexing (OFDM) scheme has been chosen as the transmission scheme for most digital television, digital radio and wireless standards. In this scheme the sub-carriers form a set of functions that are orthogonal to each other, that is, the integral of the product between any two of these functions within the interval of a symbol is null. The orthogonality ensures that the inter-symbolic interference in the frequencies of the sub-carriers is null. This paper presents an stochastic analysis of the OFDM signal, to obtain general expressions for its autocorrelation and power spectral density formulas. The resulting power spectral density is exactly what is produced by practical spectrum analyzers.**

***Keywords*— OFDM, stochastic analisys.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a variant of a technique known as Frequency Division Multiplexing (FDM), which was widely used, for many years, by telecommunications companies in analog systems for long distance transmission. It is also known as Discrete Multitone Modulation (DMT) and aims at transmitting multiple signals in different frequencies [1].

The OFDM scheme uses channel coding, which is a technique for correcting transmission errors, giving rise to the acronym COFDM. The technique typically requires a fast Fourier transform (FFT), which is implemented in a digital signal processor (DSP) or directly in an integrated circuit (IC) [2].

The OFDM has been used in radio broadcasting, communication networks and computer networks. Its main performance attribute is its robustness to multipath effect, which leads to "ghost"in the reception with analog television, as well as to the frequency selective fading, in mobile communications [3].

An OFDM baseband signal is a result from the combination of many orthogonal subcarriers, with the data of each subcarrier independently moduled with the use of any QAM (Quadrature Amplitude Modulation) or PSK (Phase Shift Keying) technique. This signal is used to modulate a main carrier for radiofrequency broadcasting.

Some advantages come from the use of OFDM, including high spectral efficiency, robustness against multipath and burst noise, as well as robustness to slow phase distortion and fading. This is achieved by combining OFDM with techniques of error correction, adaptive equalization, data interleaving and reconfigurable modulation. Additionally, the COFDM (Coded Orthogonal Frequency Division Multiplexing) has a spectrum which is typically uniform, which leads to some advantages regarding cochannel interference [4].

II. DESCRIPTION OF OFDM

The ISDB-T uses the technique of Orthogonal Frequency Division Multiplexing (OFDM) as a transmission scheme. The subcarriers form a set of functions that are orthogonal to each other, that is, the integral of the product between any two of these functions within the interval of a symbol is null.

This orthogonality ensures that the inter-symbolic interference in the frequencies of the subcarriers be null. Figure 1 illustrates the effect of the orthogonality. On the other hand, the orthogonality also allows for the narrowest possible band occupied by the OFDM signal, which, in turn, makes the signal fit into a 6 MHz pass-band channel.

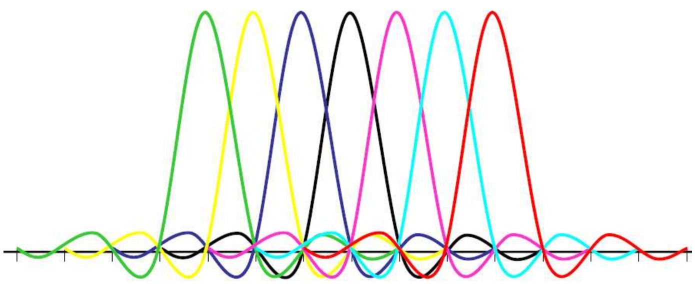

Fig. 1. Orthogonality between the carriers of the OFDM system.

In Figure 1, the frequency of the main subcarrier (f_c) equals the inverse of the duration of the symbol ($\frac{1}{T_S}$). In the 8K mode, f_c is 837 054 Hz and, in the 2K mode, it is equal to

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3,348.214 Hz. The values calculated for f_c result from the need to maintain the orthogonality between the subcarriers.

Each subcarrier is modulated in QPSK, 16-QAM or 64-QAM by one of the v bit sets mapped by the block mapper. For every set of v bits (2, 4, or 6 bits) there is a given state of phase/amplitude of the subcarrier. During a symbol (T_S), the states of the subcarriers remain unchanged. In the next symbol, they will acquire new states due to the new sets of v bits that are found in the input of the modulators of each subcarrier.

It is worth mentioning that the state of the subcarriers, within the transmission of a symbol, possesses the information of the frequency spectrum isolated from the OFDM signal. To convert this information to time domain, the IFFT is used. All these operations are digitally performed by means of digital processors. The obtained OFDM signal is in digital format and ready to be injected in the next block, in which the guard interval is inserted [5].

