Economic Analysis on Passive Optical Networks Using Markov Chain and Monte Carlo Simulation

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Abstract—**Passive Optical Networks (PON) supports actual and near future traffic demand expected for fixed broadband access networks. The increase of the traffic demand requirements have led costumers for demanding more reliable services forcing operators to invest more in infrastructure, e.g., operators are investing to reduce extra costs caused by undesired equipment malfunction, i.e., repair costs and disconnection time penalties. This paper aims to verify the economical feasibility of different protection schemes for PONs through CAPEX and OPEX analysis. A geometric model is used to describe PON deployment area and a Markov cost model solved with Monte Carlo simulation is used to compute the failure-associated costs in order to find the best protection scheme in terms of implementation cost savings and investments return. Results point PON architecture having protection as the most attractive alternative in terms of reliability and cost savings.**

Keywords — PON, protection, Markov, Monte Carlo, CAPEX, OPEX.

I. Introduction

The Passive Optical Networks (PON) technologies are able to provide and handle high traffic demand expected for today and near future. Following the increase in terms of traffic requirements the operators are starting to deal with new customers, e.g., business/commercial, who are becoming even more exigent and costly in terms of penalties. The new customer profile has driven operators to invest in reliable solutions, e.g., protection topologies for Feeder Fiber (FF) and Distribution Fiber (DF), aiming to strict respect the service level agreement (SLA) in order to reduce failures and undesired revenue losses.

The reduction in revenues is related to non-planned investment on both capital expenditures (CAPEX), e.g., in terms of infrastructure through replacement of died equipment, and operational expenditures (OPEX), e.g., in terms of penalty fee associated with the time that a customer is turned-off due to network/equipment malfunction.

In the literature, paper [1] presents and evaluates by a comprehensive assessment of CAPEX and OPEX a cost-efficient protection for TDM PONs based on sharing FF ducts between Optical Line Terminal (OLT) and Remote Node (RN) with backup fibers. Results confirm the benefit of the proposed way to deploy protection, which causes significant reduction of Total Cost of Ownership (TCO), i.e., CAPEX and OPEX, compared to the unprotected case in all of the considered scenarios (rural, urban, and dense urban). Paper [2] presents a comprehensive cost analysis for fiber access networks including both CAPEX and OPEX. Results show that for business users the TCO in protection topologies may be lower than in some unprotected topologies. On the other hand, papers [1] and [2] did not take into account geometric models to represent the scenarios. Paper [3] assessed OPEX for PONs in terms of both expected repair cost and expected penalty cost using Markov cost model based on a geometric model describing the area under study. Results show that the expected penalty cost accounts for the major part of these OPEX, growing remarkably in sparse scenarios and when business clients are considered. However, it should be noticed that during the investigation this paper did not considered CAPEX impact, which cannot be neglected [4].

In this paper we propose a set of math models based on Markov Chain Monte Carlo in order to assess the investment cost necessary to reduce CAPEX and OPEX. We compare three different PON topologies: No protection, protection in the Feeder Fiber (FF), and protection in the FF and Distribution Fiber (DF), illustrated respectively in Fig. 1 (a), (b) and (c). We simulate an urban scenario using Markov chain to represent the equipment failure rate and Monte Carlo simulation to repeat the event N times. Accordingly to our knowledge this is the first effort to assess the TCO through simulation in order to provide the most cost effective PON topology respecting network reliability.

The paper is organized as follows. In Section II, we present the methodology used to compute the CAPEX and OPEX for PONs. In Section III, we introduce the math models adopted in the study. In Section IV, we present a case study with all the assumptions used to compute the costs. Section V summarizes the main findings from this paper by presenting the CAPEX and OPEX for all PONs topologies. Finally, Section VI concludes the paper.