Figure 2 shows the difference between the transmission techniques with single carrier, used in the ATSC (Advanced Television Systems Committee) standard, and multicarrier, used in the DVB (Digital Video Broadcasting), ISDB (Integrated Services Digital Broadcasting) and ISDB-Tb (*Brazilian Digital Television System*) standards. The figure shows the the spectra $S(\omega)$, in which ω_k if the frequency of the k-th subcarrier (in radians per second), $\mathrm{W_{SC}}$ is the passband for the signal modulated with single carrier and $\mathrm{W_{MC}}$ is the total passband for a transmission with N carriers [6].

Defining the spectrum of each deterministic modulated carrier in OFDM by $F_k(\omega)$, the spectrum of the composed signal is given by

$$S(\omega) = \sum_{k=1}^{N} F_k(\omega). \quad (1)$$

As a consequence, the transmitted signal is written as

$$s(t) = \sum_{l=-\infty}^{\infty} \sum_{k=1}^{N} b_{kl} e^{j\omega_k (t - lT_S)} f(t - lT_S), \quad (2)$$

in which b_{kl} is the l-th information *bit* in the k-th subcarrier, $f(t)$ represents the pulse waveform for the transmitted signal, and T_S is the symbol interval. By taking discrete samples of the signal $s(t)$, the resulting equation represents the inverse discrete Fourier transform of the signal. Hence, for demodulating the OFDM signal, it suffices to obtain the direct Fourier transform of the received signal.

This is useful for a preliminary analysis, but do not represent the measurement of a real signal. First, there is no negative power, as most engineers know. Second, real signals are random, not deterministic. Therefore, it is important to deal with practical signals and power spectral densities, instead of Fourier transform of deterministic waveforms.

The power spectrum density is useful to analyze the channel transmission effects. For example, a multipath channel, which is typical in digital television transmission, leads to a selective attenuation in some range of frequencies, affecting the signal reception. Figure 3 shows the effect of the channel filtering, caused by multipath, for instance, in the transmission with single carrier and OFDM.

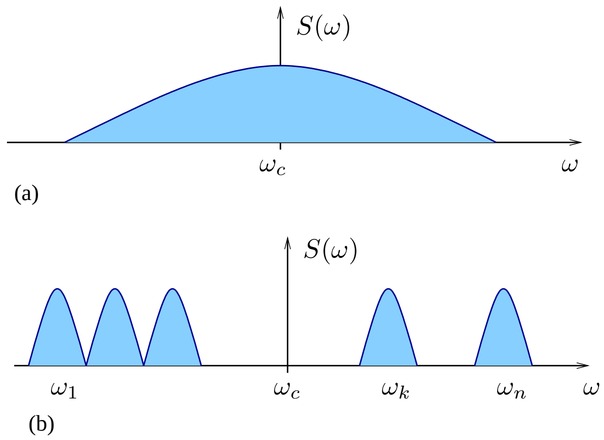

Fig. 2. Transmission techniques: (a) Single carrier and (b) OFDM.

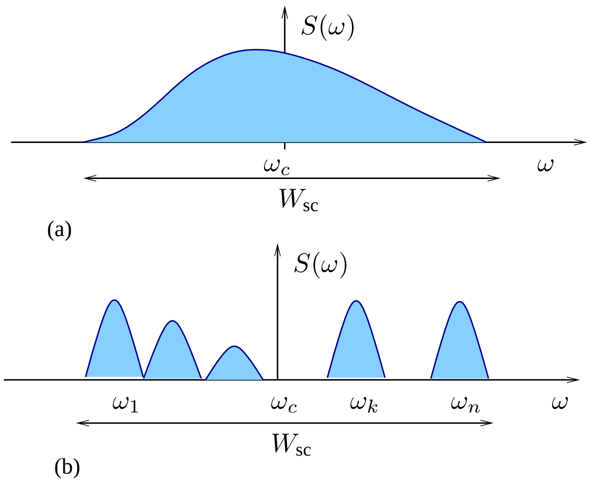

Fig. 3. Channel effect: (a) Single carrier and (b) OFDM.

As can be observed in Figure 3, the effect of the attenuation caused by the channel transfer function $H(\omega)$ is smaller for the OFDM transmission, since it affects only some subcarriers, whose information rates are slow. As a consequence, the received signal does not need equalization or it only needs simple equalization for each subcarrier. The single carrier system, by its turn, requires a complex adaptive equalization subsystem.

The OFDM signal is affected by fast time variant channels or by the frequency offset of some carrier. Additionally, the signal tends to have high peak-to-average ratio.

The OFDM technique is used in ADSL (Asymmetric Digital Subscriber Line) connections that follows the G.DMT (ITU G.922.1) standard and is also employed in wireless local area networks, including the IEEE 802.11a/g standard, or HIPERLAN/2, and the data wireless transmission WiMax system. This technique is used in Europe and in other localities in which the standards Eureka 147 Digital Audio Broadcast (DAB) and Digital Radio Mondiale (DRM) were adopted for digital radio transmission as well as for television transmission in DVB standard.

For digital television transmission, one argues that the COFDM systems, used in Japanese and European standards, have robustness to multipath higher than the 8-VSB, used in the American standard. The first 8-VSB digital television receivers had signal reception problems in urban areas, however, most recent receivers have evolved in this aspect. The 8-VSP modulation requires a smaller transmitted power, which is interesting for small cities. However, in urban areas or with geographical accidents, the COFDM provides a better reception [7].

Generally, the TV systems uses 2,000 to 8,000 carriers, which can be modulated with QPSK (Quadrature Phase Shift Keying), 16-QAM or 64-QAM. As a consequence, each carrier transports a relatively low bit rate. Besides, as each part of the signal is transported by a carrier in a different frequency, this provides certain noisy immunity with respect to interference in specific frequencies, since only part of the information is affected.

The modulated symbols in QAM or QPSK are transformed with the Inverse Fast Fourier Transform (IFFT) and positioned in the time domain in orthogonal carriers. Upon receiving the signal, the receiver must only perform the FFT in the clocks of received signal blocks to obtain the transmitted signal. Guard intervals are inserted between the bits.

III. COFDM TRANSMISSION

The terrestrial channel is different from satellite and cable channels, and is exposed to multipath effects, as a result of the surface topography and the presence of buildings. During the DVB-T development, the mobile reception was not an exigency, but the system can handle the signal variation due to the movement of transmitters/receivers.

The system that best fits the needs for such channel conditions is the COFDM. COFDM manages very well the multipath problem, and offers a high level of frequency economy, allowing a better utilization of the available bandwidth [8]. The COFDM technique is based on the utilization of several carriers which transport the signal in FDM (Frequency Division Multiplexing) sub-channels with 6, 7 or 8 MHz. In this way, each carrier transports a limited bit rate. The interference between the carriers is avoided by making them orthogonal. That occurs when the separation between the carriers is the inverse of the period over which the receiver will accomplish the signal demodulation [9]. In the DVB standard, 1 705 (in the 2K mode) or 6 817 (um the 8K mode) carriers are used. To increase the immunity to external interferences, several codification techniques are employed, which includes a pseudo-random exchange of the useful load among the carriers.

For a given configuration of the above mentioned parameters, the bits are assembled to form a word. Each word modulates a carrier, during the time of one symbol (T_S). The set of words of all the carriers in a given time interval T_S is called a COFDM symbol. Thus, the COFDM symbol defined for the DVB-T consists in any of the 6,817 carriers of the 8K mode or the 1 705 carriers of the 2K mode. In the DVB-T system, the OFDM symbols are combined to form a transmission frame, which consists in 68 consecutive symbols. Four consecutive transmission frames form a super-frame [8].

The beginning of each COFDM symbol is preceded by an interval called guard band. The aim of this interval is to guarantee the immunity to echoes and reflections, so that if, for example, an echo is in the guard band, it does not affect the decoding of useful data transmitted on the symbols. The guard band is a cyclic continuation of the symbol and its length in relation to the duration of a symbol can assume four different values: 1/4, 1/8, 1/16 or 1/32 of symbol time [10].

To achieve synchronism and phase control, pilot-signals are inserted. These pilots are transmitted with an amplitude 1/0.75 larger than the amplitude of the others carriers, with the aim of making them particularly robust to the transmission effects. The two end carriers (0 and 1 704 for the 2K mode and 0 and 6 816 for the 8K mode) play this role. Another 43 carriers in the 2K mode and 175 for the 8K mode are used as continuous pilot. For the remaining carriers, some words are used in predefined sequences to act as dispersed pilot signals, which are used to estimate the transmission carrier (and neighboring carriers) characteristics [11]. Some carriers are used to send a TPS control signal which identifies the transmission channel parameters, such as modulation type and number of carriers.