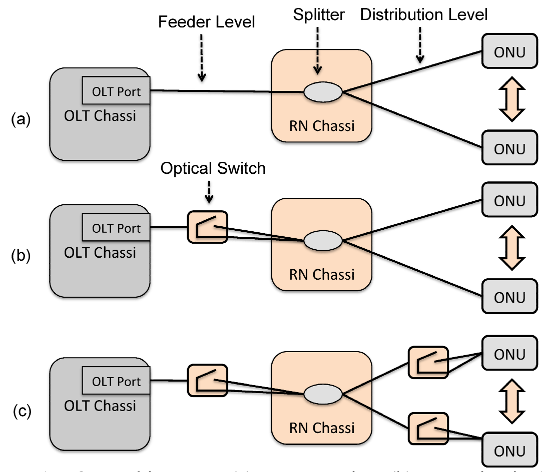

Figure 1: PON architectures. (a) No Protection, (b) Protection in the FF, (c) Protection in the FF and DF.

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II. METHODOLOGY

This article aims to evaluate the economic feasibility of three different PON topologies at a dense urban scenario through their total cost of ownership (TCO) analysis. The methodology is divided into three steps. The first one is the *Geometric Model,* in which we define the investigated scenario and its network topology. In this step we follow the Manhattan Model that is an analytical model widely used to compute fiber length [5]. We assume that all streets are connected using one divider street, i.e., an orthogonal cross-point connecting two streets [5], and that topology is composed by the number of subscribers, represented by the number of optical network units (ONU), and the distance between two adjacent subscribers. More details regarding this step are provided in Section III.A.

The second step is defining the *Failure Associated Costs* using a finite-state continuous-time Markov chain and Monte Carlo Simulation (MCMC). In the Markov chain, each state definition is in function of the type and number of failed equipment, their distance to the CO and the number of affected subscribers [3]. Additionally, the failure rates of a state are given by the equipment failure rate; and the repair rate of a state is calculated as the inverse of the total sum of the time to travel up to the equipment location and mean time to repair. In order to use MCMC we adapted the model presented in [6] to simulate the topology in a period *T*. More details regarding this step are provided in Section III.B.

The last step is obtaining the *Total Cost of Ownership*, calculated as a result of the total sum of *capital expenditures* (CAPEX) and *operational expenditures* (OPEX). More details regarding this step are provided in Section III.C.

III. MATH MODELS

A. Geometric Model

The objective of this section is to describe how the associated clients and the distance from the equipment to the Central Office (CO) are determined. The *Geometric Model* is based on Manhattan Model [5] and assumes a uniform distribution of subscribers over a regular grid and follows the PON architecture with two levels, i.e., feeder and distribution levels. The first is Feeder Level which has the CO at the center, N is the number of blocks in a row and L is the distance between adjacent blocks. Following the same logic, the second level is a Distribution Level, where a Remote Node (RN) is located at the center, n is the number of ONUs in a row of a block and l is the distance between adjacent residences, i.e., $L = n \times l$. It can be observed in Figure 2.

The protection scheme at the Feeder Level requires the addition of $(N-1) * N$ fiber trenchs. In case of a distribution level protection scheme this addition is as follows $[(n-1) \times n \times N2$. For cost savings, operators deploy fibers using the existing ducts where is feasible [5].

In this model each subscriber is associated with an ONU, and each block is associated with a RN chassis, i.e., each block has n^2 subscribers. Additionally, the number of subscribers served by splitters depend on the RN and the splitting ratio (SR), and the number of splitters in one block is given by $\left\lceil \frac{n^2}{SR} \right\rceil$.

Each OLT chassis is associated to a maximum number of OLT ports and each one of that is connected to a splitter. So, the subscribers served by an OLT Chassis are the ones served by the OLT ports and splitter related to that. The subscribers served by a feeder fiber trench are determined by the blocks connected to it and in the same way the subscribers served by a distribution trench are determined by the associated ONUs.The equipment locations have two coordinates, i.e., a vertical (P_v) and horizontal (P_h) at the levels. Those positions vary with the type of equipment. Moreover, through the coordinates is possible to extract the distance from the equipment to the Central Office.