IV. OFDM WITH RANDOM SIGNALS

A more general expression for the power spectral density of the orthogonal frequency division scheme can be obtained using the theory of modulation with stochastic processes. The resulting power spectrum represents the very signal which the engineer observes when operating a spectrum analyzer.

The OFDM signal is the sum of randomly modulated signals

$$s(t) = \sum_{k=1}^{N} s_k(t), \tag{3}$$

in which the quadrature modulated signals $s_k(t)$ can be written as

$$s_k(t) = b_k(t)\cos(\omega_k t + \phi) + d_k(t)\sin(\omega_k t + \phi). \tag{4}$$

The random modulating signals $b(t)$ and $d(t)$ can be correlated or uncorrelated and the phase ϕ is a random variable uniformly distributed in a interval of 2π radians and independent of $b(t)$ and $d(t)$.

A. Quadrature Modulation with Random Signals

The quadrature modulation scheme (QUAM) uses sine and cosine orthogonality properties to allow the transmission of two different signals in the same carrier, which occupies a bandwidth that is equivalent to the AM signal. The information is transmitted by both the carrier amplitude and phase.

It is possible to write the modulated signal as

$$s_k(t) = \sqrt{b_k^2(t) + d_k^2(t)}\cos\left(\omega_k t - \tan^{-1}\left[\frac{d_k(t)}{b_k(t)}\right] + \phi\right), \tag{5}$$

in which the modulating signal, or amplitude resultant, can be expressed as

$$a_k(t) = \sqrt{b_k^2(t) + d_k^2(t)} \tag{6}$$

and the phase resultant is given by

$$\theta_k(t) = -\tan^{-1}\left[\frac{d_k(t)}{b_k(t)}\right]. \quad (7)$$

The autocorrelation function for the OFDM signal can be computed using the definition of the expected value operator $E[\cdot]$,

$$R_S(\tau) = E[s(t)\cdot s(t+\tau)]. \quad (8)$$

Substituting Equations 3 and 4 into 8, expanding the product and using the linearity property of the expected value, gives

$$\begin{aligned} R_S(\tau) = &\sum_{k=1}^{N}\sum_{i=1}^{N} E[b_k(t)b_i(t+\tau) \\ &\cdot \cos(\omega_k t+\phi)\cos(\omega_i(t+\tau)+\phi) \\ &+ d_k(t)d_i(t+\tau)\sin(\omega_k t+\phi)\sin(\omega_i(t+\tau)+\phi) \\ &+ b_k(t)d_i(t+\tau)\cos(\omega_k t+\phi)\sin(\omega_i(t+\tau)+\phi) \\ &+ b_i(t+\tau)d_k(t)\cos(\omega_i(t+\tau)+\phi)\sin(\omega_k t+\phi)]. \end{aligned} \quad (9)$$

Using trigonometric and orthogonality properties, and collecting terms which represent known autocorrelation functions, it follows that

$$\begin{aligned} R_S(\tau) = &\sum_{k=1}^{N}\left[\frac{R_{B_k}(\tau)+R_{D_k}(\tau)}{2}\right]\cos\omega_k\tau \\ &+ \sum_{k=1}^{N}\left[\frac{R_{BD_k}(\tau)-R_{DB_k}(\tau)}{2}\right]\sin\omega_k\tau. \end{aligned} \quad (10)$$

Considering zero mean uncorrelated modulating signals,

$$\begin{aligned} R_{BD_k}(\tau) &= E[b_k(t)d_k(t+\tau)] = 0 \\ R_{DB_k}(\tau) &= E[b_k(t+\tau)d_k(t)] = 0, \end{aligned} \quad (11)$$

, the resulting autocorrelation can be written as

$$R_S(\tau) = \sum_{k=1}^{N}\frac{R_{B_k}(\tau)}{2}\cos\omega_c\tau + \frac{R_{D_k}(\tau)}{2}\cos\omega_c\tau. \quad (12)$$