The distance from a block to the scenario Center ($D_{q \to CC}$) is defined by equation (1) using parameters mentioned before.

$$D_{q \to CC} = [N - (A+B) - 1]L,$$
$$\text{where: } A = \begin{cases} P_h, if P_h < \frac{N}{2} \\ N - P_h - 1, if P_h \geq \frac{N}{2} \end{cases} \quad B = \begin{cases} P_v, if P_v < \frac{N}{2} \\ N - P_v - 1, \quad if P_v \geq \frac{N}{2} \end{cases} \tag{1}$$

The distance from a Feeder Fiber Step to the center of scenario ($D_{FFS \to CC}$) is calculated through equation (2).

$$D_{FFS \to CC} = \begin{cases} \left[N - (A+B) - \frac{3}{2}\right] L, if P_h \neq \frac{N}{2} \\ \left(\frac{N - 2B - 1}{2}\right) L, if P_h = \frac{N}{2} \end{cases},$$
$$\text{where: } A = \begin{cases} P_h, if P_h < \frac{N}{2} \\ N - P_h, if P_h > \frac{N}{2} \end{cases} \quad B = \begin{cases} P_v, if P_v < \frac{N}{2} \\ N - P_v - 1, if P_v \geq \frac{N}{2} \end{cases} \tag{2}$$

The distance from a protection feeder fiber step to the Center of scenario ($D_{PFFS \to CC}$) is given by equation (3).

$$D_{FFS \to CC} = \begin{cases} \left[N - (A+B) - \frac{3}{2}\right] L, if P_h \neq \frac{N}{2} \\ \left(\frac{N - 2B - 1}{2}\right) L, if P_h = \frac{N}{2} \end{cases},$$
$$\text{where: } A = \begin{cases} P_h, if P_h < \frac{N}{2} \\ N - P_h, if P_h > \frac{N}{2} \end{cases} \quad B = \begin{cases} P_v, if P_v < \frac{N}{2} \\ N - P_v - 1, if P_v \geq \frac{N}{2} \end{cases} \tag{3}$$

Through equations (1), (2) and (3), we obtain the distance of all equipments to the center of the scenario. To ONUs, Distribution Fiber Steps and Protection Distribution Fiber Steps the distance to CO is the result of the sum of equations (1) with adapted (1), (2) and (3) equations, respectively. For the adaptations, N is replaced by n, as the number of

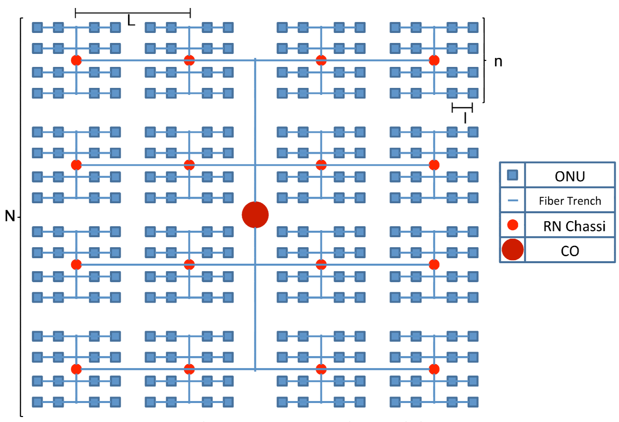

Figure 2: Geometric Model

TABLE I - Scenario Parameters[2][3][7][8]

Parameters	Value
N	10
n	10
l (km)	1/24
SR	01:32
$N_{OLT/C}$	72
Business Users Penalty(US$/h)	650
Residential Users Penalty(US$/h)	10
Crew Salary(US$/h)	190

residences in a row of the block; L is replaced by l, for the distance between two adjacent residences. P_h and P_v represent the equipment positions in the block. Due to their location at the center of the block we use the equation (1) to obtain the distance of splitters and RN chassis to the center. In case of OLT ports and OLT chassis the distance to the center is null.