The carrier power is given by the following formula

$$P_S = R_S(0) = \sum_{k=1}^{N}\frac{P_{B_k}+P_{D_k}}{2}. \quad (13)$$

The power spectrum density is obtained by applying the Fourier transform to the autocorrelation function (Wiener-Khintchin theorem), which gives

$$\begin{aligned} S_S(\omega) = &\frac{1}{4}\sum_{k=1}^{N}[S_{B_k}(\omega+\omega_k)+S_{B_k}(\omega-\omega_k) \\ &+ S_{D_k}(\omega+\omega_k)+S_{D_k}(\omega-\omega_k)] \\ &+ \frac{j}{4}\sum_{k=1}^{N}[S_{BD_k}(\omega+\omega_k)-S_{BD_k}(\omega-\omega_k) \\ &+ S_{DB_k}(\omega-\omega_k)-S_{DB_k}(\omega+\omega_k)], \end{aligned} \quad (14)$$

in which $S_{B_k}(\omega)$ and $S_{D_k}(\omega)$ represent the respective power spectrum densities for $b_k(t)$ and $d_k(t)$; $S_{BD_k}(\omega)$ is the cross-spectrum density between $b_k(t)$ and $d_k(t)$; $S_{DB_k}(\omega)$ is the cross-spectrum density between $d_k(t)$ and $b_k(t)$.

For uncorrelated signals, the previous formula can be simplified to

$$\begin{aligned} S_S(\omega) = &\sum_{k=1}^{N}\frac{1}{4}[S_{B_k}(\omega+\omega_k)+S_{B_k}(\omega-\omega_k) \\ &+ S_{D_k}(\omega+\omega_k)+S_{D_k}(\omega-\omega_k)]. \end{aligned} \quad (15)$$

The quadrature amplitude modulator diagram, which can produce the QAM, as well as, the QPSK modulated signals, is shown in Figure 4.

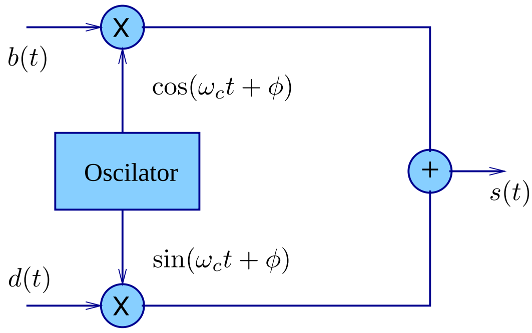

Fig. 4. Block diagram for the quadrature modulator.

B. Quadrature Modulation with a Digital Signal

Quadrature amplitude modulation using a digital signal is usually called QAM. The computation of the autocorrelation function and the power spectrum density follows the same rules previously established.

The modulating signals for the QAM scheme are

$$b(t) = \sum_{n=-\infty}^{\infty} b_n p(t-nT_b) \quad (16)$$

and

$$d(t) = \sum_{n=-\infty}^{\infty} d_n p(t-nT_b). \quad (17)$$

The modulated carrier is similar to the QUAM's

$$s(t)_{QAM} = b(t)\cos(\omega_c t+\phi) + d(t)\sin(\omega_c t+\phi). \quad (18)$$

The constellation diagram for a 4QAM signal is shown in Figure 5, in which the symbols $b_n = \{A, -A\}$ and $d_n = \{A, -A\}$. It can be shown, using the same previous methods, that the modulated signal power for this special case is given by $P_S = A^2/2$. This signal is also known as $\pi/4$-QPSK and is largely used in schemes for mobile cellular communication systems. If the constellation is rotated by $\pi/4$ it produces another QAM modulation scheme, which presents more ripple than the previous one, because one of the carriers shuts off whenever the other carrier is transmitted.

Figure 6 shows the constellation diagram for a 16QAM signal, whose points have coordinates $b_n = \{-3A, -A, A, 3A\}$,

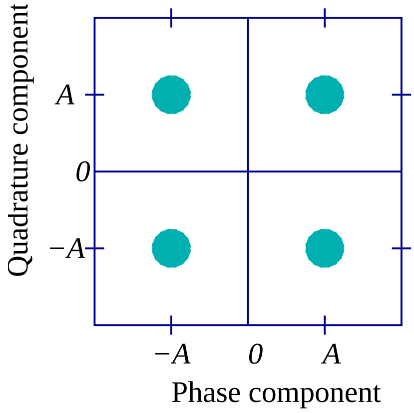

Fig. 5. Constellation diagram for the 4QAM signal.