B. Markov Model and Monte Carlo Model

In this section we present the models to compute the failure-associated costs in a determined period *T*. That cost is obtained adapting the methodology presented in [6] of a continuous-time Markov chain and Monte Carlo Simulation.

The states definitions are given by the number and type of failed equipment, the distance from the equipments to the CO and the number of subscribers affected by the failure.

The cost models are included by the Markov reward model, where each state has an associated reward. In this case, the related costs of failed equipment repair (C_i^{repair}) and penalty costs ($C_i^{penalty}$) are given by the equations (4) and (5).

$$C_i^{penalty} = NB_{sub}^{fail} \times \Pr_{\text{bus}} + NR_{sub}^{fail} \times \Pr_{\text{res}} \tag{4}$$

$$C_i^{Repair} = Sal + \sum C_{ij}\lambda_{ij} \tag{5}$$

Where NB_{sub}^{fail} and $\Pr_{\text{bus}}$ are respectively the number of business subscribers affected by failure and business penalty rate agreed in SLA. Furthermore, NR_{sub}^{fail} and $\Pr_{\text{res}}$ are the number of residential subscribers affected by failures and the residential penalty rates agreed in SLA, respectively.

The parameter *Sal* is the repair crew's salary, C_{ij} is the repair cost of failed equipment in transition from state *i* to state *j*, and finally, λ_{ij} is the transition rate from state *i* to state *j*.

The repair rate of a state is calculated as the inverse of the sum of travel time to the equipment location and mean time to repair. If there is more than one failed equipment, the one that saves more penalty costs in less time is repaired first.

For our period analysis, we adapted [6] making a Monte Carlo trial be concluded just as the state transition number gets the maximum number of transitions in a time interval ($Num_{máx}^{trans}$). That is possible because the equipments have a low failure rate and a high repair rate, having low probability of simultaneous failures in network. So it is expected the repair occurs just after the failure, so that $Num_{máx}^{trans}$ can be determined through equation (6).

$$Num_{\max}^{trans} = 2 \times \frac{T_d}{\frac{10^9}{\sum Eq_{FIT}}} \tag{6}$$

Where T_d is the desired time interval, in hours, $\sum Eq_{FIT}$ is the sum of all equipments failure rates and $\frac{10^9}{\sum Eq_{FIT}}$ represents the mean time between two failures.

C. Total Cost of Ownership (TCO)

In this section we present the models to compute the capital expenditures (CAPEX) and the operational expenditures (OPEX) costs. Through those models we obtain the Total Cost of Ownership (TCO) for the fixed broadband operators.

There is no standard of what costs are included in CAPEX and OPEX but it is widely assumed that CAPEX is composed by the infrastructure costs, e.g., components prices and installations costs; and OPEX as operational costs, e.g., failure reparation, failure penalties, service maintenance, among others.

For CAPEX analysis it is taking into account the total cost of equipment presented in equation (7).

$$C_{Eq}^{Tot} = \sum_{k=0}^{5} N_k^{Eq} \times Pr_k^{Eq} \tag{7}$$

Where *k* classifies the type of equipment varying from 0 to 5, being 0 an ONU, 1 a splitter, 2 a Remote Node Chassis, 3 OLT Port, 4 OLT Chassis and 5 a Switch. *N* is the quantity of one type of equipment and *Pr* is the price of it; Additionally, for the installation cost it is considered the time to install ($T_{inst}[i]$), the travel time to the equipment ($\frac{dis[i]}{vel}$), and the crew salary (Sal) as shown in the equation (8).

$$C_{inst}^{Eq} = \sum_{i=0}^{NumEq-1} \left(\left(T_{inst}[i] + \frac{dis[i]}{vel} \right) \times Sal \right) \times pair_{fiber} \tag{8}$$

Where, $Pair_{fiber}$ is considered 0 if *i* is a fiber step, and 1 if it is not.

In order to calculate the fiber costs it is considered two equations. One to obtain the fiber cost (C_{inst}^{Fiber}), and another to calculate the deployment cost (C_{trench}). In order to get the fiber cost, it is necessary to calculate the distance of each fiber step and multiply for its price (Pr_{fiber}^{inst}) like in equation (9).

$$C_{inst}^{Fiber} = \left[\begin{array}{l} 4 \times N^2 \times \left(\sum_{i=0}^{\frac{n}{2}-1} \sum_{j=0}^{\frac{n}{2}-1} \frac{n-2 \times i-1}{2} + \frac{n-2 \times j-1}{2} \right) + \\ 4 \times \left(\left(\sum_{i=0}^{\frac{N}{2}-1} \sum_{j=0}^{\frac{N}{2}-1} \frac{N-2 \times j-1}{2} + \frac{N-2 \times j-1}{2} \right) \right) \end{array} \right] \times Pr_{fiber}^{inst} \tag{9}$$

The deployment cost is calculated by the trenching price (Pr_{trench}) for the total length of fiber, as equation (10).

$$C_{trench} = [(n^2-1) \times l \times N^2 + (N^2-1) \times L] \times Pr_{trench} \tag{10}$$

The OPEX analysis includes a Failure Cost equation (11), which is the product of the sum of penalty and repair costs with the expected time at the state i (Z_i), derived from Monte Carlo method mentioned before, and an energy consumption cost given by equation (12).

$$C_{failure} = \sum_{i=\Omega}[C_i^{penalty} + C_i^{repair}]Z_i \quad (11)$$

$$C_{energy}^{tot} = \left[\sum_i (n_i^{eq} \times P_i^{eq})\right] \times Pr_{energy} \times t\,, \quad (12)$$

$$i \rightarrow Non-passive equipments$$

IV. CASE STUDY

This section presents a case study where the proposed methodology is applied. In this case study, we calculate the number of equipment in the scenario, number of both equipment and link failures over a period of 20 years. Finally we present the overall cost to deploy and operate different Passive Optical Networks (PONs).

We assume a city with 10,000 residences based on Manhattan Model with 1/24km distance between two residences. The study consists in a comparison of three types of PON architectures. The first without protection, the second is protected at the feeder fiber level, and the third is protected at the feeder and distribution levels.

The parameters used in equations are presented in Table I and the costs used to calculate CAPEX and OPEX are presented in Table II. Additionally, we assume C_{ij} as 30% of the equipment value, and the failure rates of a state were obtained through the equipment failure rates extracted from [2, 7].

V. RESULTS

The results of the simulations performed over the case study are presented in this section. Results present the CAPEX and OPEX investments to operate a PON for the next 20 years. For CAPEX we present the investment costs of infrastructure, installation and equipment acquisition, whereas for OPEX we present the investment costs of reparation, penalty and energy consumption. Moreover, we present a sensitive analysis varying the cost of the most expensive elements in the topology, e.g., trenching and business penalty.

A. Capex and Opex

Figure 3 shows the CAPEX for the PON considering the investment for the network operation during 20 years of network lifetime. It is evident that the PON topology of both having *No Protection* and having *Protection in the FF* are about 2 times cheaper than a fully protected topology. The main reason for such CAPEX saving is that the fully protected topology requests extra trenching, which is the predominant expense in terms of capital investments, i.e., FF covers long distances to connect only the CO and RN, while DF connects the splitter with 32 ONUs representing a higher proportion in terms of investments. Moreover, we observe that the topology based on *Protection in the FF* does not affect the CAPEX in a significant manner compared with *No Protection* topology.