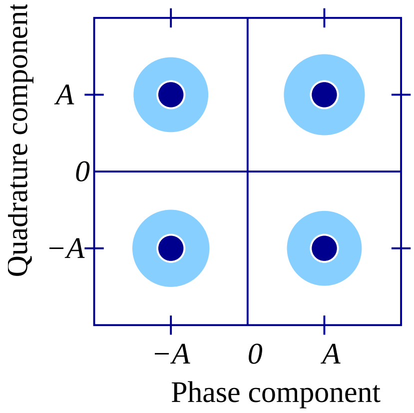

Fig. 7. Constellation diagram for a 4QAM signal with additive noise.

for the abscissa, and $d_n = \{-3A, -A, A, 3A\}$, for the ordinate.

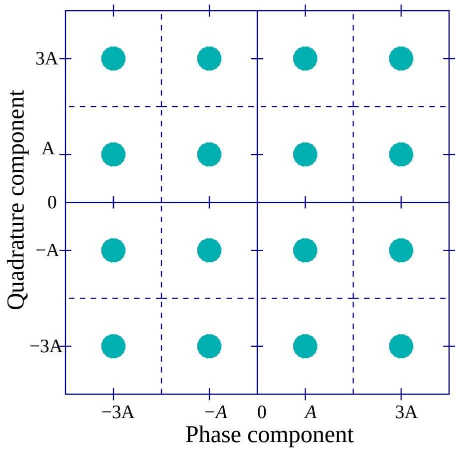

Fig. 6. Constellation diagram for the 16QAM signal.

The efficient occupation of the signal space renders the QAM technique more efficient than ASK and PSK, in terms of bit error probability versus transmission rate. On the other hand, the scheme is vulnerable to non-linear distortion, which occurs in power amplifiers on board satellite systems. It is evident that other problems can occur and degrade the transmitted signal. It is important that the communication engineer could identify them, in order to establish the best combat strategy.

Figure 7 shows a constellation diagram in which the symbols are contaminated by additive Gaussian noise [12]. The circles illustrate, ideally, the uncertainty region surrounding the signal symbols.

An interesting analogy can be made between the noise effect and the heating process. Heating is the result of an increase in the kinetic energy of the system. If the points on the constellation diagram are heated, they acquire kinetic energy and move from their original position. Because the movement is rapid and random is it more difficult to perfectly identify their positions.

The symbol error probability is, usually, controlled by the smaller distance between the constellation symbols. The formulas for the error probability produce curves that decrease exponentially, or quasi-exponentially, with the signal to noise ratio.

The probability of error for the 4QAM signal is [13]

$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E_b}{N_0}}\right), \quad (19)$$

which can be simplified to

$$P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right), \quad (20)$$

in which E_b is the pulse energy and N_0 represents the noise power spectral density and $\operatorname{erfc}(\cdot)$ is the complementary error function,

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt. \quad (21)$$

For M symbols, the error probability is given by

$$P_e = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right)\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right]^2. \quad (22)$$

For most purposes, the formula can be approximated by [13].

$$P_e \approx 2\left(1 - \frac{1}{\sqrt{M}}\right)\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right). \quad (23)$$

C. The Baseband Digital Signal

This section presents a mathematical formulation for the digital signal, including the computation of the autocorrelation function and the power spectrum density.

The digital signal, which can be produced by the digitization of the speech signal, or directly generated by a computer hooked to the Internet, or other equipment, can be mathematically expressed as

$$m(t) = \sum_{k=-\infty}^{\infty} m_k p(t - kT_b), \quad (24)$$

where m_k represents the k-th randomly generated symbol from the discrete alphabet, $p(t)$ is the pulse function that shapes the transmitted signal and T_b is the bit interval.

Autocorrelation for the Digital Signal

The autocorrelation function for signal $m(t)$, which can be non-stationary, is given by the formula

$$R_M(\tau, t) = E[m(t)m(t+\tau)]. \quad (25)$$

Substituting $m(t)$ into Formula 25,

$$R_M(\tau, t) = E\left[\sum_{k=-\infty}^{\infty}\sum_{\infty}^{i=\infty} m_k p(t-kT_b) m_j p(t+\tau-iT_b)\right]. \quad (26)$$

The expected value operator applies directly to the random signals, because of the linearity property, giving

$$R_M(\tau, t) = \sum_{k=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} E\left[m_k m_j\right] p(t-kT_b)p(t+\tau-iT_b). \quad (27)$$

In order to eliminate the time dependency, the time average is taken in Equation 27, producing

$$R_M(\tau) = \frac{1}{T_b}\int_0^{T_b} R_M(\tau, t)dt, \quad (28)$$

or, equivalently

$$R_M(\tau) = \frac{1}{T_b}\int_0^{T_b}\sum_{k=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} E\left[m_k m_j\right] \cdot p(t-kT_b)p(t+\tau-iT_b)dt. \quad (29)$$

Changing the integral and summation operations, it follows that

$$R_M(\tau) = \frac{1}{T_b}\sum_{k=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} E\left[m_k m_j\right] \cdot \int_0^{T_b} p(t-kT_b)p(t+\tau-iT_b)dt. \quad (30)$$

Defining the discrete autocorrelation as

$$R(k-i) = E\left[m_k m_j\right], \quad (31)$$

the signal autocorrelation can be written as

$$R_M(\tau) = \frac{1}{T_b}\sum_{k=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} R(k-i) \cdot \int_0^{T_b} p(t-kT_b)p(t+\tau-iT_b)dt. \quad (32)$$

For a rectangular pulse, with independent and equiprobable symbols, the autocorrelation function is given by

$$R_M(\tau) = A^2\left[1-\frac{|\tau|}{T_b}\right][u(\tau+T_b)-u(\tau-T_b)], \quad (33)$$

where T_b is the bit interval, A represents the pulse amplitude.
Figure 8 shows that this function has a triangular shape. Its maximum occurs at the origin (signal power) and is equal to A^2. The autocorrelation decreases linearly with the time interval and reaches zero at time T_b.

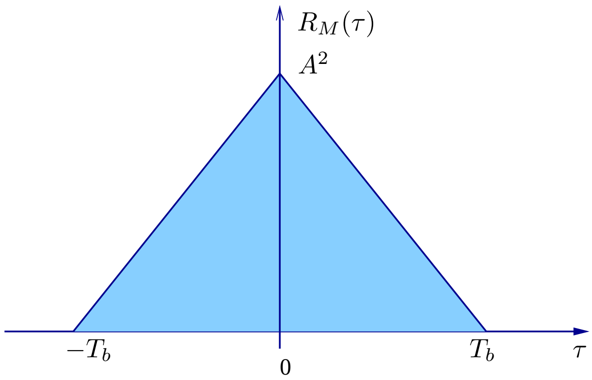

Fig. 8. Autocorrelation for the digital signal.

Power Spectrum Density for the Digital Signal

The power spectrum density for the digital signal can be obtained by derivating $R_M(\tau)$ twice and taking the Fourier transform of the result. Therefore

$$\frac{d^2R_M(\tau)}{d\tau^2} = \frac{A^2}{T_b}\delta(\tau+T_b) + \frac{A^2}{T_b}\delta(\tau-T_b) - \frac{2A^2}{T_b}\delta(\tau). \quad (34)$$

and the corresponding Fourier transform can be written as

$$(j\omega)^2 S_M(\omega) = \frac{A^2}{T_b}e^{j\omega T_b} + \frac{A^2}{T_b}e^{-j\omega T_b} - \frac{2A^2}{T_b}. \quad (35)$$

Using Euler trigonometric relations, and after a few algebraic manipulations, one can find

$$S_M(\omega) = A^2 T_b\left[\frac{\sin\left(\frac{\omega T_b}{2}\right)}{\frac{\omega T_b}{2}}\right]^2. \quad (36)$$

The graphic of $S_M(\omega)$ is sketched in Figure 9. The first null is a usual measure of the bandwidth, and is given by $\omega_M = 2\pi/T_b$.

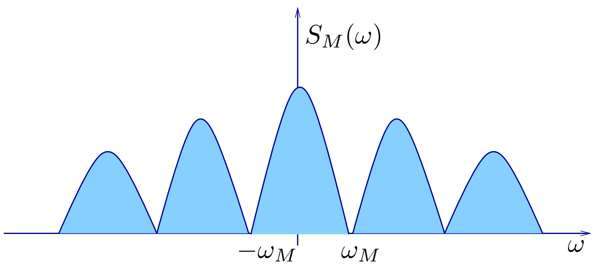

Fig. 9. Power spectrum density for the random digital signal.

V. CONCLUSIONS

An stochastic analysis of the Orthogonal Frequency Division Multiplexing (OFDM) scheme, which is the choice of most digital television, digital radio and wireless standards, was provided. The sub-carriers of an OFDM system form a set of functions that are orthogonal to each other, and this was used to simplify the analysis.

The stochastic analysis provided general expressions for the autocorrelation and power spectral density functions. The resulting power spectral density can be used to solve practical problems with the aid of spectrum analyzers.

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