Figure 4 shows the OPEX investment to operate the PON and DF over 20 years. We observe that *Protection in the FF* and *Protection in the FF and DF* save almost the same amount in OPEX, i.e., around 995US$/User/Year, compared with *No Protection*. However, the CAPEX investment to deploy the *Protection in the FF and DF* does not pay off. Comparing with the topology with *No Protection* the investment in protection reduces the penalty cost, which decreases the OPEX about 17%. Regarding all OPEX metrics the energy consumption highlights as the most expensive compared with reparation and penalty costs. Energy consumption represents about 95US$ in all topologies. The topology based on *Protection in the FF* highlights as the most reasonable option to guarantee reliability for the users and to reduce extra expenses.

B. Sensitivity Analysis

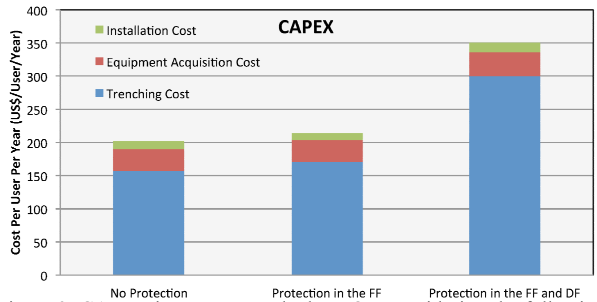

Figure 3: CAPEX investment to deploy PON considering the following topologies: No Protection, Protection in the FF, and Protection in the FF and DF.

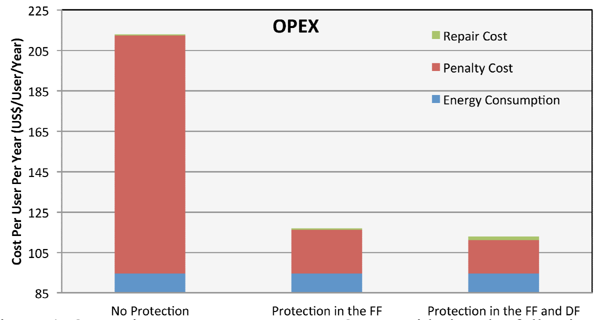

Figure 4: OPEX investments to operate PON considering the following topologies: No Protection, Protection in the FF, and Protection in the FF and DF.

TABLE II - Parameters Used to Calculate CAPEX and OPEX[2][3][7][8]

	Cost (US$)	Installation Time (min)	Failure Rate (FIT)	Mean Time to Repair(h)	Energy Consumption(W)
ONU	350	60	256	1	5
Splitter	50	10	120	1	0
RN Chassi	700	10	666	1	0
OLT port	7600	10	256	1	1197
OLT Chassi	4500	30	500	1	0
Optical Switch	50	10	200	2	0
Fiber	160/km				
Trench	68500/km		570/km	7	0

This subsection reveals the impact caused in the overall conclusions by the variation of the most costly elements in terms of CAPEX and OPEX. For CAPEX we vary the trenching cost in a range of 50%, i.e., 7000US$ [8] up to 130.000US$ [9], whereas for OPEX we also vary the Business penalty fee in a range of 50%, i.e., 100US$ [3] up to 1200US$ [2].

Figure 5 presents the additional investment in trenching to upgrade a No Protection topology towards Protection in the FF topology. The results are expressed comparing extra CAPEX. We observe that the extra investment in protection is returned through OPEX reduction along the years. Moreover, for the trenching investment from 7000US$/Km up to 84000US$/Km and business penalty cost of 100US$/hour the return is 100%, and considering the trenching cost equals to 130000US$/Km the ratio between additional CAPEX investment versus OPEX savings represent an extra investment of 7US$/User/Year to guarantee protection and to improve the network reliability. In the case of Protection in the FF and DF the savings in OPEX does not compensate CAPEX investments, e.g., considering trenching equals to 7000US$/Km the additional CAPEX investment is about 22US$/User/Year and the OPEX savings is about 15US$/User/Year.

Figure 6 shows the business penalty cost variation versus the OPEX for *No Protection, Protection in the FF, and Protection in the FF and DF* topologies. The business penalty cost is varied in a range between 100US$/hour up to 1200US$/hour. From Figure 6, it becomes evident that *No Protection* topology is not reliable, which results in significant profit losses, e.g., business penalty fee of 1200US$User/Year increases operational costs in up to 86% the operational costs. Moreover, we observe that protection dramatically reduces OPEX revenues compared to the *No Protection* topology.

From this sensitivity analysis it becomes evident that *Protection in the FF* gives the best ratio between CAPEX and OPEX. Moreover, we observe that for any value of business penalty cost, which is represented by x in Figure 6, the P_FF is always the most attractive, e.g., for x equals to 5000US$/hour the cost per user per year for NP, P_FF and P_FF&DF are respectively US$901.41, US$160.76 and US$121.71 per user per year.

VI. CONCLUSION

This paper presents a comprehensive methodology for simulate and compute the capital expenditures (CAPEX) and operational expenditures (OPEX) of passive optical Networks (PON) as well as for investigating the business viability to invest in reliable PON topologies through the usage of protection topologies. The paper focuses on three PON topologies: *No Protection*, *Protection in the Feeder Fiber*, and *Protection in the Feeder Fiber and Distribution Fiber*. Moreover, the math models are used in a case study to compute the most attractive PON option respecting cost and reliability. Results show that *Protection in the Feeder Fiber* is the best option to be deployed considering capital and operational expenditures. Moreover, results also show that trenching and business penalty costs are the highest expenses for PONs.

The sensitivity analysis show that depending on the trenching cost, i.e., lower or equal to than 84000US$/Km, the investment in feeder fiber protection can be 100% recovered through OPEX over 20 years. Additionally, results show that the business penalty cost makes *No Protection* topology uneconomical for Cities having strict regulation and high penalty cost.

For future work we are willing to apply the math models for different fiber-based topologies, i.e., fiber to the building and fiber to the cabinet. Moreover, we want to add copper-based technologies in the last mile aiming to verify the impact on CAPEX and OPEX.

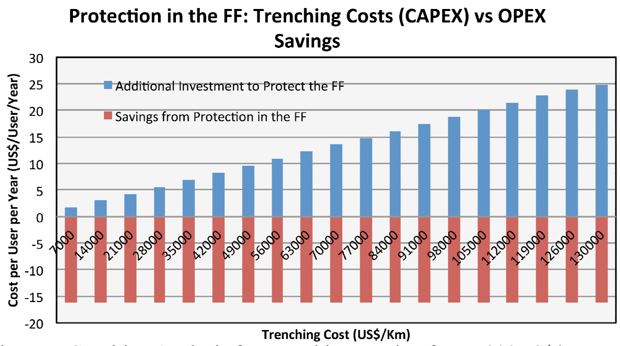

Figure 5: Sensitive Analysis for Trenching varying from 7000US$/Km up to 130.000US$/Km.

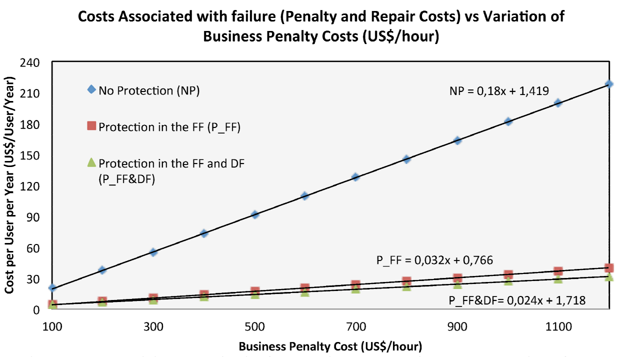

Figure 6: Sensitive Analysis for Business Penalty Cost varying from 200US$/hour up to 1200US$/hour.

VII. ACKNOWLEDGMENT

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